# AVL Trees

# Objectives

- To know what an AVL tree is (§20.1).
- To understand how to rebalance a tree using the LL rotation, LR rotation, RR rotation, and RL rotation (§20.2).
- To know how to design the AVLTree class (§20.3).
- To insert elements into an AVL tree (§20.4).
- To implement node rebalancing (§20.5).
- To delete elements from an AVL tree (§20.6).
- To implement the AVLTree class (§20.7).
- To test the AVLTree class (§20.8).
- To analyze the complexity of search, insert, and delete operations in AVL trees (§20.9).

# Why AVL Tree?

The search, insertion, and deletion time for a binary tree is dependent on the height of the tree. In the worst case, the height is O(n). If a tree is *perfectly balanced*, i.e., a complete binary tree, its height is . Can we maintain a perfectly balanced tree? Yes. But it will be costly to do so. The compromise is to maintain a well-balanced tree, i.e., the heights of two subtrees for every node are about the same.

# What is an AVL Tree?

AVL trees are well-balanced. AVL trees were invented by two Russian computer scientists G. M. Adelson-Velsky and E. M. Landis in 1962. In an AVL tree, the difference between the heights of two subtrees for every node is 0 or 1. It can be shown that the maximum height of an AVL tree is O(logn).

# Balance Factor/Left-Heavy/Right-Heavy

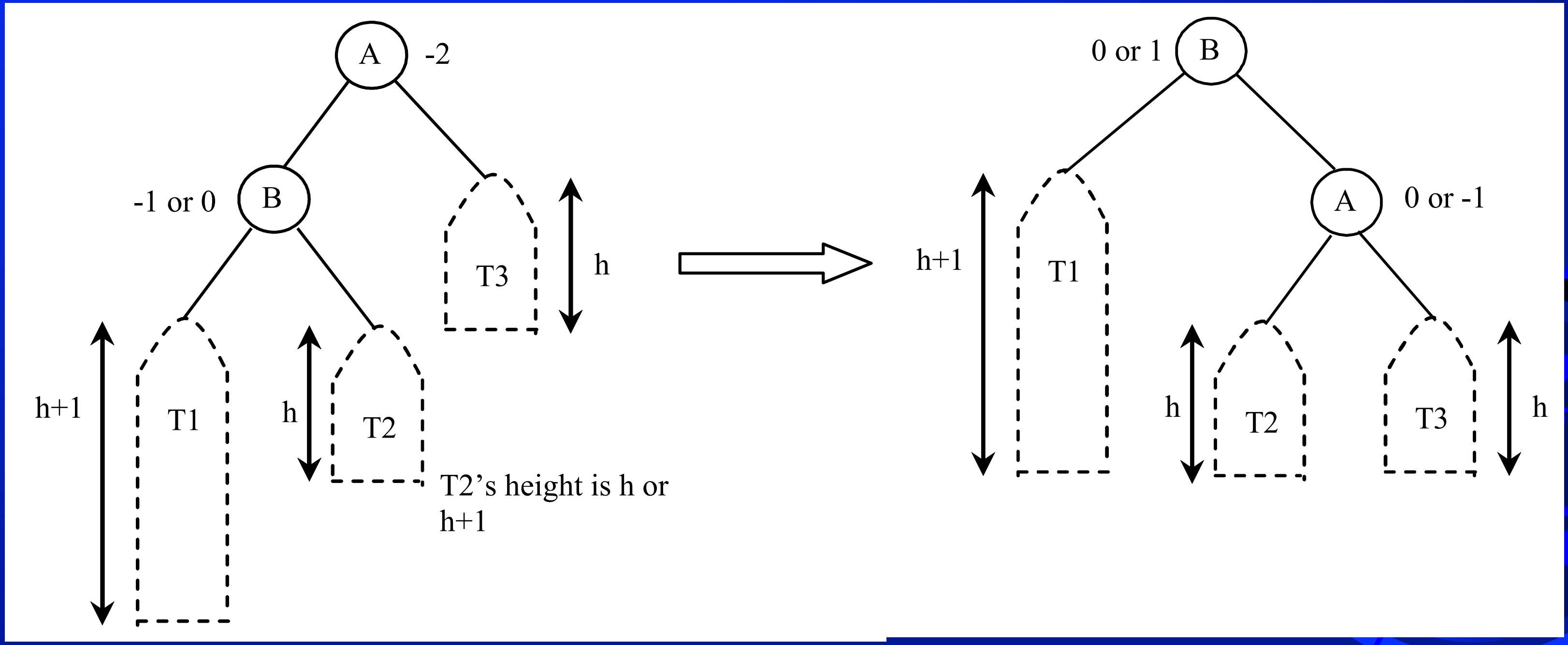
The process for inserting or deleting an element in an AVL tree is the same as in a regular binary search tree. The difference is that you may have to rebalance the tree after an insertion or deletion operation. The *balance factor* of a node is the height of its right subtree minus the height of its left subtree. A node is said to be *balanced* if its balance factor is -1, 0, or 1. A node is said to be *left-heavy* if its balance factor is -1. A node is said to be *right-heavy* if its balance factor is +1.

# Balancing Trees

If a node is not balanced after an insertion or deletion operation, you need to rebalance it. The process of rebalancing a node is called a *rotation*. There are four possible rotations.

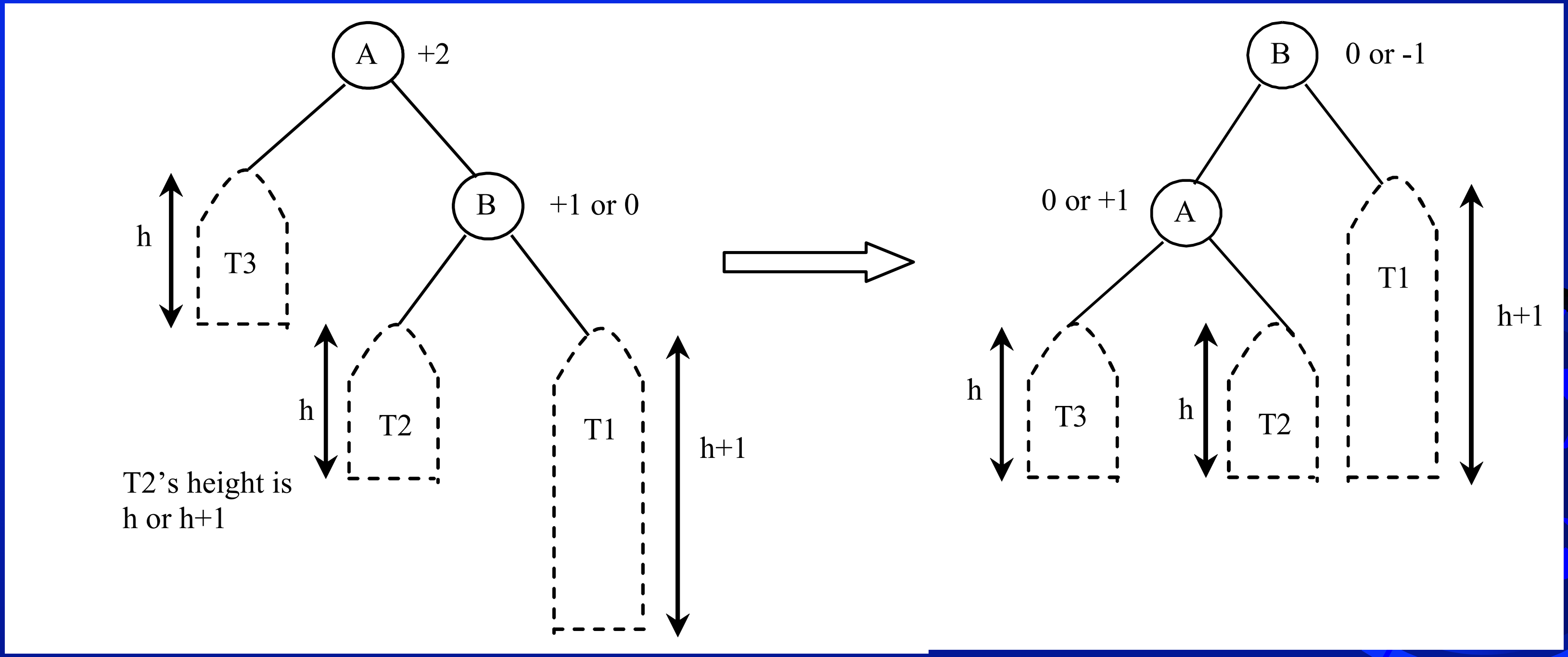
# LL imbalance and LL rotation

**LL Rotation**: An *LL imbalance* occurs at a node A such that A has a balance factor -2 and a left child B with a balance factor -1 or 0. This type of imbalance can be fixed by performing a single right rotation at A.



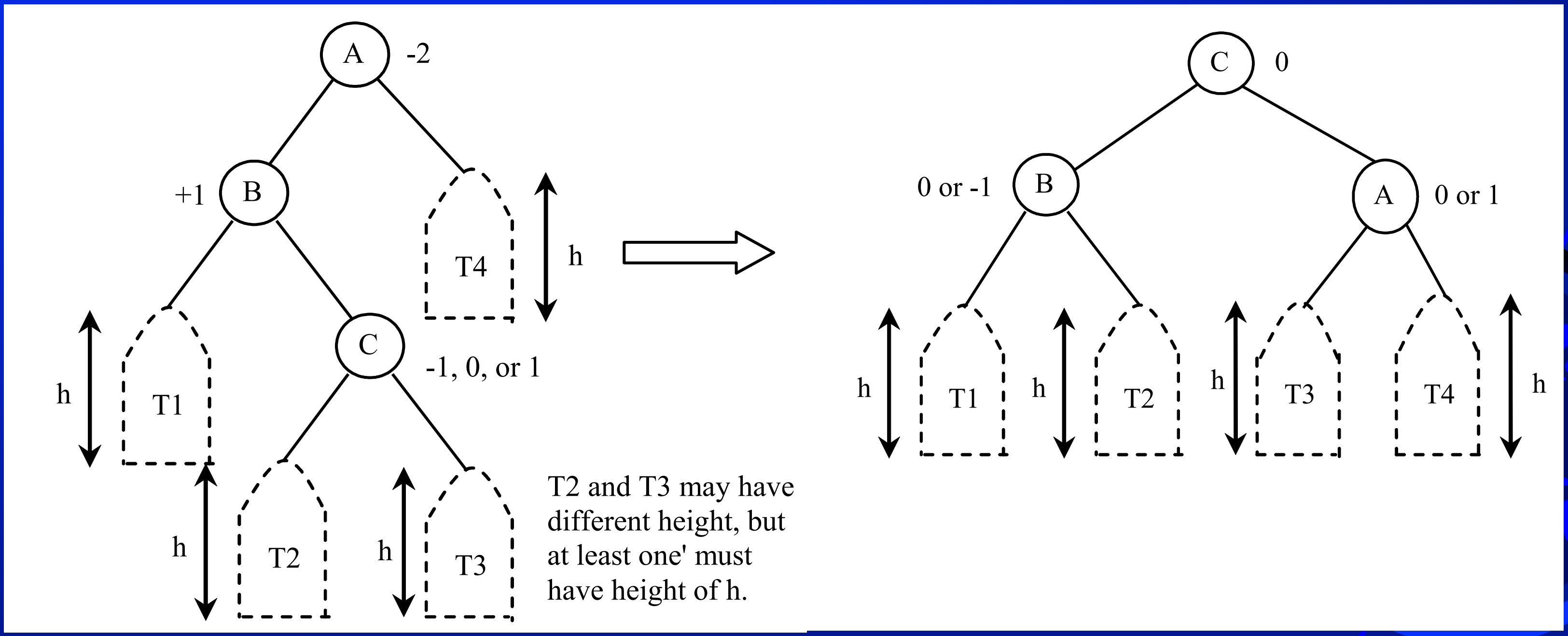
# RR imbalance and RR rotation

**RR Rotation**: An *RR imbalance* occurs at a node A such that A has a balance factor +2 and a right child B with a balance factor +1 or 0. This type of imbalance can be fixed by performing a single left rotation at A.



# LR imbalance and LR rotation

**LR Rotation**: An *LR imbalance* occurs at a node A such that A has a balance factor -2 and a left child B with a balance factor +1. Assume B's right child is C. This type of imbalance can be fixed by performing a double rotation at A (first a single left rotation at B and then a single right rotation at A).



# RL imbalance and RL rotation

**RL Rotation**: An *RL imbalance* occurs at a node A such that A has a balance factor +2 and a right child B with a balance factor -1. Assume B's left child is C. This type of imbalance can be fixed by performing a double rotation at A (first a single right rotation at B and then a single left rotation at A).