Development of a Verified, Efficient Checker for SAT Proofs

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OVERVIEW

Boolean Satisfiability (SAT) solvers are proliferating and useful.

PROBLEM: How can we trust their claims of unsatisfiability?

SOLUTION:

- SAT Solver emits a proof, p_0
- DRAT-trim (from Marijn Heule) processes p_0, creating smaller proof p_1 that includes hints
- Verified ACL2 program checks p_1

This talk is high-level, avoiding details such as “RAT” and “DRAT”.

OUTLINE

Outline

THE PROBLEM

Boolean Satisfiability (SAT) solvers are proliferating and useful.

- They verify unsatisfiability of a Boolean *formula*, represented as a list of *clauses* (each a disjunction of *literals*).
- Example of unsatisfiable formula:

```
(
 (1 2 -3) ; 1 OR 2 OR (not 3)
 (-1)     ; (not 1)
 (-2 -3)  ; (not 2) OR (not 3)
 (3)      ; 3
)
```

But how can we *trust* SAT solvers?

Outline

Towards a Solution (1)

Modern SAT solvers [2] emit *proofs*!

- Proof step: **Add a clause** to the formula (conjunction of clauses) that **preserves satisfiability**.
  - Eventually add the empty clause.
  - So final formula is unsatisfiable.
  - So input formula must be unsatisfiable!
- Also legal: proof steps that **delete a clause** from the formula.
  - Clearly preserves satisfiability.

TOWARDS A SOLUTION (2)

But how do we know that these “proofs” are valid?

We check them with software programs called *checkers*!

But how do we know that a checker is *sound*? Inspection?

- Key property: clause addition preserves satisfiability
- Checkers (e.g., DRAT-trim) are typically simpler than solvers...
  - ... but not *that* simple, and *inspection is error-prone*.

TOWARDS A SOLUTION (3)

Wetzler proved soundness of an ACL2-based solution [6, 5, 4].
I'll explain our "**[lrat-4]**" and "**[lrat-5]**" versions of soundness:

```
(implies (and (formula-p formula)
              (refutation-p$ proof formula))
         (not (satisfiable formula)))
```

```
(let ((formula
       (mv-nth 1 (proved-formula cnf-file clrat-file
                                 chunk-size debug
                                 nil ; incomplete-okp
                                 ctx state))))
  (implies formula
           (not (satisfiable formula))))
```

; Print proved formula, to `diff` *against input formula:*

```
(defmacro print-formula (formula &optional filename)
  ...)
```

Towards a Solution (4)

Problem: Efficiency.

On one example:

- DRAT-trim: 1.5 seconds
- Verified checker [5]: ~ 1 week

NOTE:

- Wetzler's ITP 2013 checker [5] was intended to be a proof of concept, not an efficient tool.
- He did some preliminary work towards increasing efficiency (no timings reported).

OUTLINE

A Sequence of Checkers (1)

1. **[rat]** Nathan's ITP 2013 RAT checker [5]: no deletion
2. **[drat]** Support deletion (thus implementing DRAT)
3. **[lrat-1]** Avoid search and delete clauses efficiently, using *fast-alists* (applicative hash tables) and a *linear* proof format, and with soundness proved from scratch
4. **[lrat-2]** Shrink fast-alists to keep formulas small
5. **[lrat-3]** Minor tweak to formula data-structure
6. **[lrat-4]** Use **stobjs** for assignments
7. **[lrat-5]** Support incremental file reading using improved `read-file-into-string`; verify improved soundness theorem

A SEQUENCE OF CHECKERS (2)

This table shows times (in seconds) for some checker runs (including parsing), on examples provided by Marijn Heule. Test “R_4_4_18” is the one that took a week with Wetzler’s ITP 2013 checker.

benchmark	[lrat-1] *(fast-alist)*	[lrat-3] *(shrink)*	[lrat-4] *(stobjs)*	[lrat-5] *(incremental)*
uuf-100-3	0.09	0.03	0.05	0.01
tph6[-dd]	3.08	0.57	0.33	0.33
R_4_4_18	164.74	5.13	2.23	2.24
transform	25.63	6.16	5.81	5.82
Schur_161_5_d43	5341.69	2355.26	840.04	259.82

NOTE: For the last (Schur) example: 4.3 minutes for checker adds little to the DRAT-trim time of 20 minutes.

A SEQUENCE OF CHECKERS (3)

This project illustrates the interplay between ACL2 as a programming language and as a theorem prover:

- Optimize the program for efficiency.
- Deal with proving correctness for the optimizations.

Profiling was very useful.

Plan: Our **[lrat-5]** checker will be used in the 2017 SAT competition.

Time comparison on a set of examples (courtesy of Marijn Heule and J Moore):

```
DRAT-trim         210223 seconds
[lrat-5] checker   20811 seconds
```

Outline

RELATED WORK

- [1] The *Linear RAT* (LRAT) proof format and its use in our ACL2 checker, as well as a corresponding Coq-based checker (which takes 10 minutes on one example compared to our 9 seconds)
- [3] An Isabelle development using a refinement framework that (independently of our work) produces an efficient verified checker

Outline

Conclusion

There is now an efficient formally verified SAT checker!

- On a large example, its time of 4.3 minutes (including parsing) adds relatively little to the DRAT-trim time of 20 minutes.

These checkers are available in the community books under `books/projects/sat/lrat/`:

[rat] `projects/sat/proof-checker-itp13/`
[drat] `projects/sat/lrat/early/drat/`
[lrat-1] `projects/sat/lrat/early/rev1/`
[lrat-2] `projects/sat/lrat/early/rev2/`
[lrat-3] `projects/sat/lrat/list-based/`
[lrat-4] `projects/sat/lrat/stobj-based/`
[lrat-5] `projects/sat/lrat/incremental/`

Outline

References

A much more detailed (but somewhat outdated – no mention of [lrat-5]) version of this talk is available on the ACL2 seminar website.

A preprint of a paper on this work (with Heule, Hunt, and Wetzler) is at:
`http://www.cs.utexas.edu/users/kaufmann/papers/lrat-preprint/index.html`.

The final slide has references for citations in this talk.

Thank you for your attention!

[1] Luís Cruz-Filipe, Marijn Heule, Warren Hunt, Matt Kaufmann, and Peter Schneider-Kamp. Efficient certified RAT verification. In *CADE 2017*. To appear.

[2] Marijn Heule, Warren A. Hunt Jr., and Nathan Wetzler. Verifying refutations with extended resolution. In Maria Paola Bonacina, editor, *Automated Deduction - CADE-24 - 24th International Conference on Automated Deduction, Lake Placid, NY, USA, June 9-14, 2013. Proceedings*, volume 7898 of *LNCS*, pages 345–359. Springer, 2013.

[3] Peter Lammich. Efficient verified (UN)SAT certificate checking. In *CADE 2017*. To appear, 2017.

[4] Nathan Wetzler. Supplemental material for a paper appearing in interactive theorem proving 2013 [RAT proof-checker]. https://github.com/acl2/acl2/tree/master/books/projects/sat/proof-checker-itp13/, Accessed: December 2016.

[5] Nathan Wetzler, Marijn J.H. Heule, and Jr. Warren A. Hunt. Mechanical verification of SAT refutations with extended resolution. In *ITP 2013*, volume 7998 of *LNCS*, pages 229–244. Springer, 2013.

[6] Nathan David Wetzler. *Efficient, Mechanically-Verified Validation of Satisfiability Solvers*. PhD thesis, University of Texas at Austin, 2015.