Text Categorization

Rocchio, kNN,
and Bayesian Methods

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Using Relevance Feedback (Rocchio)

- Relevance feedback methods can be adapted for text categorization.
- Use standard TF/IDF weighted vectors to represent text documents (normalized by maximum term frequency).
- For each category, compute a *prototype* vector by summing the vectors of the training documents in the category.
- Assign test documents to the category with the closest prototype vector based on cosine similarity.

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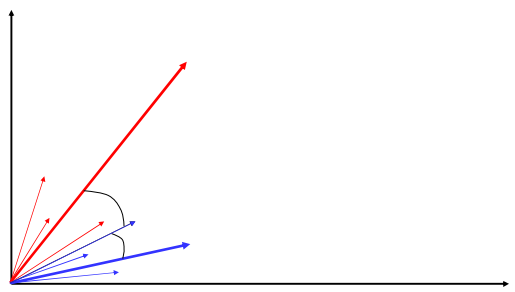
Illustration of Rocchio Text Categorization



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Rocchio Text Categorization Algorithm (Training)

Assume the set of categories is $\{c_1, c_2, \ldots c_n\}$
For i from 1 to n let $\mathbf{p}_i = <0, 0, \ldots, 0>$ *(init. prototype vectors)*
For each training example $<x, c(x)> \in D$
  Let $\mathbf{d}$ be the frequency normalized TF/IDF term vector for doc x
  Let $i = j$: $(c_j = c(x))$
  (sum all the document vectors in c_i to get $\mathbf{p}_i$)
  Let $\mathbf{p}_i = \mathbf{p}_i + \mathbf{d}$

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Rocchio Text Categorization Algorithm (Test)

Given test document x
Let $\mathbf{d}$ be the TF/IDF weighted term vector for x
Let $m = -2$ *(init. maximum cosSim)*
For i from 1 to n:
  (compute similarity to prototype vector)
  Let $s = \text{cosSim}(\mathbf{d}, \mathbf{p}_i)$
  if $s > m$
    let $m = s$
    let $r = c_i$ *(update most similar class prototype)*
Return class r

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Rocchio Properties

- Does not guarantee a consistent hypothesis.
- Forms a simple generalization of the examples in each class (a *prototype*).
- Prototype vector does not need to be averaged or otherwise normalized for length since cosine similarity is insensitive to vector length.
- Classification is based on similarity to class prototypes.

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Rocchio Time Complexity

- Note: The time to add two sparse vectors is proportional to minimum number of non-zero entries in the two vectors.
- Training Time: $O(|D|(L_d + |V_d|)) = O(|D| L_d)$
  where L_d is the average length of a document in D and $|V_d|$ is the average vocabulary size for a document in D.
- Test Time: $O(L_t + |C||V_t|)$
  where L_t is the average length of a test document and $|V_t|$ is the average vocabulary size for a test document.
  - Assumes lengths of $\mathbf{p}_i$ vectors are computed and stored during training, allowing cosSim($\mathbf{d}$, $\mathbf{p}_i$) to be computed in time proportional to the number of non-zero entries in $\mathbf{d}$ (i.e. $|V_t|$)

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Nearest-Neighbor Learning Algorithm

- Learning is just storing the representations of the training examples in D.
- Testing instance x:
  - Compute similarity between x and all examples in D.
  - Assign x the category of the most similar example in D.
- Does not explicitly compute a generalization or category prototypes.
- Also called:
  - Case-based
  - Memory-based
  - Lazy learning

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K Nearest-Neighbor

- Using only the closest example to determine categorization is subject to errors due to:
  - A single atypical example.
  - Noise (i.e. error) in the category label of a single training example.
- More robust alternative is to find the k most-similar examples and return the majority category of these k examples.
- Value of k is typically odd to avoid ties, 3 and 5 are most common.

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Similarity Metrics

- Nearest neighbor method depends on a similarity (or distance) metric.
- Simplest for continuous *m*-dimensional instance space is *Euclidian distance*.
- Simplest for *m*-dimensional binary instance space is *Hamming distance* (number of feature values that differ).
- For text, cosine similarity of TF-IDF weighted vectors is typically most effective.

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3 Nearest Neighbor Illustration
(Euclidian Distance)

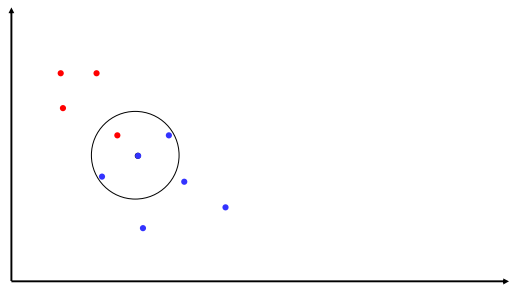

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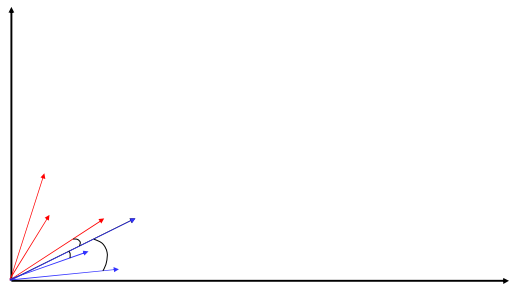
Illustration of 3 Nearest Neighbor for Text



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K Nearest Neighbor for Text

Training:
For each each training example $\langle x, c(x) \rangle \in D$
Compute the corresponding TF-IDF vector, $\mathbf{d}_x$, for document x

Test instance y:
Compute TF-IDF vector $\mathbf{d}$ for document y
For each $\langle x, c(x) \rangle \in D$
Let $s_x = \text{cosSim}(\mathbf{d}, \mathbf{d}_x)$
Sort examples, x, in D by decreasing value of s_x
Let N be the first k examples in D. (*get most similar neighbors*)
Return the majority class of examples in N

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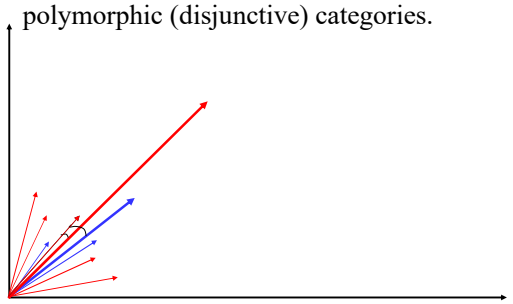
Rocchio Anomoly

- Prototype models have problems with polymorphic (disjunctive) categories.



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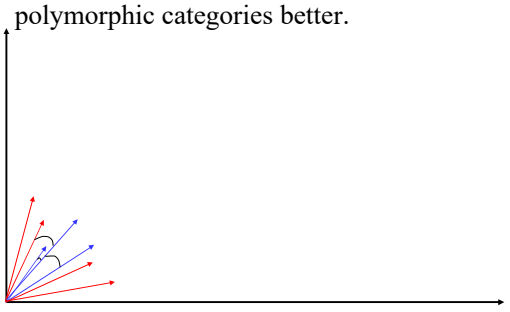
3 Nearest Neighbor Comparison

- Nearest Neighbor tends to handle polymorphic categories better.



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Nearest Neighbor Time Complexity

- Training Time: O($|D|\ L_d$) to compose TF-IDF vectors.
- Testing Time: O($L_t + |D||V_t|$) to compare to all training vectors.
  - Assumes lengths of $\mathbf{d}_x$ vectors are computed and stored during training, allowing cosSim($\mathbf{d}$, $\mathbf{d}_x$) to be computed in time proportional to the number of non-zero entries in $\mathbf{d}$ (i.e. $|V_t|$)
- Testing time can be high for large training sets.

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Nearest Neighbor with Inverted Index

- Determining k nearest neighbors is the same as determining the k best retrievals using the test document as a query to a database of training documents.
- Use standard VSR inverted index methods to find the k nearest neighbors.
- Testing Time: O($B|V_t|$)
  where B is the average number of training documents in which a test-document word appears.
- Therefore, overall classification is O($L_t + B|V_t|$)
  - Typically $B << |D|$

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Bayesian Methods

- Learning and classification methods based on probability theory.
- Bayes theorem plays a critical role in probabilistic learning and classification.
- Uses *prior* probability of each category given no information about an item.
- Categorization produces a *posterior* probability distribution over the possible categories given a description of an item.

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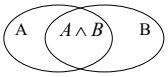
Axioms of Probability Theory

- All probabilities between 0 and 1

  $0 \le P(A) \le 1$
- True proposition has probability 1, false has probability 0.

  P(true) = 1 P(false) = 0.
- The probability of disjunction is:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$



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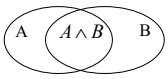
Conditional Probability

- P(A | B) is the probability of A given B
- Assumes that B is all and only information known.
- Defined by:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)}$$



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Independence

- A and B are *independent* iff:

$$P(A \mid B) = P(A)$$
$$P(B \mid A) = P(B)$$

These two constraints are logically equivalent

- Therefore, if A and B are independent:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)} = P(A)$$

$$P(A \wedge B) = P(A)P(B)$$

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Joint Distribution

- The joint probability distribution for a set of random variables, $X_1,\ldots,X_n$ gives the probability of every combination of values (an n-dimensional array with v^n values if all variables are discrete with v values, all v^n values must sum to 1): $P(X_1,\ldots,X_n)$

positive

	circle	square
red	0.20	0.02
blue	0.02	0.01

negative

	circle	square
red	0.05	0.30
blue	0.20	0.20

- The probability of all possible conjunctions (assignments of values to some subset of variables) can be calculated by summing the appropriate subset of values from the joint distribution.

$$P(red \wedge circle) = 0.20 + 0.05 = 0.25$$
$$P(red) = 0.20 + 0.02 + 0.05 + 0.3 = 0.57$$

- Therefore, all conditional probabilities can also be calculated.

$$P(positive \mid red \wedge circle) = \frac{P(positive \wedge red \wedge circle)}{P(red \wedge circle)} = \frac{0.20}{0.25} = 0.80$$

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Probabilistic Classification

- Let Y be the random variable for the class which takes values $\{y_1,y_2,\ldots y_m\}$.
- Let X be the random variable describing an instance consisting of a vector of values for n features $\langle X_1,X_2\ldots X_n\rangle$, let x_k be a possible value for X and x_{ij} a possible value for X_i.
- For classification, we need to compute $P(Y=y_i \mid X=x_k)$ for $i=1\ldots m$
- However, given no other assumptions, this requires a table giving the probability of each category for each possible instance in the instance space, which is impossible to accurately estimate from a reasonably-sized training set.
  - Assuming Y and all X_i are binary, we need 2^n entries to specify $P(Y=\text{pos} \mid X=x_k)$ for each of the 2^n possible x_k's since $P(Y=\text{neg} \mid X=x_k) = 1 - P(Y=\text{pos} \mid X=x_k)$
  - Compared to $2^{n+1} - 1$ entries for the joint distribution $P(Y,X_1,X_2\ldots X_n)$

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Bayes Theorem

$$P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}$$

Simple proof from definition of conditional probability:

$$P(H \mid E) = \frac{P(H \wedge E)}{P(E)} \quad \text{(Def. cond. prob.)}$$

$$P(E \mid H) = \frac{P(H \wedge E)}{P(H)} \quad \text{(Def. cond. prob.)}$$

$$P(H \wedge E) = P(E \mid H)P(H)$$

QED: $P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}$

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Bayesian Categorization

- Determine category of x_k by determining for each y_i

$$P(Y = y_i \mid X = x_k) = \frac{P(Y = y_i)P(X = x_k \mid Y = y_i)}{P(X = x_k)}$$

- $P(X=x_k)$ can be determined since categories are complete and disjoint.

$$\sum_{i=1}^{m} P(Y = y_i \mid X = x_k) = \sum_{i=1}^{m} \frac{P(Y = y_i)P(X = x_k \mid Y = y_i)}{P(X = x_k)} = 1$$

$$P(X = x_k) = \sum_{i=1}^{m} P(Y = y_i)P(X = x_k \mid Y = y_i)$$

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Bayesian Categorization (cont.)

- Need to know:
  - Priors: $P(Y=y_i)$
  - Conditionals: $P(X=x_k \mid Y=y_i)$
- $P(Y=y_i)$ are easily estimated from data.
  - If n_i of the examples in D are in y_i then $P(Y=y_i) = n_i / |D|$
- Too many possible instances (e.g. 2^n for binary features) to estimate all $P(X=x_k \mid Y=y_i)$.
- Still need to make some sort of independence assumptions about the features to make learning tractable.

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Generative Probabilistic Models

- Assume a simple (usually unrealistic) probabilistic method by which the data was generated.
- For categorization, each category has a different parameterized generative model that characterizes that category.
- **Training**: Use the data for each category to estimate the parameters of the generative model for that category.
  - **Maximum Likelihood Estimation (MLE)**: Set parameters to maximize the probability that the model produced the given training data.
  - If M_λ denotes a model with parameter values λ and D_k is the training data for the kth class, find model parameters for class k (λ_k) that maximize the likelihood of D_k:

$$\lambda_k = \underset{\lambda}{\operatorname{argmax}}\, P(D_k \mid M_\lambda)$$

- **Testing**: Use Bayesian analysis to determine the category model that most likely generated a specific test instance.

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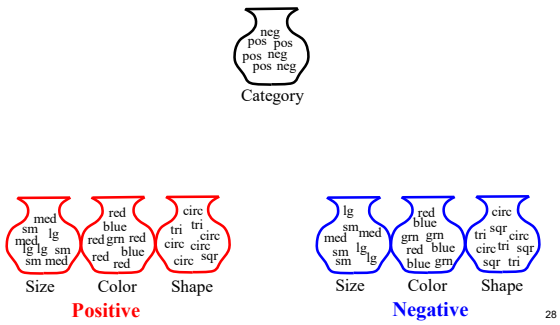
Naïve Bayes Generative Model



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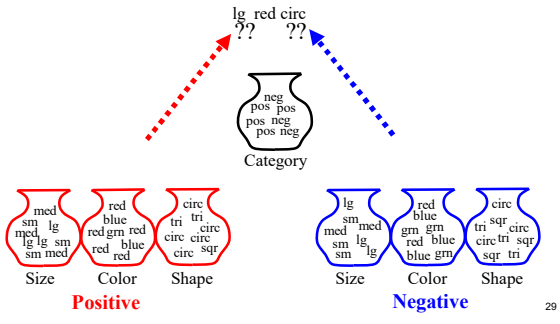
Naïve Bayes Inference Problem



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Naïve Bayesian Categorization

- If we assume features of an instance are independent **given the category** (***conditionally independent***).

$$P(X \mid Y) = P(X_1, X_2, \cdots X_n \mid Y) = \prod_{i=1}^{n} P(X_i \mid Y)$$

- Therefore, we then only need to know $P(X_i \mid Y)$ for each possible pair of a feature-value and a category.
- If Y and all X_i and binary, this requires specifying only $2n$ parameters:
  - $P(X_i\text{=true} \mid Y\text{=true})$ and $P(X_i\text{=true} \mid Y\text{=false})$ for each X_i
  - $P(X_i\text{=false} \mid Y) = 1 - P(X_i\text{=true} \mid Y)$
- Compared to specifying 2^n parameters without any independence assumptions.

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Naïve Bayes Example

Probability	positive	negative
P(*Y*)	0.5	0.5
P(small | *Y*)	0.4	0.4
P(medium | *Y*)	0.1	0.2
P(large | *Y*)	0.5	0.4
P(red | *Y*)	0.9	0.3
P(blue | *Y*)	0.05	0.3
P(green | *Y*)	0.05	0.4
P(square | *Y*)	0.05	0.4
P(triangle | *Y*)	0.05	0.3
P(circle | *Y*)	0.9	0.3

Test Instance:
<medium ,red, circle>

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Naïve Bayes Example

Probability	positive	negative
P(*Y*)	0.5	0.5
P(medium | *Y*)	0.1	0.2
P(red | *Y*)	0.9	0.3
P(circle | *Y*)	0.9	0.3

Test Instance:
<medium, red, circle>

P(positive | *X*) = P(positive)*P(medium | positive)*P(red | positive)*P(circle | positive) / P(*X*)
0.5 * 0.1 * 0.9 * 0.9
= 0.0405 / P(*X*) = 0.0405 / 0.0495 = 0.8181

P(negative | *X*) = P(negative)*P(medium | negative)*P(red | negative)*P(circle | negative) / P(*X*)
0.5 * 0.2 * 0.3 * 0.3
= 0.009 / P(*X*) = 0.009 / 0.0495 = 0.1818

P(positive | *X*) + P(negative | *X*) = 0.0405 / P(*X*) + 0.009 / P(*X*) = 1

P(*X*) = (0.0405 + 0.009) = 0.0495

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Estimating Probabilities

- Normally, probabilities are estimated based on observed frequencies in the training data.
- If *D* contains n_k examples in category y_k, and n_{ijk} of these n_k examples have the *j*th value for feature X_i, x_{ij}, then:

$$P(X_i = x_{ij} \mid Y = y_k) = \frac{n_{ijk}}{n_k}$$

- However, estimating such probabilities from small training sets is error-prone.
- If due only to chance, a rare feature, X_i, is always false in the training data, $\forall y_k : P(X_i = \text{true} \mid Y = y_k) = 0$.
- If X_i=true then occurs in a test example, *X*, the result is that $\forall y_k$: $P(X \mid Y = y_k) = 0$ and $\forall y_k$: $P(Y = y_k \mid X) = 0$

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Probability Estimation Example

Ex	Size	Color	Shape	Category
1	small	red	circle	positive
2	large	red	circle	positive
3	small	red	triangle	negative
4	large	blue	circle	negative

Probability	positive	negative
P(*Y*)	0.5	0.5
P(small | *Y*)	0.5	0.5
P(medium | *Y*)	0.0	0.0
P(large | *Y*)	0.5	0.5
P(red | *Y*)	1.0	0.5
P(blue | *Y*)	0.0	0.5
P(green | *Y*)	0.0	0.0
P(square | *Y*)	0.0	0.0
P(triangle | *Y*)	0.0	0.5
P(circle | *Y*)	1.0	0.5

Test Instance *X*:
<medium, red, circle>

P(positive | *X*) = 0.5 * 0.0 * 1.0 * 1.0 / P(X) = 0

P(negative | *X*) = 0.5 * 0.0 * 0.5 * 0.5 / P(X) = 0

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Smoothing

- To account for estimation from small samples, probability estimates are adjusted or *smoothed*.
- Laplace smoothing using an *m*-estimate assumes that each feature is given a prior probability, *p*, that is assumed to have been previously observed in a "virtual" sample of size *m*.

$$P(X_i = x_{ij} \mid Y = y_k) = \frac{n_{ijk} + mp}{n_k + m}$$

- For binary features, *p* is simply assumed to be 0.5.

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Laplace Smothing Example

- Assume training set contains 10 positive examples:
  - 4: small
  - 0: medium
  - 6: large
- Estimate parameters as follows (if *m*=1, *p*=1/3)
  - P(small | positive) = (4 + 1/3) / (10 + 1) = 0.394
  - P(medium | positive) = (0 + 1/3) / (10 + 1) = 0.03
  - P(large | positive) = (6 + 1/3) / (10 + 1) = 0.576
  - P(small or medium or large | positive) = 1.0

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Naïve Bayes for Text

- Modeled as generating a bag of words for a document in a given category by repeatedly sampling with replacement from a vocabulary $V = \{w_1, w_2, \ldots w_m\}$ based on the probabilities $P(w_j \mid c_i)$.
- Smooth probability estimates with Laplace *m*-estimates assuming a uniform distribution over all words ($p = 1/|V|$) and $m = |V|$
  - Equivalent to a virtual sample of seeing each word in each category exactly once.

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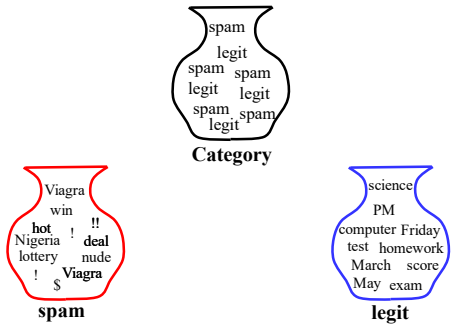
Naïve Bayes Generative Model for Text



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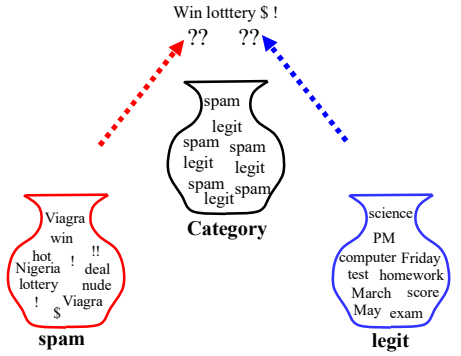
Naïve Bayes Classification



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Text Naïve Bayes Algorithm (Train)

Let V be the vocabulary of all words in the documents in D
For each category $c_i \in C$
  Let D_i be the subset of documents in D in category c_i
  $P(c_i) = |D_i| / |D|$
  Let T_i be the concatenation of all the documents in D_i
  Let n_i be the total number of word occurrences in T_i
  For each word $w_j \in V$
    Let n_{ij} be the number of occurrences of w_j in T_i
    Let $P(w_j \mid c_i) = (n_{ij} + 1) / (n_i + |V|)$

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Text Naïve Bayes Algorithm (Test)

Given a test document X
Let n be the number of word occurrences in X
Return the category:

$$\underset{c_i \in C}{\operatorname{argmax}} P(c_i) \prod_{i=1}^{n} P(a_i \mid c_i)$$

where a_i is the word occurring the ith position in X

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Underflow Prevention

- Multiplying lots of probabilities, which are between 0 and 1 by definition, can result in floating-point underflow.
- Since $\log(xy) = \log(x) + \log(y)$, it is better to perform all computations by summing logs of probabilities rather than multiplying probabilities.
- Class with highest final un-normalized log probability score is still the most probable.

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Naïve Bayes Posterior Probabilities

- Classification results of naïve Bayes (the class with maximum posterior probability) are usually fairly accurate.
- However, due to the inadequacy of the conditional independence assumption, the actual posterior-probability numerical estimates are not.
  - Output probabilities are generally very close to 0 or 1.

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Evaluating Categorization

- Evaluation must be done on test data that are independent of the training data (usually a disjoint set of instances).
- *Classification accuracy*: c/n where n is the total number of test instances and c is the number of test instances correctly classified by the system.
- Results can vary based on sampling error due to different training and test sets.
- Average results over multiple training and test sets (splits of the overall data) for the best results.

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N-Fold Cross-Validation

- Ideally, test and training sets are independent on each trial.
  - But this would require too much labeled data.
- Partition data into N equal-sized disjoint segments.
- Run N trials, each time using a different segment of the data for testing, and training on the remaining N–1 segments.
- This way, at least test-sets are independent.
- Report average classification accuracy over the N trials.
- Typically, $N = 10$.

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Learning Curves

- In practice, labeled data is usually rare and expensive.
- Would like to know how performance varies with the number of training instances.
- *Learning curves* plot classification accuracy on independent test data (*Y* axis) versus number of training examples (*X* axis).

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N-Fold Learning Curves

- Want learning curves averaged over multiple trials.
- Use *N*-fold cross validation to generate *N* full training and test sets.
- For each trial, train on increasing fractions of the training set, measuring accuracy on the test data for each point on the desired learning curve.

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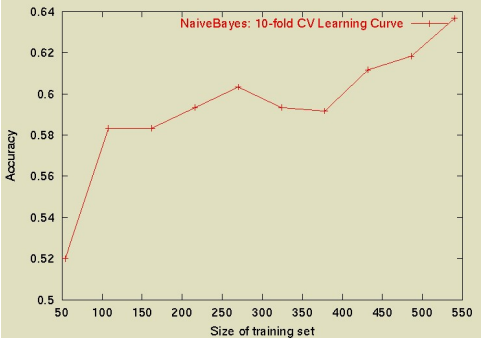
Sample Learning Curve (Yahoo Science Data)



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