Verifying Quantum Fourier Transform Using Bit-Vector Abstractions

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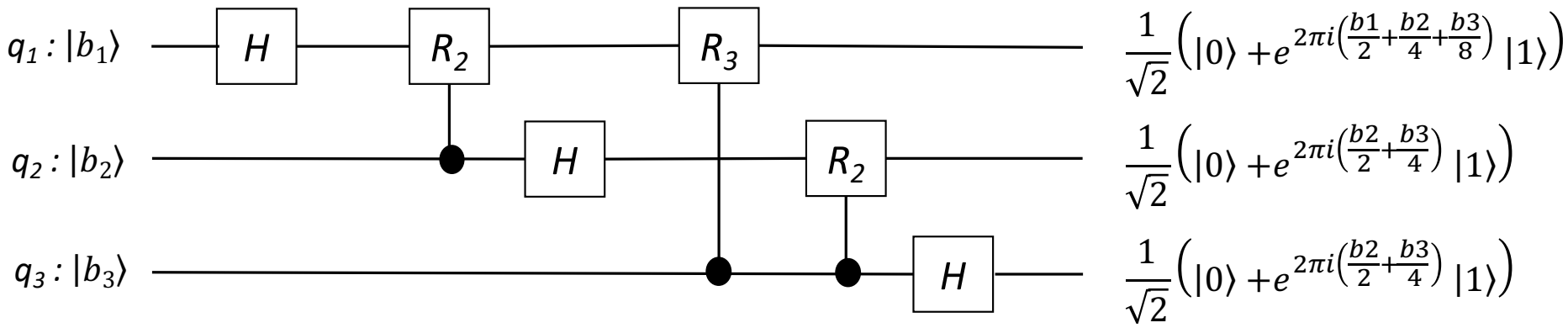
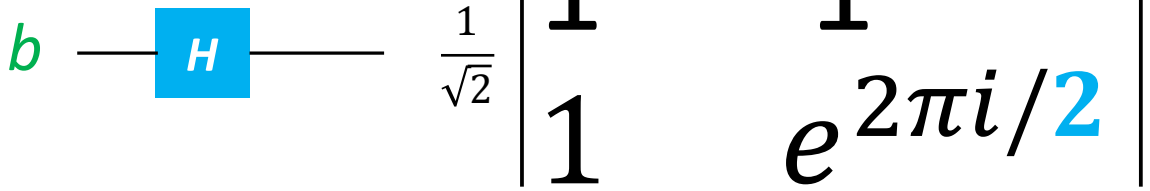
3-Qubit Quantum Fourier Transform (QFT)

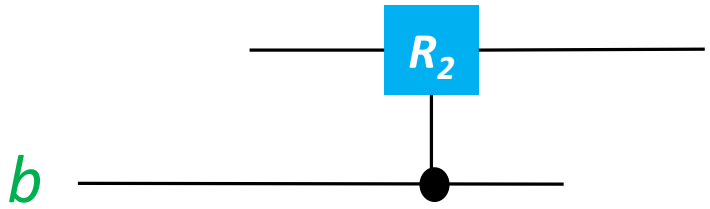


Rotational Abstraction: Hadamard Gate



Abs: 0.*b*

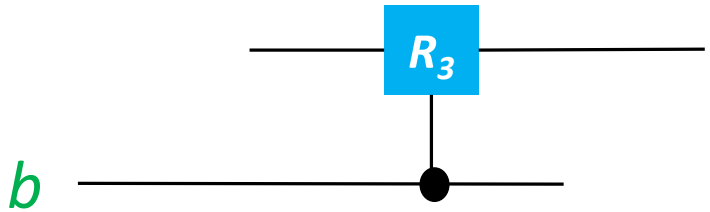
Rotational Abstraction: Control Rotation Gate



$$\begin{vmatrix} 1 & 0 \\ 0 & e^{2\pi i/4} \end{vmatrix}$$

Abs: 0.0*b*

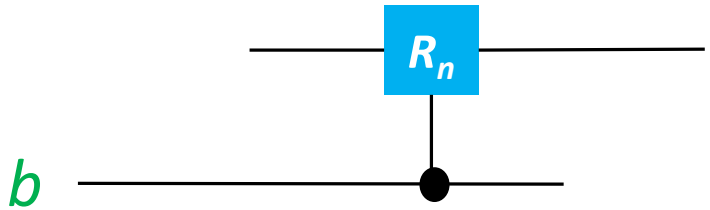
Rotational Abstraction: Control Rotation Gate



$$\begin{vmatrix} 1 & 0 \\ 0 & e^{2\pi i/8} \end{vmatrix}$$

Abs: 0.00*b*

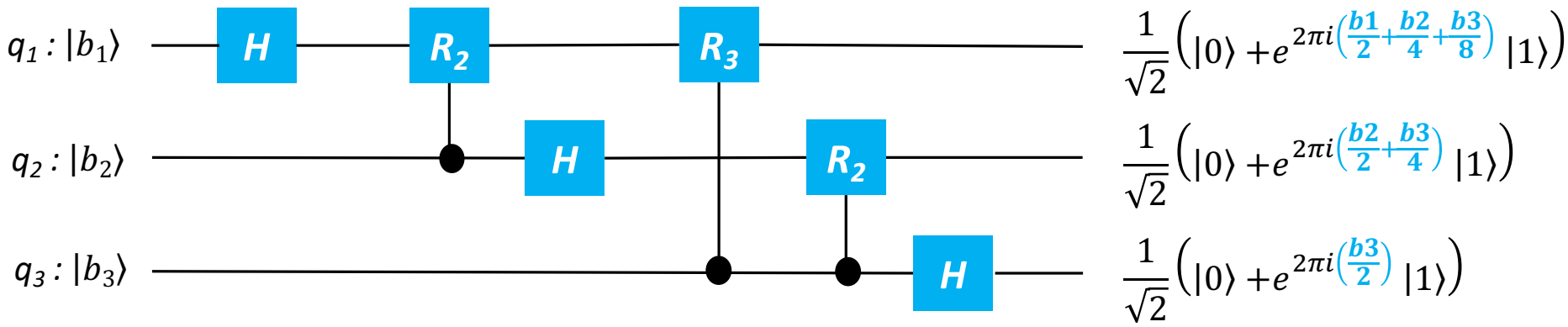
Rotational Abstraction: Control Rotation Gate



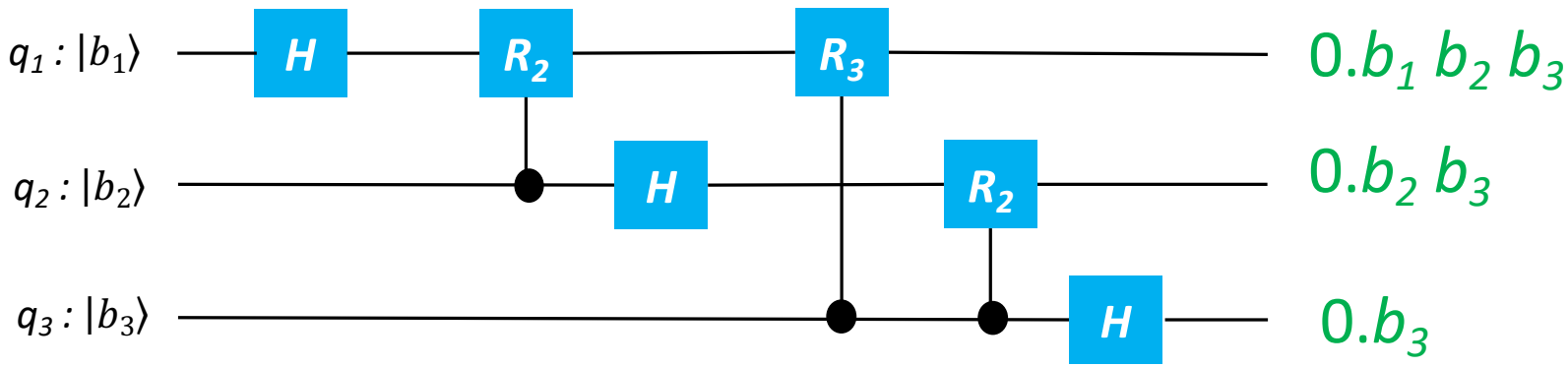
$$\begin{vmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^n} \end{vmatrix}$$

Abs: 0.0…0*b*

Rotational Abstraction: QFT



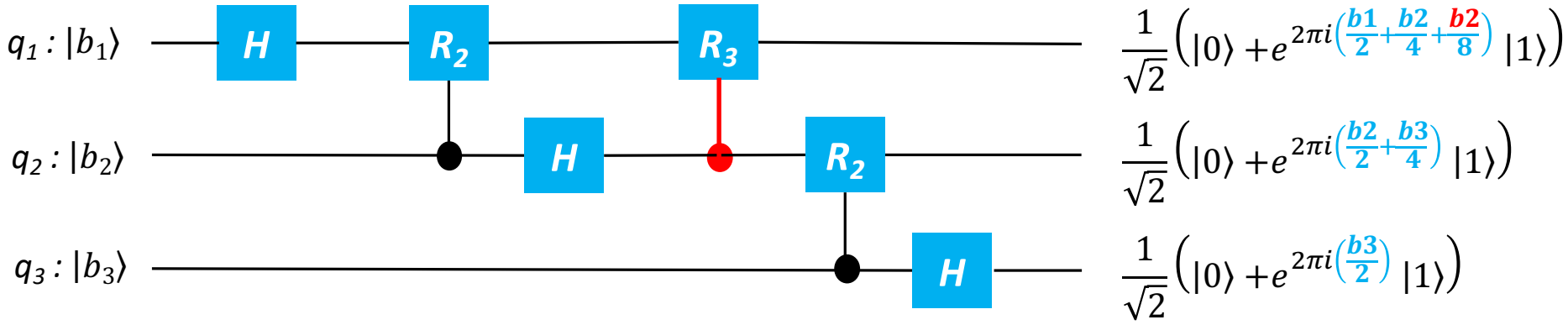
Rotational Abstraction: QFT

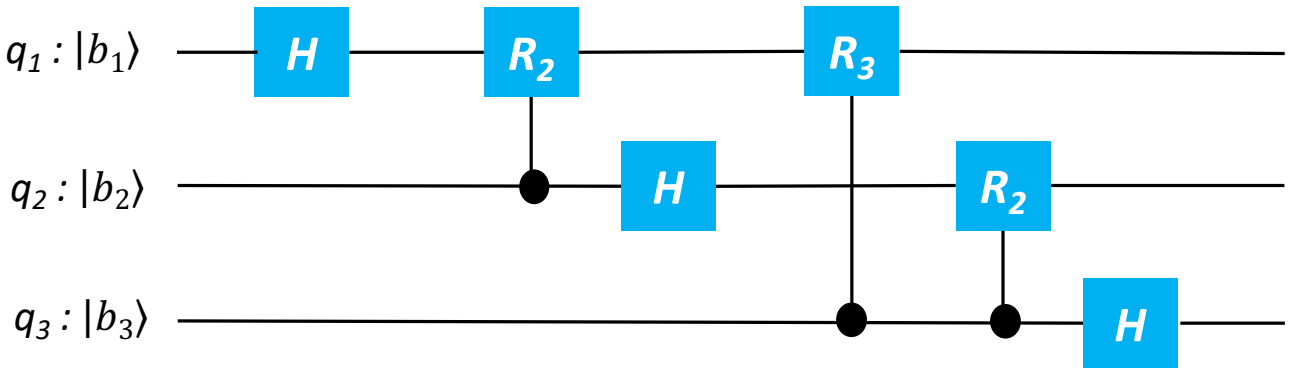


QFT Verification Results

	Verification Time [seconds]		
#Qubits	Correct Circuit	Control Gate Error	Incorrect Gate Error
128	0.04s	0.02s	0.04s
256	0.19s	0.08s	0.06s
512	0.26s	0.2s	0.2s
1,024	1.37s	1.29s	0.92s
2,048	9.85s	9.47s	5.87s
4,096	95.75s	79.68s	53.57s
8,192	1109.0s	639.57s	643.9s
10,000	**2379.88s**	**1524.65s**	**1568.79s**

Gate Control Error

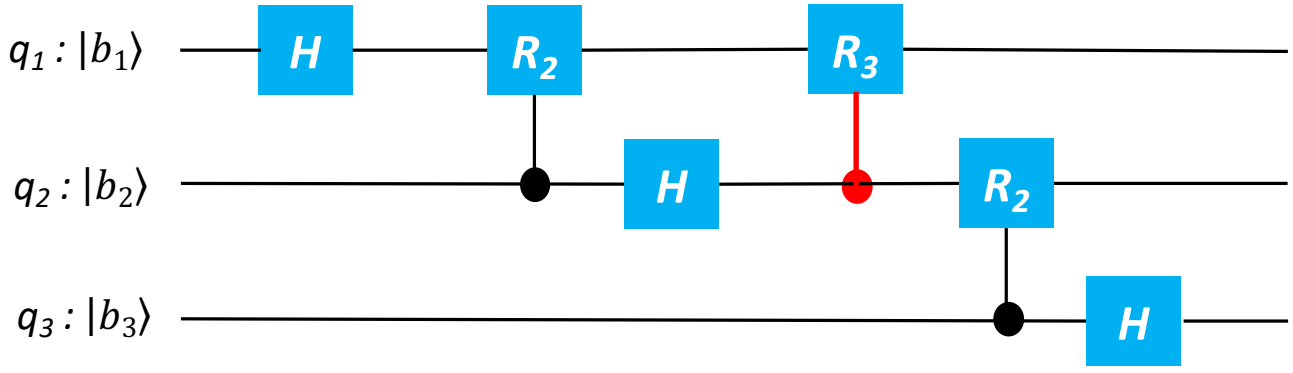




$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b1}{2}+\frac{b2}{4}+\frac{b3}{8}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b2}{2}+\frac{b3}{4}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b3}{2}\right)}|1\rangle\right)$$

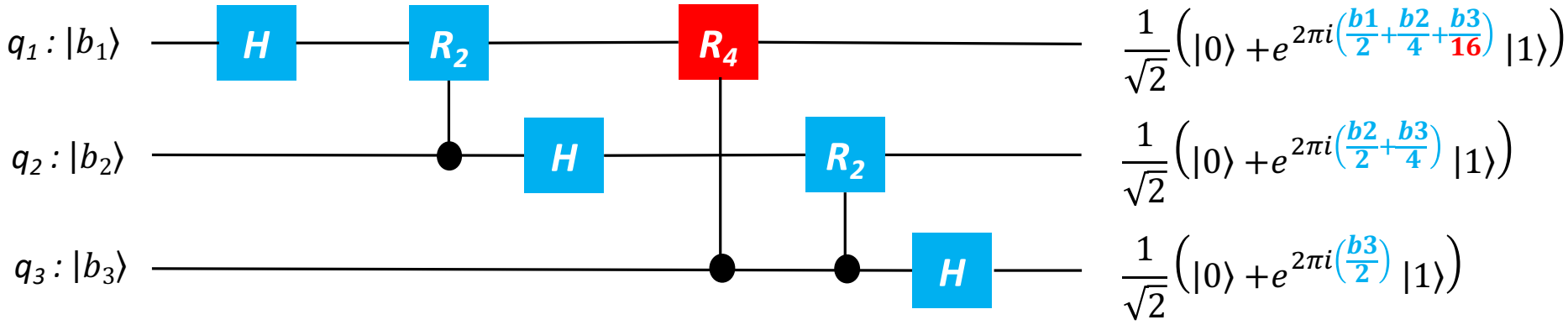


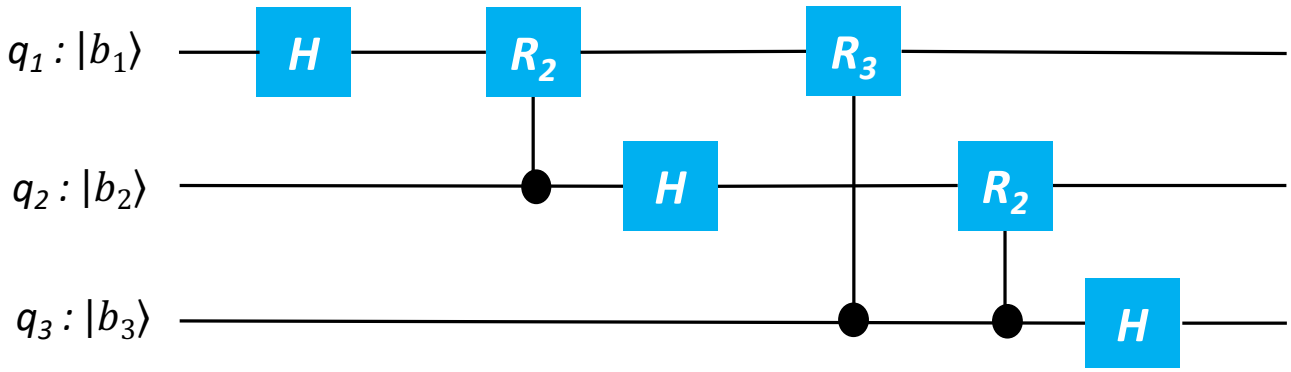
$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b1}{2}+\frac{b2}{4}+\frac{b2}{8}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b2}{2}+\frac{b3}{4}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b3}{2}\right)}|1\rangle\right)$$

Incorrect Gate Error

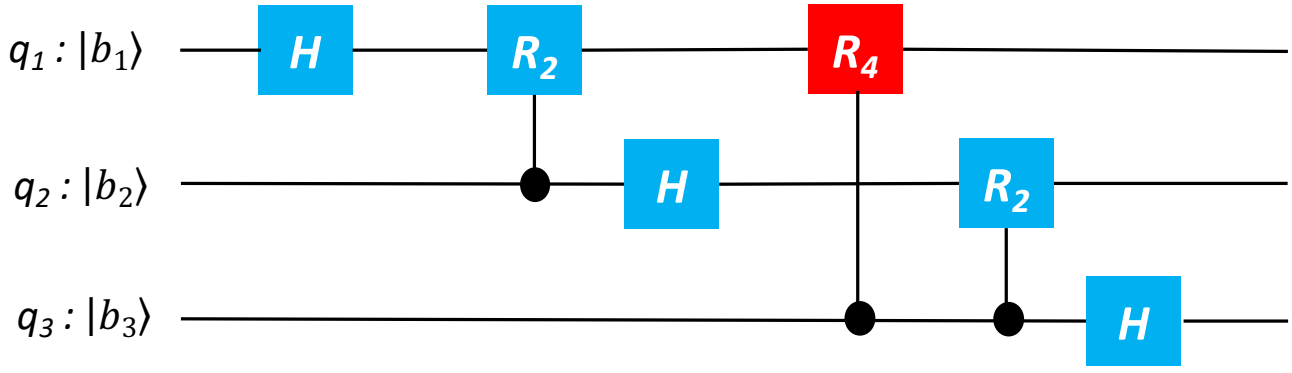




$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b1}{2}+\frac{b2}{4}+\frac{b3}{8}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b2}{2}+\frac{b3}{4}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b3}{2}\right)}|1\rangle\right)$$



$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b1}{2}+\frac{b2}{4}+\frac{b3}{16}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b2}{2}+\frac{b3}{4}\right)}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{b3}{2}\right)}|1\rangle\right)$$

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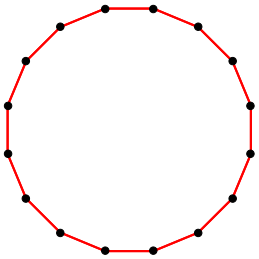
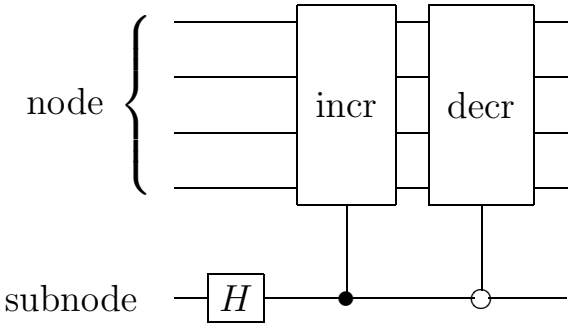
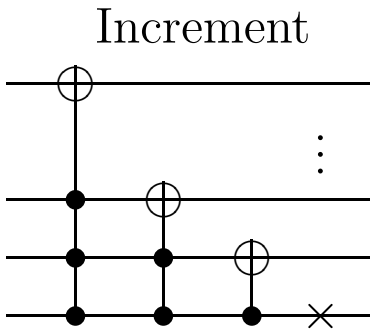
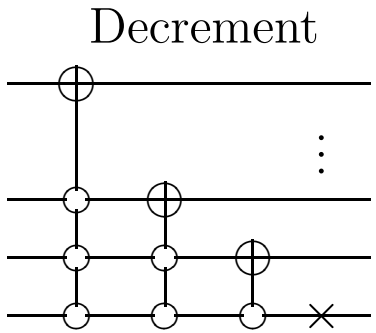
Quantum Walks

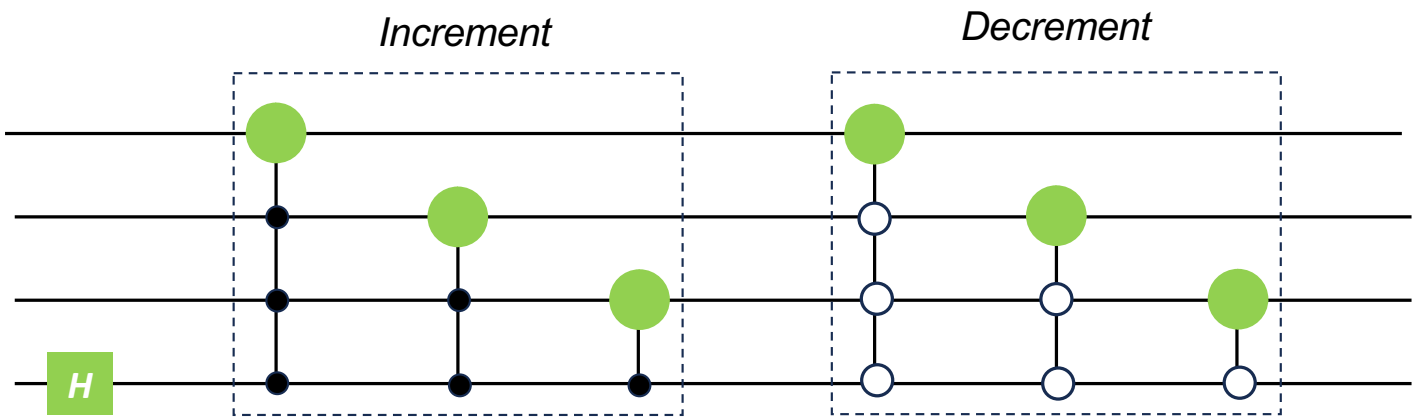








Superposition Abstraction



Quantum Walk Verification Results

	Verification Time [seconds]		
#Qubits	Correct Circuit	Control Gate Error	Incorrect Gate Error
128	0.41s	0.25s	0.19s
256	1.48s	0.86s	0.56s
512	4.26s	2.97s	2.10s
1,024	38.49s	10.18	36.84s
2,048	1,186.69s	40.81s	213.31s
4,096	10,486.36s	180.04s	320.45s
5,000	17,184.68**s**	286.20**s**	2,099.02s

What We Propose To Do

Verified Generators Using ACL2

1. Unify Rotational Abstraction and Superposition Abstraction
2. Analyze commonly used circuits for other abstractions.
3. Develop Verified Generators in ACL2