Combining ACL2 and Mathematica for the symbolic simulation of digital systems

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Symbolic simulation

- Symbolic simulation - semi-formal technique based on numeric simulation :
  - some inputs are valued, others stay symbolic
  - results: arithmetic and logical expressions

- Problems:
  - Symbolic expressions become very large with the number of simulation cycles
  - The simulation tree grows exponentially (with conditional statements when the condition is a symbolic term)
  - The results are unreadable

Constrained Symbolic Simulation

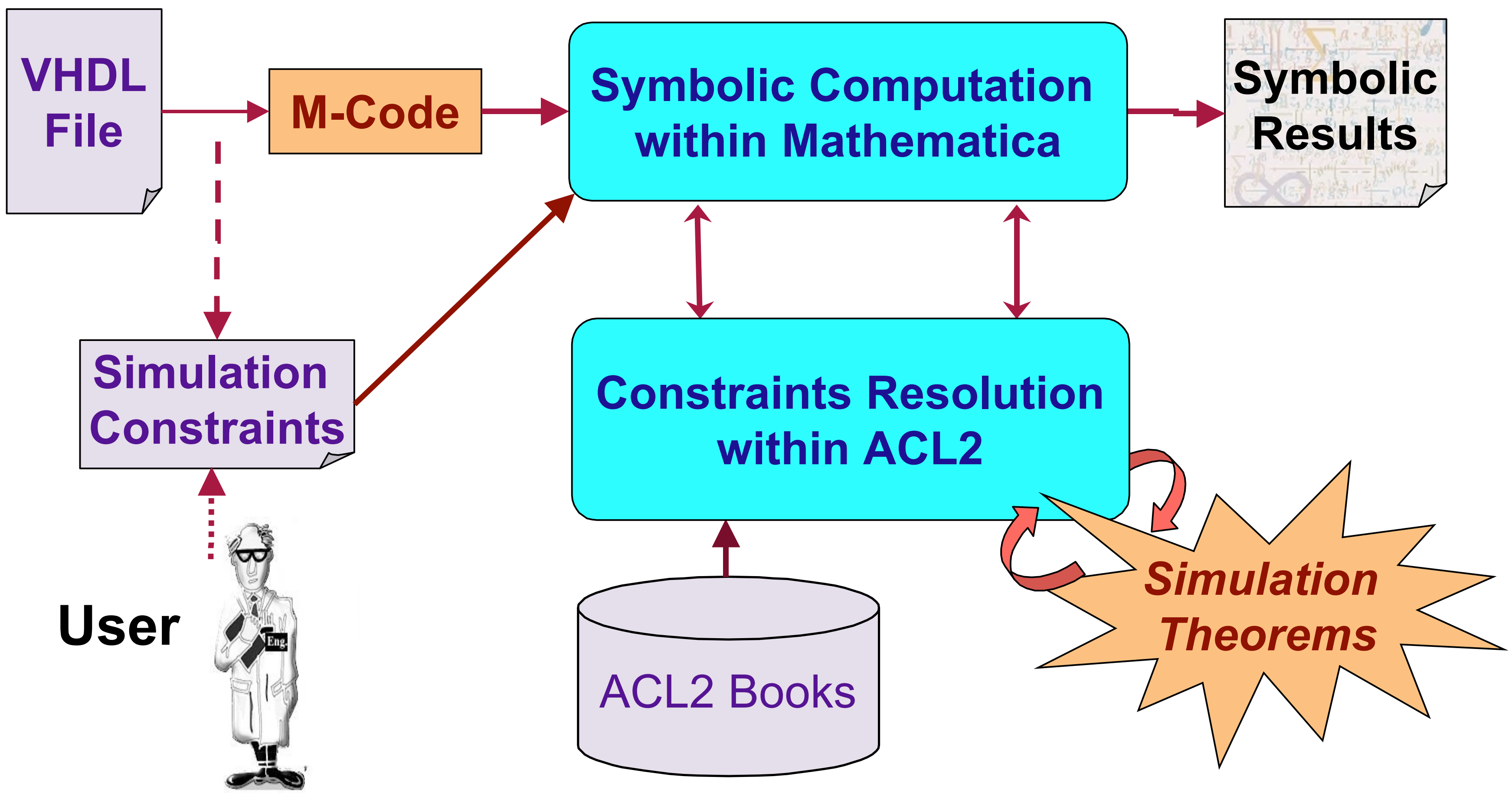
Two aspects of the simulation:

algebraic computation -> Mathematica

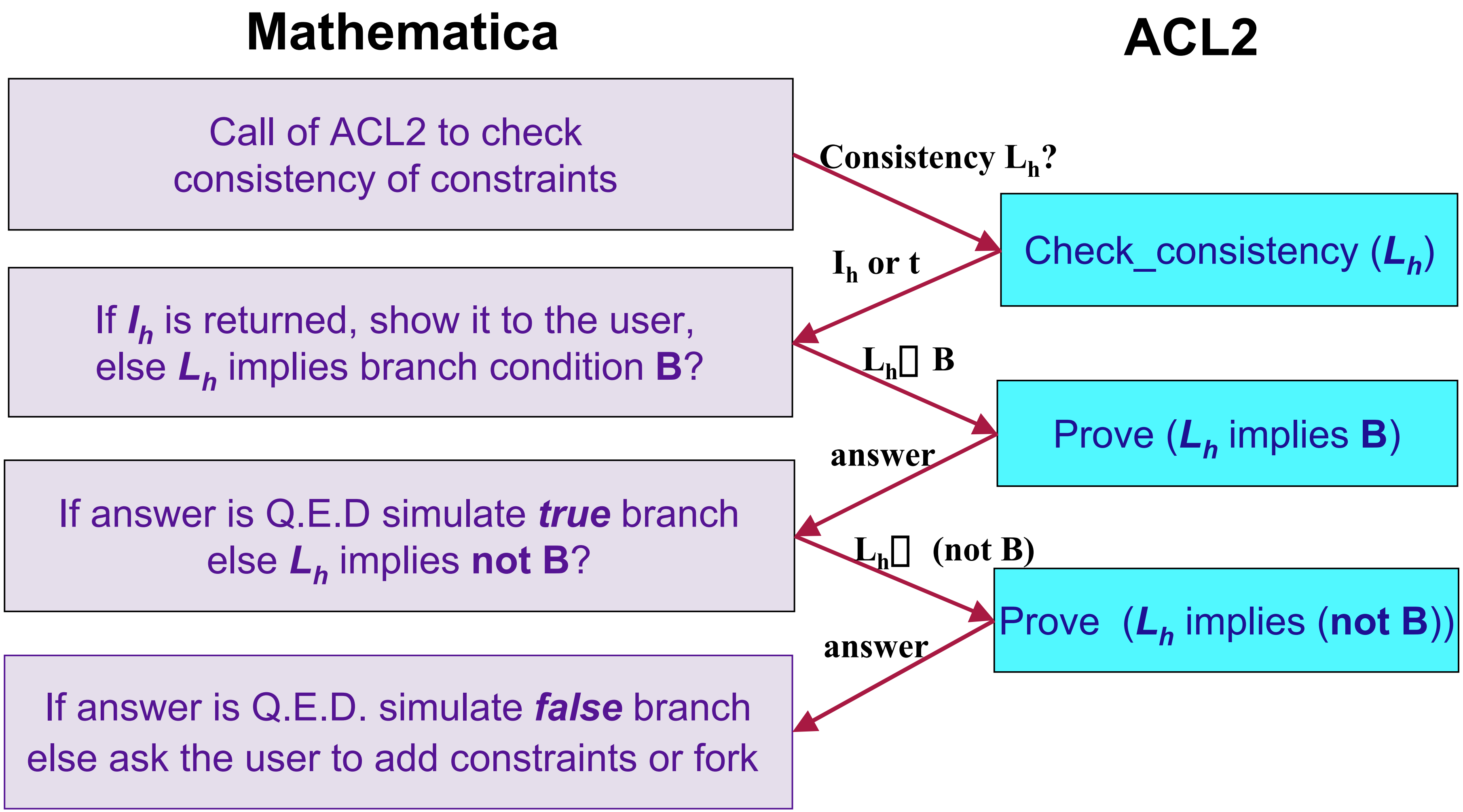
branching decision -> ACL2

- Constraints:
  - data type restrictions,
  - inequalities and equalities between expressions composed of design variables or input signals and arithmetic operators

Constrained Symbolic Simulation



Branch Decision Scheme



Checking constraints consistency

- *check_consistency* function
  - Uses :
    - *tool1-fn* from /books/misc/expander
    - *consistency* function

```
(defun check-consistency (Lh state)
   (if (true-listp Lh)
       (cond ((endp Lh) (value  nil))
             (t (mv-let (erp val state)
                    (tool1-fn Lh state nil t nil t t)
                    (if erp
                       (value nil)
                       (if (nth 1 val)
                           (value t)
                           (consistency Lh nil 1 state))))))
       (value nil)))
```

Checking constraints consistency

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
    - otherwise a ***minimal set of contradictory constraints*** (*minimal = any strict subset is satisfiable*)

I_h nil

L_h

$$(\underbrace{C_1}_{t} \ldots C_{k-1}\ C_k\ C_{k+1} \ldots C_n)$$

Checking constraints consistency

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L_h

$(\underbrace{C_1 \ldots C_{k-1}}_{t}\ C_k\ C_{k+1} \ldots C_n)$

Checking constraints consistency

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
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I_h L_h

nil $(\underbrace{C_1 \ldots C_{k-1}\, C_k}_{f}\; C_{k+1} \ldots C_n)$

Checking constraints consistency

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
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      (*minimal = any strict subset is satisfiable*)

I_h nil

L_h

$(C_1 \ldots C_{k-1}\, C_k\, C_{k+1} \ldots C_n)$

$\underbrace{C_1 \ldots C_{k-1}\, C_k}_{f}$

Checking constraints consistency

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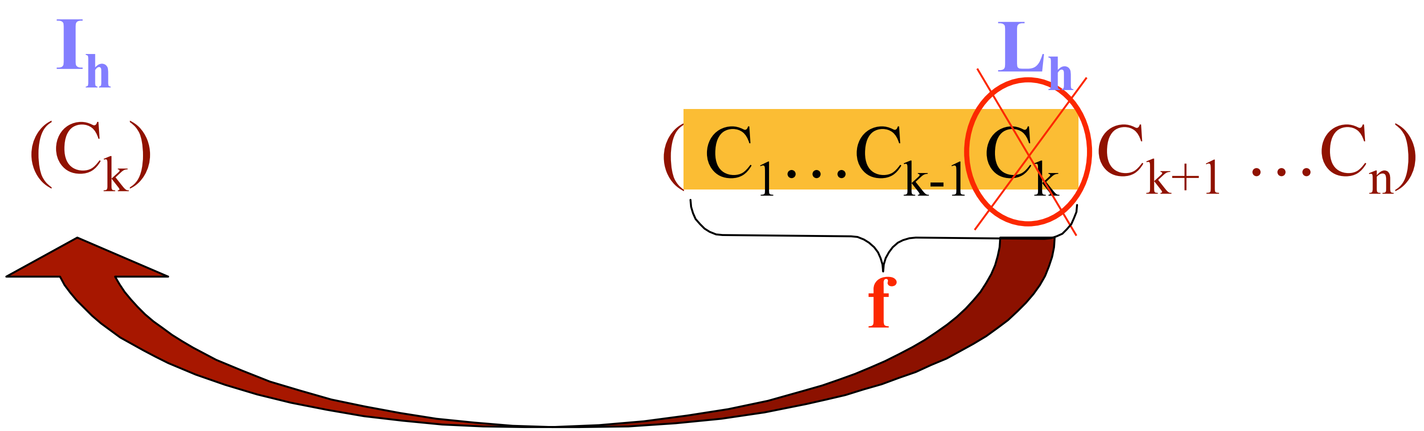

Checking constraints consistency

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
    - otherwise a ***minimal set of contradictory constraints*** (*minimal = any strict subset is satisfiable*)

I_h L_h

(C_k) $(C_1 \ldots C_{k-1}\ C_{k+1}\ \ldots C_n)$

f -> stop

Checking constraints consistency

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
    - otherwise a ***minimal set of contradictory constraints***
      (*minimal = any strict subset is satisfiable*)

I_h (C_k) → t

L_h $(C_1 \ldots C_{k-1}\ C_{k+1} \ldots C_n)$

Checking constraints consistency

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
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I_h L_h

(C_k) $(C_1 \ldots C_{k-1}\ C_{k+1} \ldots C_n)$

t …

Checking constraints consistency Example

- *consistency* function
  - Returns, for an inconsistent set of constraints
    - ***nil*** in case of errors
    - otherwise a ***minimal set of contradictory constraints*** (*minimal = any strict subset is satisfiable*)

L_h= ((< a b) (integerp a) (< c a) (< c d) (< b c))

I_h= ((< b c) (< c a) (< a b))

ACL2 - Mathematica communication

- Communication via a pipe
- Initialized by Mathematica with *callAcl2* function
- Mathematica gets the last line of the ACL2 response
- One ACL2 session during the whole simulation

```
callAcl2["(defthm foo (equal x x) : rule-classes nil)"]
FOO

callAcl2["(defthm foo (not (equal x x) ): rule-classes nil)"]
******** FAILED ******** See :DOC failure ******** FAILED
   ****
```

ACL2 - Mathematica communication

- Theorems:

```
callAcl2["(mv-let (erp val state)
                  (defthm foo (implies L B))
                  (declare (ignore val))
                  (if erp
                      (value nil)
                      (value T)))"]
```

- Functions:

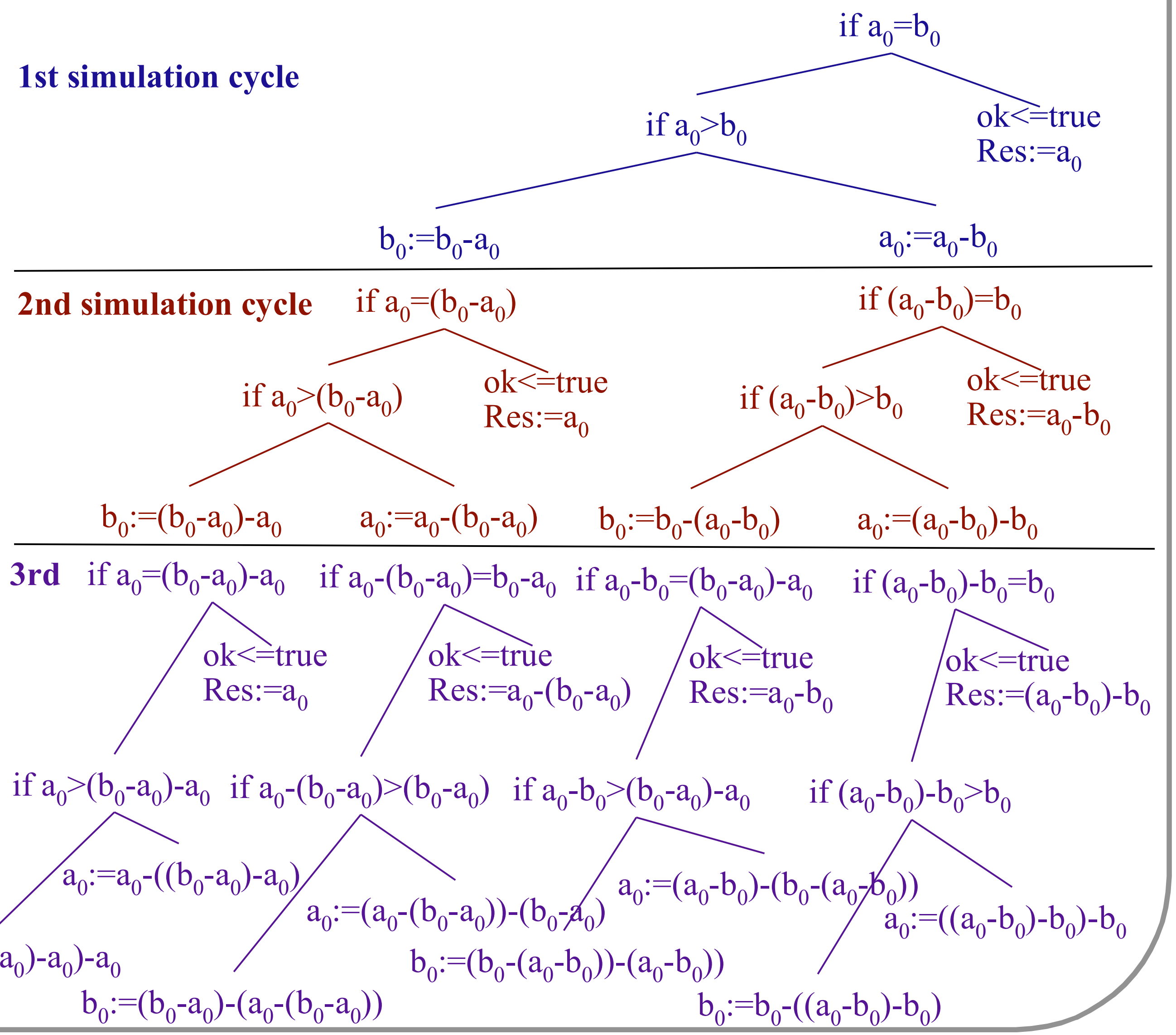
```
callAcl2["(mv-let (erp val state)
                  (check_consistency L)
                  (if erp
                      (value nil)
                      (value val)))"]
```

Euclid's GCD Algorithm

```
P1: process begin
    wait until clk='1';
    if RST='1' then
       a0:=a;
       b0:=b;
       ok<=False;
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

Euclid's GCD Algorithm

```
P1: process begin
    wait until clk='1';
    if RST='1' then
       a0:=a;
       b0:=b;
       ok<=False;
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```



Reduction of the execution tree

```
Constraints:
    a=3n, b=n, n∈𝒩*

P1: process begin
    wait until clk='1';
    if RST='1' then
        a0:=a;
        b0:=b;
        ok<=False;
    elsif a0=b0 then
        ok<=True;
        res<=a0;
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        a0:=a0-b0;
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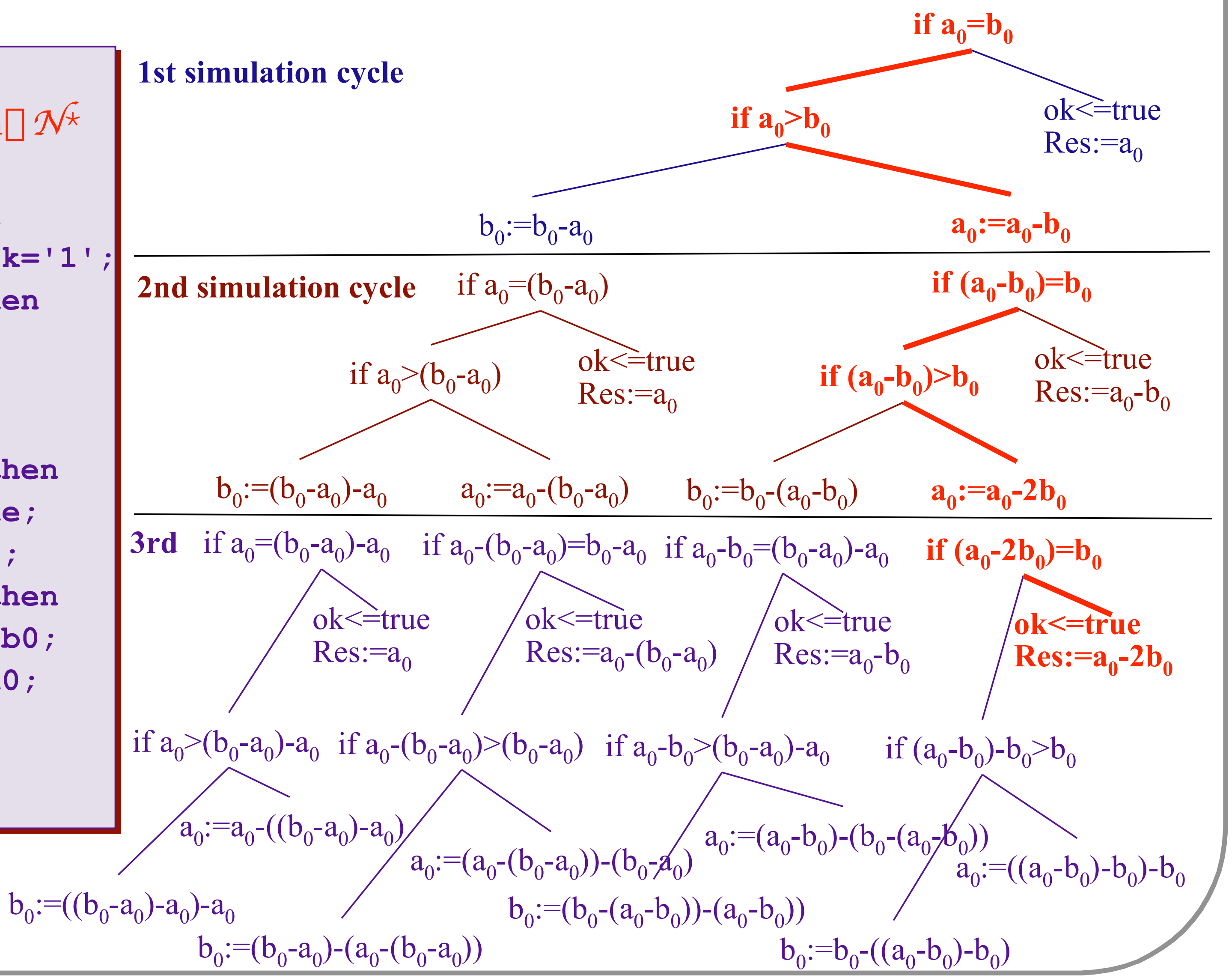
```



Reduction of the execution tree

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```
P1: process begin
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    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

if $a_0=b_0$

```
callAcl2["(mv-let (erp val state)
         (check_consistency
           ((integerp n) (< 0 n)))
         (if erp (value nil)
             (value val)))"]
```

Reduction of the execution tree

Constraints:

a=3n, b=n, n∈$\mathcal{N}^*$

```
P1: process begin
    wait until clk='1';
    if RST='1' then
       a0:=a;
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    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

T

if $a_0=b_0$

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callAcl2["(mv-let (erp val state)
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            ((integerp n) (< 0 n)))
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Reduction of the execution tree

Constraints:

$a=3n,\ b=n,\ n \in \mathcal{N}^*$

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    elsif a0=b0 then
        ok<=True;
        res<=a0;
    elsif a0>b0 then
        a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

if $a_0=b_0$

```
callAcl2["(mv-let (erp val state)
         (defthm thm1
            (implies (and (integerp n)(< 0 n))
                     (equal (* 3 n) n)))
         (declare (ignore val))
         (if erp (value nil)
            (value T)))"]
```

Reduction of the execution tree

Constraints:

a=3n, b=n, n$\in\mathcal{N}^*$

```
P1: process begin
    wait until clk='1';
    if RST='1' then
       a0:=a;
       b0:=b;
       ok<=False;
    elsif a0=b0 then
         ok<=True;
         res<=a0;
    elsif a0>b0 then
         a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

nil

if $a_0=b_0$

```
callAcl2["(mv-let (erp val state)
        (defthm thm1
           (implies (and (integerp n)(< 0 n))
                    (equal (* 3 n) n)))
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           (value T)))"]
```

Reduction of the execution tree

Constraints:

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          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

if $a_0=b_0$

```
callAcl2["(mv-let (erp val state)
          (defthm thm1-neg
             (implies (and (integerp n)(< 0 n))
                      (not (equal (* 3 n) n))))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

Reduction of the execution tree

Constraints:

a=3n, b=n, n∈$\mathcal{N}$*

```
P1: process begin
    wait until clk='1';
    if RST='1' then
        a0:=a;
        b0:=b;
        ok<=False;
    elsif a0=b0 then
            ok<=True;
            res<=a0;
    elsif a0>b0 then
            a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

if $a_0=b_0$

T

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callAcl2["(mv-let (erp val state)
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            (implies (and (integerp n)(< 0 n))
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Reduction of the execution tree

Constraints:

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P1: process begin
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       a0:=a;
       b0:=b;
       ok<=False;
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

if $a_0=b_0$

if $a_0>b_0$

```
callAcl2["(mv-let (erp val state)
          (defthm thm2
             (implies (and (integerp n)(< 0 n))
                      (> (* 3 n) n)))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

Reduction of the execution tree

Constraints:

a=3n, b=n, n∈$\mathcal{N}^*$

```
P1: process begin
    wait until clk='1';
    if RST='1' then
        a0:=a;
        b0:=b;
        ok<=False;
    elsif a0=b0 then
           ok<=True;
           res<=a0;
    elsif a0>b0 then
           a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

1st simulation cycle

if $a_0=b_0$

if $a_0>b_0$

T

```
callAcl2["(mv-let (erp val state)
          (defthm thm2
             (implies (and (integerp n)(< 0 n))
                      (> (* 3 n) n)))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

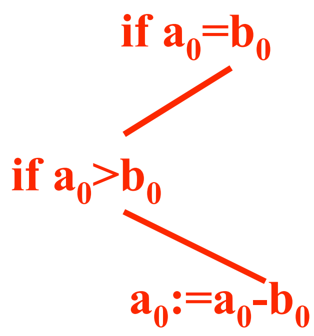
Reduction of the execution tree

Constraints:

a=3n, b=n, n∈𝒩*

```
P1: process begin
     wait until clk='1';
     if RST='1' then
         a0:=a;
         b0:=b;
         ok<=False;
     elsif a0=b0 then
            ok<=True;
            res<=a0;
     elsif a0>b0 then
            a0:=a0-b0;
     else b0:=b0-a0;
     end if;
end process P1;
```

1st simulation cycle



Reduction of the execution tree

Constraints:

$a=3n,\ b=n,\ n \in \mathcal{N}^*$

```
P1: process begin
    wait until clk='1';
    if RST='1' then
       a0:=a;
       b0:=b;
       ok<=False;
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

2nd simulation cycle

if $a_0=b_0$

if $a_0>b_0$

$a_0:=a_0-b_0$

if $(a_0-b_0)=b_0$

```
callAcl2["(mv-let (erp val state)
          (defthm thm3
             (implies (and (integerp n)(< 0 n))
                      (equal (* 2 n) n)))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

Reduction of the execution tree

Constraints:

a=3n, b=n, n□$\mathcal{N}^*$

```
P1: process begin
    wait until clk='1';
    if RST='1' then
       a0:=a;
       b0:=b;
       ok<=False;
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

2nd simulation cycle

if $a_0=b_0$

if $a_0>b_0$

$a_0:=a_0-b_0$

if $(a_0-b_0)=b_0$

nil

```
callAcl2["(mv-let (erp val state)
          (defthm thm3
             (implies (and (integerp n)(< 0 n))
                      (equal (* 2 n) n)))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

Reduction of the execution tree

```
Constraints:
    a=3n, b=n, n∈𝒩*

P1: process begin
    wait until clk='1';
    if RST='1' then
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    elsif a0=b0 then
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          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

2nd simulation cycle

if $a_0=b_0$

if $a_0>b_0$

$a_0:=a_0-b_0$

if $(a_0-b_0)=b_0$

```
callAcl2["(mv-let (erp val state)
          (defthm thm3-neg
             (implies (and (integerp n)(< 0 n))
                      (not (equal (* 2 n) n))))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

Reduction of the execution tree

```
Constraints:
    a=3n, b=n, n∈𝒩*

P1: process begin
    wait until clk='1';
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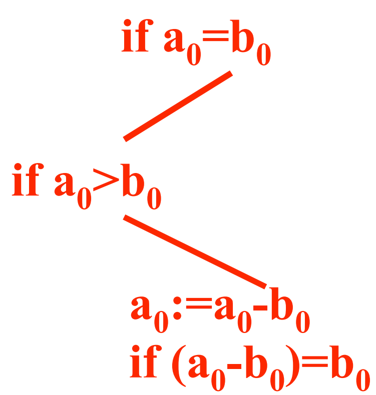
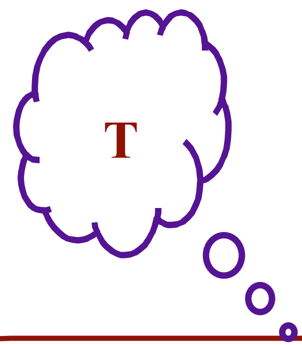
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

2nd simulation cycle






```
callAcl2["(mv-let (erp val state)
         (defthm thm3-neg
            (implies (and (integerp n)(< 0 n))
                     (not (equal (* 2 n) n))))
         (declare (ignore val))
         (if erp (value nil)
            (value T)))"]
```

Reduction of the execution tree

```
Constraints:
    a=3n, b=n, n∈𝒩*

P1: process begin
     wait until clk='1';
     if RST='1' then
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     elsif a0=b0 then
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           res<=a0;
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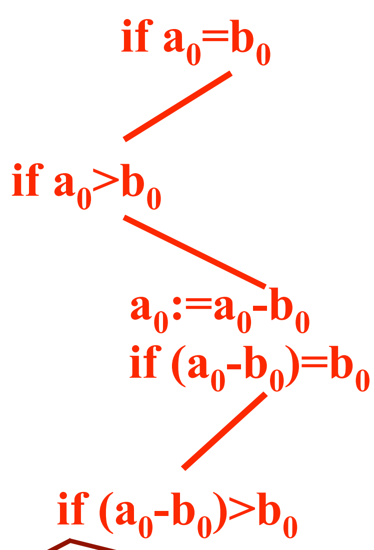
     else b0:=b0-a0;
     end if;
end process P1;
```

2nd simulation cycle




```
callAcl2["(mv-let (erp val state)
          (defthm thm4
             (implies (and (integerp n)(< 0 n))
                      (> (* 2 n) n)))
          (declare (ignore val))
          (if erp (value nil)
             (value T)))"]
```

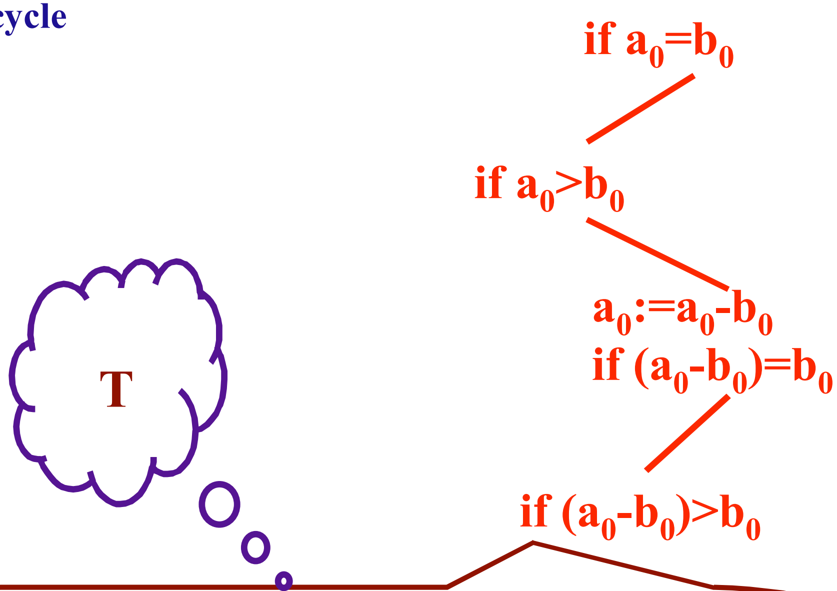
Reduction of the execution tree

Constraints:

a=3n, b=n, n∈$\mathcal{N}$*

```
P1: process begin
    wait until clk='1';
    if RST='1' then
        a0:=a;
        b0:=b;
        ok<=False;
    elsif a0=b0 then
            ok<=True;
            res<=a0;
    elsif a0>b0 then
            a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

2nd simulation cycle




```
callAcl2["(mv-let (erp val state)
          (defthm thm4
             (implies (and (integerp n)(< 0 n))
                        (> (* 2 n) n)))
          (declare (ignore val))
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```

Reduction of the execution tree

```
Constraints:
    a=3n, b=n, n∈𝒩*

P1: process begin
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       ok<=False;
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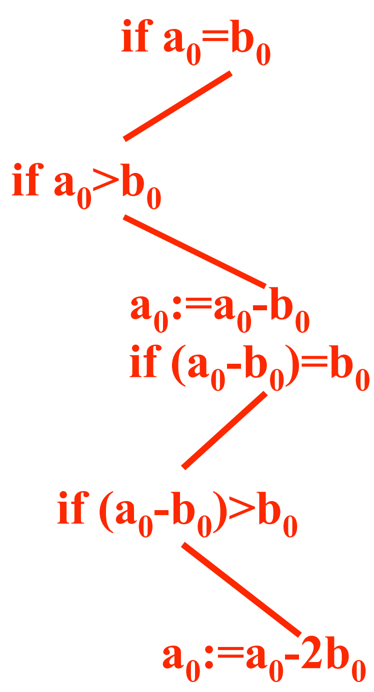
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

2nd simulation cycle



Reduction of the execution tree

***Constraints*:**

a=3n, b=n, n∈$\mathcal{N}^*$

```
P1: process begin
    wait until clk='1';
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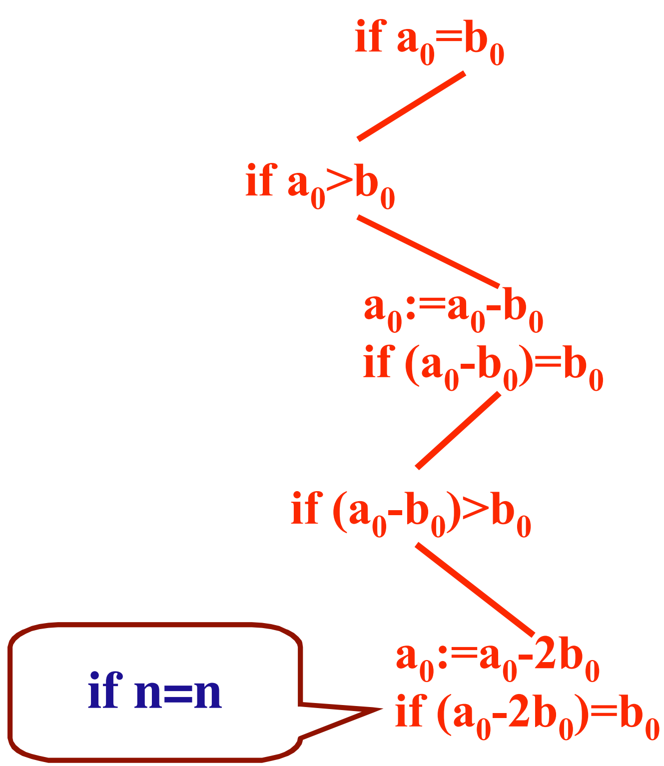
    elsif a0=b0 then
          ok<=True;
          res<=a0;
    elsif a0>b0 then
          a0:=a0-b0;
    else b0:=b0-a0;
    end if;
end process P1;
```

3rd simulation cycle

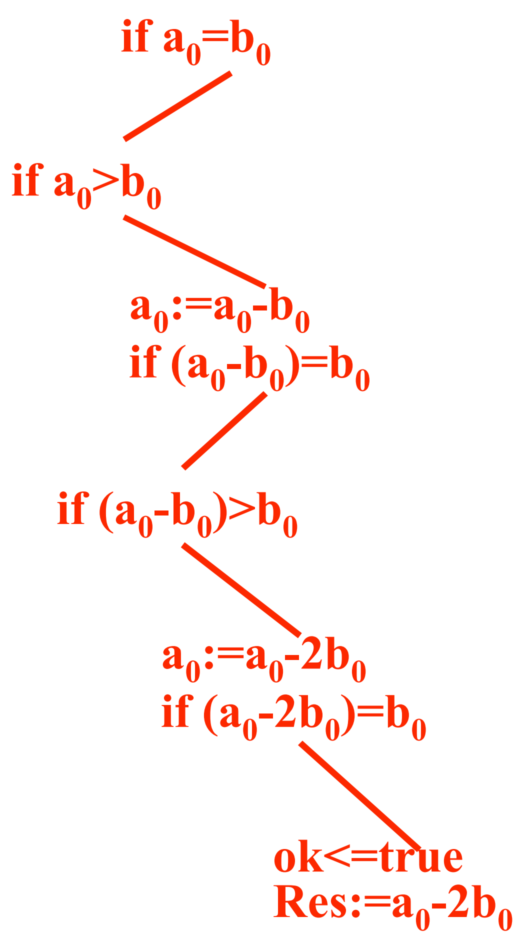


Reduction of the execution tree

```
Constraints:
    a=3n, b=n, n∈𝒩*

P1: process begin
     wait until clk='1';
     if RST='1' then
        a0:=a;
        b0:=b;
        ok<=False;
     elsif a0=b0 then
           ok<=True;
           res<=a0;
     elsif a0>b0 then
           a0:=a0-b0;
     else b0:=b0-a0;
     end if;
end process P1;
```

3rd simulation cycle



Symbolic evaluations of assertions

- the assert statement assures that *bool_expr* is never violated
- *Label1* is translated into a variable that can remain symbolic during one or more simulation cycles
- If it evaluates to false at cycle C, the simulation path is a counter example

VHDL	Mathematica *if* function
Label1: assert bool_expr	If[bool_expr
report “message”	,ChangeVar[Label1,True]
severity severitylevel;	,ChangeVar[Label1,False]
	,If[CallACL2[bool_expr]
	,ChangeVar[Label1,True]
	,ChangeVar[Label1,False]
	,Label1]]

Conclusion

- A new approach for the symbolic simulation of high level circuits specifications
- Use of typing information and user constraints to prune the execution tree
- Use of two powerful automatic systems: Mathematica and ACL2

Future works:

- Validate the approach on industrial circuits
- Extend to new VHDL subset for the system-level synthesis and SystemC

Thank you

Foreseen VHDL subset

- Level 1 synthesizable subset
  - Concurrent statements:
    - Signal assignment
    - Component instantiation
    - Processes : single clock synchronized processes
  - Sequential statements:
    - Variable and signal assignments
    - *if-then-else*, *case* conditionals, *for-loop* statements
  - Types
    - Scalar data types: integer, bit, boolean, character
    - Subtypes defined on integer, bit vector type
  - Hierarchy: components do not contain combinatorial processes

Simulation cycle=clock cycle