Progress Toward Fast Finite Sets and Maps in ACL2

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Outline

Introduction

Goals

- Want finite sets and maps that are:
  - Fast.
  - Functional and persistent.
  - Verified (no new raw lisp code, trust tags).
  - Logically convenient.

Existing Libraries

- `osets` and `omaps`
  - **Pros**:
    - Functional and persistent.
    - Unique representation means set equivalence is regular equality.
    - Generally doesn't expose internal implementation (users don't have to deal with `<<`).
  - **Cons**:
    - Inefficient.
- What about fast `alists`, `stobj` hash tables, or arrays?
  - Generally limited in usage (e.g. the "fast alist disipline", `stobj` single-threadeness).
  - Overhead of maintaining logical twin to raw lisp component.
  - May require global object or name.
- What about `bitsets`?
  - Limited to small sets of natural numbers.

Attempt #1: Ordered Sets with Treaps

Binary Search Trees

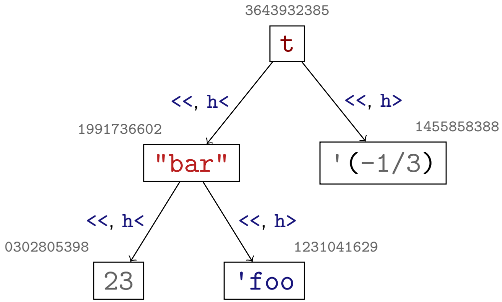
- What about binary search trees (BSTs)?
  - A natural evolution from `osets`.
  - **Challenge**: achieving a unique representation (without degrading performance).
    - Self-balancing BSTs (e.g. red-black trees, AVL trees) are generally sensitive to the order of insertion/deletion.

Treaps

- A treap (= "tree" + "heap") is a BST with an additional max heap property.
- If the BST and heap orders are total, the tree must have a unique representation.
- If the BST and heap orders are generally uncorrelated, the tree will be practically balanced.
- The BST order can use `<<`. The heap order can use a new order, `h<`, based on hash values.

Hash Function

- Based on `check-sum-obj`.
- Implementations: FNV-1a, Jenkins one-at-a-time.
- Likely could be optimized much further
- `h<` compares two objects' hash values. If they are the same, it falls back to `<<`.



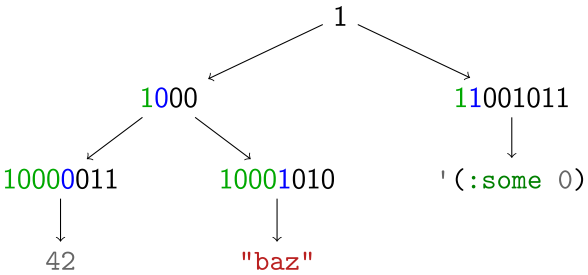
- Implemented in `books/kestrel/treeset`.
- **But wait!** There are faster data structures!

Attempt #2: Unordered Sets with Little-Endian Patricia Trees

Patricia Trees

- Hash values are 32-bit unsigned `fixnum`s.
  - Natural numbers may be viewed as bit strings. Therefore, we can consider the use of tries.
- Patricia trees (AKA binary radix trees), are a compressed form of trie.
  - They can be very fast on fixed-size integers due to "bit-twiddling" tricks (see Okasaki and Gill, *"Fast Mergeable Integer Maps"*).
  - The non-leaf structure is naturally unique.
  - No rebalancing or tree rotations necessary.
- Hash array mapped tries (HAMTs) are faster, but require arrays.

Patricia Tree Example



- Example with 8-bit hashes; previous prefix in green, branching bit in blue.
- Leaf buckets are implemented as `osets`.
- Verification is a WIP.

Conclusion

Initial Experiments

Tests on sets of size 100,000, run 10,000 times (on SBCL).

Random Elements

	osets	treaps	patricia trees
Membership	1.92	0.02	0.03
Insertion	6.80	0.40	0.03
Deletion	25.11	0.07	0.03

Consecutive Naturals

	osets	treaps	patricia trees
Membership	1.92	0.02	0.02
Insertion	12.04	0.07	0.02
Deletion	16.52	0.02	0.03