### 16.03HRS. 18 JULY 1973.

## [THEOREMS PROVED]

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[T 1 1] [ 16.03 18 JULY 1973]
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THEOREM TO BE PROVED:
[EQUAL [APPEND A [APPEND B C]] [APPEVD [APPENO A B] C]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND
[LQUAL [APPEND NIL [APPEND B C]] [APPEND [APPEND NIL B] C]]
[IMPLIES
[EQUAL [APPEND A [APPEND B C]] [APPEND [APPEND A B] C]]
[FQUAL [APPEND [CONS A1 A] [APPEND B C]] [APPEND [APPEND [CONS A1 A] B] C]I]]

WHICH IS EQUIVALENT TO:

I

FUNCTION DEFINITIONS:
[APPEND [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND $X$ [COND $Y$ T VIL] T]]]
[AND [LAMBOA [X Y] [COND X [COND Y T NIL] NIL]]]

## PRUFILE: [/[A], /ENR/ENR:]

TIME: 4.813 SECS.

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[T 1 2] [ 16.03 18 JULY 1973]
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THEOREM TO BE PROVED:
[IMPLIES [EQUAL [APPFND A B] [APPEND A C]] [EQUAL B C]]
WHICH IS EOUIVALENT TO:
[COND [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C] T]
MUST TRY INDUCTION.
INDUCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [COND [EQUAL [APPEND VIL B] [APPEND NIL C]] [EQUAL B C] T]
    [IMPLIES
        [COND [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C] T]
        [COND [EQUAL [APPEND [CONS A1 A] 3] [APPEND [CONS A1 A] C]] [EQUAL B C] T]]]
```

WHICH IS EQUIVALENT TO:
1

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
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profile: [/ENR/ENR[A], /ENR/ENR.]

TIME: 4.188 SECS.

```
[T 1 3] [ 16.04 18 JULY 1973]
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IHEOREM TO RE PROVED:
[EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]

MUST TRY INDUCTION。

INDUCT On B.

```
IHE THEOREM TO BE PROVED IS NOW:
[AND
    [EQUAL [LENGTH [APPEND A NIL]] [LENGTH [APPEND NIL A]]]
    [lMPLIES
        [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]
        [EQUAL [LENGTH [APPEND A [CONS B1 B]]] [LENGTH [APPEND [CONS B1 B] A]]]]]
WHICH IS EQUIVALENT TO:
[COND
    [tgUAL [LENGTH [APPEND A NIL]] [LENGTH A]]
    [COND [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]
    . [EQUAL [LENGTH [APPEVD A [CONS B1 3]]] [CONS NIL [LENGTH [APPEND B A]]]]
    - T]
    NIL]
```

```
THE THEOREM TO BE PROVED IS NOW:
[CUND
    [EOUAL [LENGTH [APPEND A NIL]] [LENGTH A]]
    [COND [EQUAL [LENGTH [APPEND A [CONS B1 3]]] [CONS NIL [LENGTH [APPEND A B]]]]
    . T
    - [*1]]
NIL]
```

(WURK ON ḞIRST CONJUNCT OVLY)

MUST TRY INDUCTION。

INDUCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
LCOND
    [AND [EQUAL [LENGTH [APPEND NIL NIL]] [LENGTH VIL]]
    . [IMPLIES [EQUAL [LENGTH [APPEND A NIL]] [LENGTH A]]
    [ [EQUAL [LENGTH [APPEND [CONS A1 A] NIL]] [LENGTH [CONS A1 A]]]]]
    [COND
    . [EQUAL [LENGTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]]
    - T
    - [*1]]
NILJ
```


## WHICH IS EQUIVALENT TO:

[cund
[EQUAL [LENGTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]] T [*1]]

MUST TRY INDUCTION.

INDICT OV A?.

```
THE THEOREM TO bE PROVED IS NOW:
[AND
    [COND
    - [EQUAL [l-ENGTH [APPEND NIL [CONS 31 B]]] [CONS NIL [LENGTH [APPEND NIL B]]]]
    - T
    -[*1]]
    [IMPLIES
        [COND
        - [Equal [lENgTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]]
        - T
        [*1]]
        [COND [EQUAL [LENGTH [APPEND [CONS A21 A2] [CONS B1 R]]]
                        r.CONS NIL [LENGTH [APPEND [CONS A21 A2] B]]]]
                T
                [*1]]]]
```

WHICH IS EQUIVALENT TO:
I
FUNCTION DEFINITIONS:
[APPEND [LAMBDA [X Y] [COND $X$ [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[LENGTH [LAMBDA [X] [COND X [CONS NIL [LEVGTH [CDR X]]] 0]]]
[IMPLIES [LAMBDA [X Y] [COND $X$ [COND $Y$ T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

## FERTILIZERS:

*1 = [COND [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]] NIL T]

PROFILE: [/ [B], /ENR/ENRX, /\& [A], /ENR/ENR/ENR[A2], / E.NR/ENR.]

TIME: 16.12 SECS.

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[T 1. 4] [ 16.04 18 JULY 1973]
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IHEOREM TO BE PROVED:
[EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]

MUST TRY INDUCTION.

INDUCT ON A.

IHE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [REVERSE [APPEND NIL 3]] [APPEND [REVERSE R] [REVERSE NIL]]] [IMPLIES [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]] [EQUAL [REVERSE [APPEND [COVS A1 A] R]]
[APPEND [REVERSE B] [REVERSE [CONS A1 A]]]]]]

WHICH IS EQUIVALENT TO:
LCUND [EQUAL [REVERSE B] [APPEND [REVERSE B] NIL]] [COND [EQUAL [RFVERSE [APPEVD A B]] [APPEND [REVERSE B] [REVERSE A]]] [EQUAL [APPEND [REVERSE [APPEVD A B]] [CONS A1 NIL]] [APPEND [REVERSE B] [AJPEND [REVERSE A] [CONS A1 NIL]]]] T] NILJ

FERTILIZE WITH [EQUAL [REVERSE [APPEVD A 3]] [APPEND [REVERSE B] [REVERSE A]]].

```
[COND [EQUAL [REVERSE B] [APPEND [REVERSE B] NIL]]
    [COND [EQUAL [APPEND [APPEND [REVERSE R] [REVERSE A]] [CONS A1 NIL]]
                        [APPEND [REVERSE 3] [APPEND [REVERSE A] [CONS A1. NIL]]]]
                T
        [\because1]]
    NIL]
```

(WORK UN FIRST CONJUNCT ONLY)
generalize common sueteris by replacing [zeverse b] by genrli.
THE GENERALIZED TERM IS:
[EGUAL GENRLI [APPEND GENRL1 NIL]]

MUST TRY INDUCTION.

IADuct on genrli.

IHE THEOREM TO BE PROVED IS NOW:
LCOMD
〔Av]

- [EQUAL NIL [APPEND NiL NiL]]
- [IMPLIES [EQUAL GENRL1 [APPEND GENRLI NIL]]
- [EQUAL [CONS GENRL11 GEVRL1] [APPEND [CONS GENRL11 GENRL1] NIL]J]]
[COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]
. [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]]
- T
- [*1] ]

NHJ

```
WHICH IS EQUIVALENT TO:
[COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]
    [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]]
    T
    [*1]]
```

```
GENERALI7E COMMON SUETERMS BY REPLACING [REVERSE A] BY GENRL2 AND [REVERSE B] BY
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```
    GENRLS.
THE GENERALIZED TERM IS:
[CONij [EOUAL [APPEND [APPEND GENRL3 GENRL2] [CONS A1 NIL]]
                        [APPEND GENRL3 [APPEVD GENRL'z [CONS A1 NIL]]]]
    T
    [*1]]
```

MUST Tiry IndUCTION。

INDUCT ON GENRLZ.

THE THEOREM TO bE PROVED IS NOW:

```
[AND
    [COND [EQUAL [APPEND [APPEVO NIL GENRL2] [CONS A1 NIL]]
    . [APPENO VIL [APPEND GENRL2 [CONS A1 NIL]]]]
- T
- [#1]]
[IMPLIES
[COND [EQUAL [APPEND [APPEND GENRL3 GENRL2] [CONS A1 NIL]]
                                    [APPEND GENRL3 [APPEND GENRL2 [CONS A1 NIL]J.]]
```

```
                                    [T 1 4]
[%1]] [COND [EQUAL [APPEND [APPEND [CONS GENRL31 GENRL3] GENRL2] [CONS A1 NIL]]
T
[#1]]]]
```


## WHICH IS EQUIVALENT TO:

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\(T\)
FUNCTION DEFINITIONS:
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```
[APPENU [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
```

```
[APPENU [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
```


## [REVERSE

```
[LAMBDA [X] [COVD \(X[A P P E N D[R E V E R S E[C D R X][C O N S[C A R X] N I L]] N I L]]]\)
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA \(Y X Y\) [ Y [COND \(X\) [COND Y \(T\) NIL] NIL]]]
FERTILIZERS:
```

```
*1. = [COND [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]
```

*1. = [COND [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]
NIL
NIL
T]

```
    T]
```

GENFRALIZATIONS:

```
GENRL1 = [REVERSE B]
```

GENRL3 3 [REVERSE B]
GENRL2 $=$ [REVERSE A]

PRUFILE: [/ [A], /ENR/ENRX, / \& G[GENRL1], / ENR/ENR/ENR G [GENRL3], / ENR/ENR.]

## TME: 21.38 SECS.

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[T 1 5]
[ 16,05 18 JULY 1973]
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THEDREM TO bE PROVED:
[EQJAL [LENGTH [REVERSE D]] [LENGTH D]]

MUST TRY INDUCTION.

INDUCT ON D.

The. Theorem to be proved is now:
[AND [EQUAL [LENGTH [REVERSE NIL]] [LENGTH NIL]] [IMPLIES [EQUAL [LENGTH [REVERSE D]] [LENGTH D]] [EQUAL [LENGTH [REVERSE [COVS D1 D]]] [LENGTH [CONS D1 D]J]]]

WHICH IS EQUIVALENT TO:
[CUND [EQUAL [LENGTH [REVERSE DJ] [LENGTH D]] [EQUAL [LENGTH [APPEND [REVERSE D] [CONS DI NIL]]] [CONS NIL [LENGTH D]]]「]

FERTilize with [EQUAL [LENGTH [REVERSE D]] [LENGTH D]].

IHE THEOREM TO BE PROVED IS NOW:
[CUND [EQUAL [LENGTH [APPEND [REVERSE D] [CONS DI NIL]]] [CONS Nil [LENGTH [REVERSE D]J]]
generalize common subterms by replacing [zeverse d] by genrli.

THE GENERALIZED TERM IS:
[COND [EQUAL [LENGTH [APPEND GENRL1 [CONS D1 NIL]]] [CONS NIL [LENGTH GENRL1]]] T [*1]]
must try induction.

INDUCT ON GENRLI.

```
THE THEOREM TO bE PROVED IS NOW:
[AND
    [COND [EQUAL [LENGTH [APPEND NIL [CONS DI NIL]]] [CONS NIL [LENGTH NIL]]]
    . T
    - [*1]]
    [IMPLIES
        [cund
    - [EQUAL [LENGTH [APPEND GENRL1 [CONS D1 NIL]]] [CONS NIL [LENGTH GENRL1]]]
    - T
    -[*1]]
    [CUND [EQUAL [LENGTH [APPEND [CONS GENRL11 GENRL1] [CONS D1 NIL]]]
                                    [CONS NIL [LENGTH [CONS GEVRL11 GENRL1]]]]
            T
            [*1]]]]
```

FUNCTIUN DEFINITIONS:
LREVERSE
[LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]
[LENGTH [LAMBDA [X] [COND $X$ [CONS NIL [LENGTH [CDR X]]] 0]]]
[APPEND [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA 「X Y] [COND $X$ [COND $Y$ T NIL] NIL]]]

FERTILIZERS:
*1 = [COND [EQUAL [LENGTH [REVERSE D]] [LENGTH D]] NIL T]

GENERALIZATIONS:
GENRL1 $=$ [REVERSE D]

PROFILE: [/ [D], /ENR/ENRX, /G[GENRLI],/ENR/ENR, ]

TIME: 9.438 SECS.

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[T 1. 6] [. 16.05 18 JULY 1973]
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TitEOREM TO BE PROVED:
[EQUAL [REVERSE [REVFRSE A]] A]

MUST TRY INDUCTION.

INUUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [REVERSE [REVERSE NIL]] NIL] [IMPLIES [EQUAL [REVERSE [REVERSE A]] A]
[EQUAL [REVERSE [REVERSE [CONS A1 A]]] [CONS A1 A]]J]

```
WHICH IS EQUIVALENT TO:
[COND [EQUAL [REVERSF [REVERSE A]] A]
    [EQUAL [REVERSE [APPEND [REVERSE A] [CONS A1 NIL]]] [CONS A1 A]]
    T]
```

FERTILIZE WITH [EQUAL [REVERSE [REVERSE A]] A].

```
IHE THEOREM TO BE PROVED IS NOW:
[CUND [EQUAL [REVERSE [APPEND [REVERSE A] [CONS A1 NIL]]]
    [CONS A1 [REVERSE [REVERSE A]J]]
    T
```


## [*1]]

```
GENERALIZE COMMON SUBTERMS BY REPLACING [PEVERSE A] BY GENRL1.
```

THE GENERALIZED TERM IS:
LCOND
[EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]
T
[*1]]
must try induction.
INDUCT ON GENRLI.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [CDND [EQUAL [REVERSE [APPEND NIL [CONS A1 NIL]] [CONS A1 [REVERSE NIL]]]
    - T
    [*1]]
    [IMPLIES
        [COND
    - [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]].
    - T
    - [*1]]
    [COND [EQUAL [REVERSE [APPEND [CONS GENRL11 GENRLI] [CONS A1 NIL]]]
                                    [CONS A1 [REVERSE [CONS GEVRL11 GENRL1]]]]
            T
            [*1]]]]
```

```
[COND
    [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 VIL]]] [CONS A1 [REVERSE GENRL1]]]
    [COND
    - [EQUAL [APPEND [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS GENRL11 NIL]]
    - [CONS A1 [APPEND [REVERSE GENRL1] [CONS GENRL11 NIL]]]]
. T
    - [*1]]
    T]
FERTILIZE WITH [EQUAL [REVERSE [APPEVD GEVRL1 [CONS A1 NIL]]]
    [COVS A1 [REVERSE GENRLI]]].
THE THEOREM TO BE PROVED IS NOW:
```

```
[CUND [COND [EQUAL [APPEND [CONS A1 [REVERSE GENRL1]] [CONS GENRL11 NIL]]
```

[CUND [COND [EQUAL [APPEND [CONS A1 [REVERSE GENRL1]] [CONS GENRL11 NIL]]
[CONS A1. [APPEND [REVERSE GENRL1] [CONS GENRL11 NIL]]]]
[CONS A1. [APPEND [REVERSE GENRL1] [CONS GENRL11 NIL]]]]
T
T
[*1]]
[*1]]
|
|
[*2]]

```
    [*2]]
```

WHICH IS EQUIVALENT TO:
1

## FUNCTIUN DEFINITIONS:

cREVERSE
[LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]J]
[APPEN! [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
*1 = [COND [EQUAL [REVERSE [REVERSE A]] A] NIL T]
*2 2 [COND [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]
NIL
TI

GENERALIZATIONS:
GENRL1 $=$ [REVERSE A]

PROFILE: [/ [A], /ENR/ENRX, $\quad$ / [GENRL1], /ENR/ENR/RERR , / ENR.]

TIME: 12.94 SECS.

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[T 1 7]
[ 16.05 18 JULY 1973]
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THEOREI TO BE PROVED:
[IMPLIES A [EQUAL [LAST [REVERSE A]] [CAR A]]]

WHICH IS EQUIVALENT TO:
[CUND A [EQUAL [LAST [REVERSE A]] [CAR A]] T]

MUST TRY INDUCTION.

INOUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND
[COND Nil [EQUAL [LAST [REVERSE vil]] [CAR NIL]] T]
[IMPLIES
[COND A [EQUAL [LAST [REVERSE A]] [CAR A]] T]
[COND [CONS A1 A] [EQUAL [LAST [REVERSE [CONS A1 A]]] [CAR [CONS A1 A]]] T]]]

WHICH IS EQUIVALENT TO:

```
[CONID A
    [COND [EQUAL [LAST [REVERSE A]] [CAR A]]
        [EQUAL [LAST [APPEND [REVERSE A] [CONS A1 NIL]]] A1]
        T]
```

    T]
    ```
GENERALIZE COMMON SURTERMS BY REPLACING [ZEVERSE A] BY GENRL1.
```

THE GENERALIZED TERM IS:
CCOND A
[COND [EQUAL [LAST GENRL1] [CAR A]]
[EQUAL [LAST [APPEND GENPL1 [CONS A1 NIL]]] A1]
T]
T]

```
MUST TRY INDUCTION.
```

INDUCT ON GENRLI.

```
THE THEOREM TO BE PROVED IS NOW:
[AivD
    CCOND A
    - [COND [EQUAL [LAST Nil] [CAR a]]
        [EQUAL [LAST [APPEND NIL [COVS A1 NIL]]] A1]
        T]
        T]
    [IMPLIES
        [COND A
        - [COND [EQUAL [LAST GENRL1] [CAR A]]
        - [EQUAL [LAST [APPEND GENZL1 [CONS A1 NIL]]] A1]
        \(\because \quad T]\)
        - T]
        [COND A
            [COND [EQUAL [LAST [CONS GENRL11 GENRL1]] [CAR A]]
                                    [EQUAL [LAST [APPEND [COVS GENRL11 GENRL1] [CONS A1 NIL]]] A1]
                                    T]
            Tコ]
```

```
[COND
A
[COND
    .[EQUAL [LAST GENRL1] [CAR A]]
    .[COND
- [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1]
. [COND GENRL1 [COND [APPEND GENRL1 [CONS A1 NIL]] T [EQUAL GENRL11 A1]] T]
- T]
.T]
T]
FERTILIZE WITH [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1].
THE THEOREM TO BE PROVED IS NOW:
[COND
    A
    [COND
    .[EQUAL [LAST GENRL1] [CAR A]]
    .[CONO
    - [COND GENRL1
    . [COND [APPEND GENRL1 [CONS [LAST [APPEND GENRL1 [CONS A1 NIL]]] NIL]]
    . . T
    . [EQUAL GENRL11 [LAST [APPEND GENRL1 [CONS A1 NIL]J]]]
    - i
    - [*1]]
    .T]
    T]
```

MUST TRY INDUCTION.
INDUCT ON GENRL1.

```
[AND
[COND
. A
. [CONO
- [EQUAL [LAST NIL] [CAR A]]
- [COND [COND NIL
• [COND
T
                                    [EQUAL GENRLI1 [LAST [APPEND NIL [CONS A1 NIL]]]]]
                                    T]
- -
. . T
[*1]]
T T
.T]
[IMPLIES
[COND
-4
- CCOND
- [EQUAL [LAST GENRL1] [CAR A]]
- [COND
. . COND
. . GENRL1
-. [COND [APPEND GENRL1 [CONS [LAST [APPEND GENRL1 [CONS A1 NIL]]]NIL]]
. . T
-. [EQUAL GENRL11 [LAST [APPEND GENRL1 [CONS A1 NIL]]]]]
. . T]
. .T
- .[*1]]
- T]
.T]
[COND
A
CCOND
    .[EOUAL [LAST [CONS GENRL12 GENRL1]] [CAR A]]
    .[COND
    - [COND
    . [CONS GENRL12 GENRL1]
    . .[COND
    . [APPEND [CONS GENRL12 GENRL1]
    . . [CONS [LAST [APPEND [CONS GENRLI2 GENRL1] [CONS A1 NIL]]] NIL]]
        . . T
        . [EQUAL GENRL11 [LAST [APPEND [CONS GENRL12 GENRL1] [CONS A1 NIL]]]]]
        . .T]
        . T
        . [*1]]
        .T]
        T]].]
```

WHICH IS EQUIVALENT TO:

```
FUNCTION DEFINITIONS:
crevERSE
    [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]
[LAST [LAMBDA [X] [COND X [COND [CDR X] [LAST [CDR X]] [CAR X]] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[CARARG UNDEF]
[APPEND [LAMRDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
FERTILIZERS:
*1 = [COND [EQUAL [LAST [APPEND GENRL1 [CONS A1 NILJ]] A1] NIL T]
```

GENERALIZATIONS:
GENRL1 $=$ [REVERSE A]
profile: [/ENR/ENR[A], /ENR/ENR/ENRG[GENRL1], /ENR/ t N R / ENRF, / [GENRL1] , /ENR.]

TIME: 23.13 SECS.

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[T 2 1]
[. 16.06 18 JULY
1973]
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THEOREM TO BE PROVED:
[IMPLIES [MEMBER A B] [MEMBER A [APPEND B C]]]

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WHICH IS EQUIVALENT TO:
[COND [MEMBER A B] [MEMBER A [APPEND B C]] T]
```

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:
[AND
[COND [MEMBER A NIL] [MEMBER A [APPEND NIL C]] T]
[IMPLIES [COND [MEMBER A B] [MEMBER A [APPEND B C]] T]
[COND [MEMBER A [CONS B1 B]] [MEMBER A [APPEND [CONS B1 B] C]] T]]]

## WHICH IS EQUIVALENT TO:

1

FUNCTIUN DEFINITIONS:

```
                                    [T 2 1]
    [LAmbDA [X y] [COND Y [COND [EQUAL X [CAR Y]] T [MEmbER X [CDR Y]]] NIL]]]
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [lAmbDA [X Y] [COND X [COND Y T vil] T]]]
[and [lambda [x Y] [cond X [cOND Y T NIL] NIL]]]
```

Profile: [/ENR/ENR[b], /ENR/ENR.]
IIME: 5.063 SECS.

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[T 2 2] [ 16.07 18 JULY 1973]
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THEOREM TO BE PROVED:
[IMPLIES [MEMBER A B] [MEMBER A [APPEND C B]]]

WHICH IS EQUIVALENT TO:
[COND [MEMBER A B] [MEMBER A [APPEND C B]] T]

## MUST TRY INDUCTION。

INUUCT ON C.

```
THE THEOREM TO bE PROVED IS NOW:
[AND [COND [MEMBER A B] [MEMBER A [APPEND NIL. B]] T]
    [IMPLIES [COND [MEMBER A B] [MEMBER A [APPEND C B]] T]
        [COND [MEMBER A B] [MEMBER A [APPEND [CONS C1 C] B]] T]]]
```

WHICH IS EQUIVALENT TO:
$\dagger$
FUNCTIUN DEFINITIONS:
CMEMBER
[APPEND [LAMBDA [X Y] [COND $X$ [CONS [CAR X] [APPEND [CDR X] Y]] Y]]] [IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

PROFILE: $[/ E N R / E N R[C], / E N R / E N R \cdot]$

TIME: 6.063 SECS.

```
[T 2 3]
    [ 16.07 18 JuLY 1973]
```

THEOREM TO BE PROVED:
[IMPLIES [AND [NOT [EQUAL A [CAR B]]] [MEMBER A B]] [MEMBER A [CDR B]]]

## WHICH IS EQUIVALENT TO:

[COND [EQUAL A [CAR B]] $T$ [COND [MEMBER A B] [MEMBER A [CDR B]] T]]

MUST TRY INDUCTION.

INDUCT ON B.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [COND [EQUAL A [CAR NIL]] T [COND [MEMBER A NIL] [MEMBER A [CDR NIL]] T]]
    [IMPLIES [COND [EQUAL A [CAR B]] T [COND [MEMBER A B] [MEMBER A [CDR B]] T]]
        [COND [EQUAL A [CAR [CONS 31 B]]]
            T
            [COND [MEMBER A [CONS B1 3]] [MEMBER A [CDR [CONS B1 B]]] T]]]]
```

WHICH IS EQUIVALENT TO:
1

```
[NUT [LAMBDA [X] [COND X NIL T]]]
[MEMBER
    [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[CARARG UNDEF]
[CDRARG UNDEF]
```

PROFILE: [/ENR/ENR[B], /ENR/ENR.]
TIME: 5.938 SECS.

```
[T 2 4] [. 16.07 18 JULY 1973]
THEOREM TO BE PROVED:
[IMPLIES [OR [MEMBER A B] [MEMBER A C]] [MEMBER A [APPEND B C]]]
WHICH IS EQUIVALENT TO:
[COND [MEMBER A B]
    [MEMBER A [APPEND B C]]
    [COND [MEMBER A C] [MEMBER A [APPEND B C]] T]]
```

MUST TRY INDUCTION.

INDUCT ON B.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [COND [MEMBER A NIL]
        [MEMBER A [APPEND NIL C]]
        [COND [MEMBER A C] [MEMBER A [APPEND NIL C]] T]]
        [IMPLIES [COND [MEMBER A B]
            [MEMBER A [APPEND B C]]
                            [COND [MEMBER A C] [MEMBER A [APPEND B C]] T]]
            [COND [MEMBER A [CONS B1 B]]
                        [MEMBER A [APPEND [COVS B1 B] C]]
                            [COND [MEMBER A C] [MEMBER A [APPEND [CONS B1 B] C]] T]]]]
```

```
WHICH IS EQUIVALENT TO:
```

```
FUNCTION DEFINITIONS:
cMEMBER
    [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[OR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PRUFILE: [/ENR/ENR[B], /ENR/ENR.]
IIME: 11.88 SECS.

```
[T 2 5]
[ 16.07 18 JULY 1973]
```

```
THEOREM TO BE PROVED:
[IMPLIES [AND [MEMBER A B] [MEMBER A C]] [MEMBER A [INTERSEC B C]]]
```

WHICH IS EQUIVALENT TO:
[COND [MEMBER A B] [COND [MEMBER A C] [MEMBER A [INTERSEC B C]] T] T]
MUST TRY INDUCTIUN.
INUUCT ON B.

```
THE THEOREM TO bE PROVED IS NOW:
[AND
    [COND [MEMBER A NIL] [COND [MEMBER A C] [MEMBER A [INTERSEC NIL C]] T] T]
    [IMPLIES [COND [MEMBER A B] [COND [MEMBER A C] [MEMBER A [INTERSEC B C]] T] T]
    [COND [MEMBER A [CONS B1 B]]
    [COND [MEMBER A C] [MEMBEF A [INTERSEC [CONS B1 B] C]] T]
    TJ]J
WHICH IS EQUIVALENT TO:
[COND
    [MEMBER A B]
    T
    [COND [EQUAL A B1]
        [COND [MEMBER A C] [COND [MEMBER B1 C] T [MEMBER A [INTERSEC B C]]] T]
        T]]
```

1HE THEOREM TO BE PROVED IS NOW:
[CUND
[MEMBER A B]
$\Gamma$
[COND [COND [MEMBER A C] [COND [MEMBER A C] T [MEMBER A [INTERSEC B C]]] T] T
[ $\because 13]$ ]

## WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:
LMEMBER
[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[INTERSEC [LAMBDA [ $X \quad Y]$
[COND [MEMBER [CAR X] Y]
[CONS [CAR X$]$ [INTERSEC [CDR X] Y]]
[INTERSEC [CDR X] Y]]
NILJJ]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T. NIL] T]]]

FERTILIZERS:
*1 $=$ [COND [EQUAL A B1] NIL T]

PROFILE: [/ENR/ENR[R], /ENR/ENR/ENR/ENRF//R/ENR

```
[T 2 6] [ 16,08 18 JULY 1973]
```

THEOREM TO BE PROVED:

```
[IMPLIES [OR [MEMBER A B] [MEMBER A C]] [MEMBER A [UNION B C]]]
```

WHICH IS EQUIVALENT TO:
[COND [MEMBER A B]
[MEMRER A [UNION B C]]
[COND [MEMBER A C] [MEMBER A [UNION B C]] T]]

MUST TKY INDUCTION.

INDUCT ON B.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [COND [MEMBER A NIL]
                [MEMBER A [UNION NIL C]]
        [COND [MEMBER A C] [MEMBER A [UNION NIL C]] T]]
    [IMPLIES [COND [MEMBER A B]
                                    [MEMRER A [UNION B C]]
                            [COND [MEMBER A C] [MEMBER A [UNION B C]] T]]
            [COND [MEMBER A [COVS 31 B]]
                            [MEMBER A [UNION [CONS B1 B] C]]
                            [COND [MEMBER A C] [MEMBER A [UNION [CONS B1 B] C]] T]]]]
```

WHICH IS EQUIVALENT TO:
[COND
[MEMBER A B.]
T
[CUND $\begin{gathered}\text { TMEMBER A C] }\end{gathered}$
[COND [EQUAL A B1] [COND [MEMBER B1 C] [MEMBER A [UNION B C]] T] T]]]

FERTILIZE WITH [EQUAL A B1].

```
THE THEOREM TO BE PROVED IS NOW:
[COND [MEMBER A B]
    T
    [COND [MEMBER A C]
        T
        [COND [COND [MEMBER A C] [MEMBER A [UNION B C]] T] T [*1]]]]
```

WHICH IS EQUIVALENT TO:
1

## FUNCTIUN DEFINITIONS:

[MEMBER
[LAMBDA [X Y] [COND $Y$ [COND [EQUAL X [CAR Y]] $T$ [MEMBER $X[C D R ~ Y]]] ~ N I L]]]$ [OR [LAMBDA $[X \quad Y][C O N D X T[C O N D Y T X I L]]$

LUNTON
[LAMBDA
$\left[\begin{array}{ll}X & Y\end{array}\right]$
LCOND
X
 Y] ] ]
[IMPLIES [LAMBDA [X Y] [COND X [CDND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
*1 $=[$ COND [EQUAL A B1] NIL T]

PRUFILE: [/ENNR/ENR[B]//ENR/ENR/ENR/ENRF//R/ENR . $]$

TIME: 27.06 SECS.

```
[T 2 7] [ 16.09 1.8 JULY 1973]
```

THEOREM TO BE PROVED:
[IMPLIES [SUBSET A B] [EQUAL [UNION A B] 3]]

WHICH IS EQUIVALENT TO:
[COND [SUBSET A B] [EQUAL [UNION A B] B] T]

MUST TKY INDUCTION.

INDUCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [COND [SUBSET NIL B] [EQUAL [UNION NIL B] B] T]
    [IMPLIES [COND [SUBSET A B] [EQUAL [UNION A B] B] T]
                        [COND [SUBSET [CONS A1 A] B] [EQUAL [UNION [CONS A1 A] B] B] T]]]
```

WHICH IS EQUIVALENT TO:
I
FUNCTIUN DEFINITIONS:
[SUBSET
[LAMBDA [X Y] [COND X [COND [YEMZER [CAR X] Y] [SUBSET [CDR X] Y] NIL] T]]]

```
[MEMBER
        [LAMBDA [X Y] [COND Y [COND [EQJAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[UNION
        [lAMBDA
        [X Y]
        [COND
            X
            [COND [MEMBER [CAR X] Y] [UNIOV [CDR X] Y] [CONS [CAR X] [UNION [CDR X] Y]]]
            Y]]]
[[MPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL],NIL]]]
```

PROFILE: $[/ E N R / E N R[A], / E N R / E N R$,

TIME: 6.625 SECS.

```
[T 2 8] [ 16.09 18 JULY 1973]
```

1HEOREM TO BE PROVED:
[IMPLIES [SUBSET A B] [EQUAL [INTERSEC A 3] A]]
WHICH IS EQUIVALENT TO:
[COND [SUBSET A B] [EQUAL [INTERSEC A B] A] T]
MUST TRY INDUCTION.

INDUCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [COND [SUBSET NIL B] [EQUAL [INTERSEC NIL B] NIL] T]
    [IMPLIES [COND [SUBSET A B] [EQUAL [INTERSEC A B] A] T]
    [COND [SUBSET [CONS A1 A] B]
                                    [EQUAL [INTERSEC [CONS A1 A] B] [CONS A1 A]]
                                    T]J]
```

WHICH IS EQUIVALENT TO:
1

```
cSUBSET
    [LAMBDA [X Y] [COND X [COND [YEMBER [CAR X] Y] [SUBSET [CDR X] Y] NIL] T]]]
[MEIBER
    [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[INTERSEC [LAMBDA [X Y]
    [COND X
                                    [COND [MEMBER [CAR X] Y]
                                    [COVS [CAR X] [INTERSEC [CDR X] Y]]
                                    [INTERSEC [IDR X] Y]]
                    NILJJ]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
PROFILE: [/ENR/ENR[A], / ENR/ENR/ENR,]
TIME: 8.563 SECS.
```

```
[T 2 9] [ 16.09 18 JULY 1973]
```

THEOREM TO BE PROVED:
[EQIJAL [MEMBER A B] [NOT [EQUAL [ASSOC A [PAIRLIST B C]] NIL]]]
WHICH IS EQUIVALENT TO:
[COND [ASSOC A [PAIRLIST B C]] [MEMBER A 3] [COND [MEMBER A B] NIL T]]

MUST TRY INDUCTION.

INDUCT ON C AND B.

```
THE THEOREM TO BF PROVED IS NOW:
[AND
    [AND
    .[COND [ASSOC A [PATRLIST B NIL]] [YEMBER A B] [COND [MEMBER A B] NIL T]]
    .[COND [ASSOC A [PAIRLIST NIL C]] [MEMBER A NIL] [COND [MEMBER A NIL]NIL T]]]
    [1MPLIES
    [COND [ASSOC A [PAIRLIST B C]] [MEMBER A R] [COND [MEMBER A B] NIL T]]
    [COND [ASSOC A [PAIRLIST [CONS B1 B] [CONS C1 C]]]
                [MEMBER A [CONS B1 B]]
                [COND [MEMBER A [CONS B1 Bl] N[L T]]]]
```

```
WHICH IS EQUIVALENT TO:
[CUND [ASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MFMBER A R] NIL T]]
```

```
must tiky induction.
```

INDJCT ON B.

```
THE THEOREM TO bE PROVED IS NOW:
[AN]
    [COND [ASSOC A [PAIRLIST NIL NIL]] [MEMBER A NIL] [COND [MEMBER A NIL] NIL T]]
    CIMPLIES
        [COND [ASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MEMBER A B] NIL T]]
        [COND [ASSOC A [PAIRLIST [CONS B2 8] NIL]]
            [MEMBER A [CONS B2 B]]
            [COND [MEMBER A [CONS B2 B]] NIL TJ]J]
```


## WHICH IS EQUIVALENT TO:

T

## FUIVCTIUN DEFINITIONS:

## LMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[PAIRLIST
[LAMBDA [X Y]
[COND $X$
[COND Y
[CONS [CUNS [CAR X] [CAR Y]] [PAIRLIST [CDR X] [CDR Y]]]
[CONS [CONS [CAR X] VIL] [PAIRLIST [CDR X] NIL]]]
NILJ]
LASSUC
[LAMBDA [X Y]
rconn Y
[COND [CAR Y]

NILJ]
[NOT [LAMBDA [X] [COND $X$ NIL T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND $X$ [COND Y T VIL] T]]]

PROFILE: $[/ E N R / E N R[C B], / E N R / E N R / E N R[B], / E N R / E N R$ .]

TIME: 15.0 SECS.

```
[T 3 1]
[ 16.09 18 JULY 1973]
```

THEDREM TO BE PROVED:
[EQUAL [MAPLIST [APPEND A B] C] [APPEND [YAPLIST A C] [MAPLIST B C]]]

MUST TKY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND]
[EQUAL [MAPLIST [APPEND VIL B] C] [APPEND [MAPLIST NIL C] [MAPLIST B C]]]
[IMPLIES [EQUAL [MAPLIST [APPEND A B] C] [APPEND [MAPLIST A C] [MAPLIST B C]]] [EQUAL [MAPLIST [APPEND [CONS A1 A] B] C] [APPEND [MAPLIST [CONS A1 A] C] [MAPLIST B C]I]]]

WHICH IS EQUIVALENT TO:
1

FUINCTIUN DEFINITIONS:
[APPEND [LAMBDA [X Y] [COND $X$ [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[MAPLIST
[LAMBDA [X Y] [COND $X$ [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]
[APPLY UNDEF]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

## [AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

PROFILE: [/ [A], /ENR/ENR.]

TIME: 6.25 SECS.

```
[\begin{array}{lll}{1}&{3}&{2}\end{array}]
[ 16.1 18 JULY 1973]
```

THEOREM TO BE PROVED:
[EQuAL [LENGTH [MAPLIST A B]] [LEVGTH A]]

MUST TKY INDUCTION.

INDJCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [LENGTH [MAPLIST NIL B]] [LENGTH NIL]]
[IMPLIES [EQUAL [LENGTH [MAPLIST A B]] [LENGTH A]] [EQUAL [LENGTH [MAPLIST [COVS A1 A] B]] [LENGTH [CONS A1 A]J]]]

WHICH IS EQUIVALENT TO:
1

FUNCTION DEFINITIONS:
[MAPLIST
[LAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]
[APPLY UNDEF]
[lengTh [LAMbda [X] [COND $X$ [CONS NIL [LEvGTH [CDR X]]] 0]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

# PROFILE: [/ [A], / ENR/ENR.] 

TIME: 3.438 SECS.

IHEDREM TO BE PROVED:
[EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]]

MUST TRY INDUCTION.

INDUCT ON A.

IHE THEOREM TO BE PROVED IS NOW:
LAND
[EQUAL [REVERSE [MAPLIST NIL B]] [MAPLIST [REVERSE NIL] B]] [IMPLIES
[EOJAL [REVERSE [MAPLIST A B]] [MADLIST [REVERSE A] B]]
[EQUAL [REVERSE [MAPLIST [CONS A1 A] B]] [MAPLIST [REVERSE [CONS A1 A]] B]]]]

## WHICH IS EQUIVALENT TO:

$\begin{array}{ll}\text { [COND [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]] } \\ \text { LEOUAL [APPEND [REVERSE [MAPLIST A B]] [CONS [APPLYYB A1] NIL]] } \\ \text { [M } & \text { [MAPLIST [APPEND [REVERSE A] [CONS A1 NIL]] R]] }\end{array}$

```
[CUN0 [EOUAL [APPEND [MAPLIST [REVERSE A] B] [CONS [APPLY B A1] NIL]]
                    [MAPLIST [APPFND [KEVERSE A] [CONS A1 NIL]] B]]
    T
    [*1]]
```

generalize common surterms by replacing [zeverse a] by genrli.
The generalized term is:
[COAD [EQUAL [APPEND [MAPLIST GENRL1 B] [CONS [APPLY B A1] NIL]]
[MAPLIST [APPEND GENRLI [CONS A1 NIL]] B]]
「
[*1]]
must try induction.
INDUCT ON GENRLI.
THE THEOREM TO BE PROVED IS NOW:
LAND
[COND [EQUAL [APPEND [MAPLIST NIL B] [COVS [APPLY B A1] NIL]]
[MAPLIST [APPEND NIL [CONS A1 NIL]] B]]
- T
- [*1]]
[IMPLIES
[COND [EQUAL [APPEND [MAPLIST GENRLI B] [CONS [APPLY B A1] NIL]]
[MAPLIST [APPEND GENR-1 [CONS A1 NIL]] B]]
- $\quad T$
- [*1]]
[Cuni)
[EQUAL [APPEND [MAPLIST [CONS GENRL11 GENRL1] B] [CONS [APPLY B A1] NIL]]
- [MAPLIST [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]] B]]
$\dot{T}$
[*1]]]]

```
WHICH IS EQUIVALENT TO:
r
FUNCTIUN DEFINITIONS:
[MAPLIST
    [LAMBBA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]
[APPLY UNDEF]
[REVERSE
    [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:
*1 = [COND [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]] NIL T]
GENERALIZATIONS:
GENRL1 $=$ [REVERSE A]

TIME: 12.19 SECS.

```
[T 4 1] [ 16.13 18 JULY 1973]
```

THEOREM TO BE PROVED:
[EUJAL [LIT [APPEND, A B] C D] [LIT A [LIT B C D] D]]

MUST TRY INDUCTION.

INDijCT ON A.

```
1HE THtOREM TO BE PROVED IS NOW:
[AN]
    [EGUAL [LIT [APPEND NIL B] C D] [LIT NIL [LIT B C D] D]]
    [IMPLIES
        [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]
        [匕QUAL [LIT [APPEND [CONS A1 A] B] C D] [LIT [CONS A1 A] [LIT B C D] D]נ]
```

WHICH IS EQUIVALENT TO:
[CUND
[EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]
[EQUAL [APPLY D A1 [LIT [APPEND A B] C D]] [APPLY D A1 [LIT A [LIT B C D] D]]]
T]
FERTILIZE WITH [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]].

```
LCOND [EQUAL [APPLY D A1 [LIT [APPEND A B] C D]]
                        [APPLY D A1 [LIT [APPEND A B] C D]]]
\Gamma
[*1]]
```

WHICH IS EQUIVALENT TO:
T

FUNCTION DEFINITIONS:
LAPPEND [LAMBDA $[X \quad Y][C O N D X[C O N S[C A R X][A P P E N D ~[C D R ~ X] ~ Y]] ~ Y]]] ~$ [LIT.[LAMBDA [X Y Z] [COND X [APPLY Z [CAR X] [LIT [CDR X] Y Z]] Y]]] [APPLY UNDEF]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X $Y$ ] [COND $X$ [COND $Y$ T NIL] NIL]]]

FERTILIZERS:
*1 = [COND [EQUAL [LIT [APPEND A B] C D] [LIT A [LITB C D] D]] NIL T]

PROFILE: [/ [A], /ENR/ENRX, /ENR.]

TIME: 8.313 SECS.

```
LT 4 2] [ 16.14 18 July 1973]
```

THEOREM TO BE PROVED:

```
[IMPLIES [AND [BOOLEAN A] [BOOLEAN B]]
    [EQUAL [AND [IMPLIES A B] [IMPLIES B A]] [EQUAL A B]]]
```

WHICH IS EQUIVALENT TO:
T

FUINCTIUN DEFINITIONS:
[BUOLEAN [LAMBDA [X] [COND X [EQUAL X T] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

PRUFILE: [/ENR/ENR.]

TIME: 4.313 SECS.

```
[T 4 4 3}
[ 16.14 18 JULY
1973]
```

THEOREM TO BE PROVED:
[EQuAL [ELEMENT B A] [ELEMENT [APPEND C B] [APPEND C A]]]

MUST TKY INDUCTION.

INDUCT ON C.

THE THEOREM TO BE PROVED IS NDW:
[AND [EQUAL [ELEMENT B A] [ELEMENT [APPENU NIL B] [APPEND NIL A]]] [IMPLIES [EQUAL [ELEMENT B A] [ELEMENT [APPEND C B] [APPEND C A]]] [EQUAL [ELEMENT B A] [ELEMENT [APPEND [CONS C1 C] R] [APPEND [CONS C1 C] A]J]]]

WHICH IS EQUIVALENT TO:
1

FUINCTIUN DEFINITIONS:
celement
[LAMBDA $[X Y][C O N D Y[C O N D X[E L E M E N T[C D R X][C D R ~ Y]][C A R ~ Y]] ~ N I L]]]$
LAPPEND [LAMBDA [X Y] [COVD $X$ [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLItS [LAMBDA [X Y] [COND $X$ [COND $Y$ T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [CUND Y T NIL] NIL]]]

## PROFILE: [/ [C]; /ENR/ENR.]

TIME: 5.25 SECS.

```
[T 4 4] [ [ 16.14 18 JULY 1973]
THEOREM TO BE PROVED:
[IMPLIES [ELEMENT B A] [MEMBER [ELEMENT B A] A]]
WHICH IS EQUIVALENT TO:
[CUND [ELEMENT B A] [MEMBER [ELEMENT B A] A] T]
```

MUST TRY INDUCTION.
INDUCT ON B AND A.

THE THEOREM TO BE PROVED IS NOW:
[AND [AND [COND [ELEMENT NIL A] [MEMBER [ELEMENT NIL A] A] T] [COND [ELEMENT 3 NIL] [MEMBER [ELEMENT B NIL] NIL] T]]
[IMPLIES [COND [ELEMENT B A] [MEMBER [ELEMENT B A] A] T] [COND [ELEMENT [CONS B1 B] [CONS A1 A]] [MEMBER [ELEMENT [CONS B1 B] [CONS A1 A]] [CONS A1 A]] T]] ]

WHICH IS EQUIVALENT TO:
1

## [ELEMENT

[LAMRDA [X Y] [COND Y [COND $X$ [ELEMENT [CDR X] [CDR Y]] [CAR Y]] NIL]]]
[MEMBER
[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[IMPLiES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

PRUFILE: [/ENR/ENR[BA], /ENR/ENR/ENR.]

TIME: 6.188 SECS.

```
[T 4 5]
[ 16.14 18 JULY 1973]
```

IHEOREM TO BE PROVED:
[EQUAL [CDRN $C$ [APPEND A B]] [APPEND [CDRV C A] [CDRN [CDRN A C] B]]]

MUS゙T TRY INDUCTION.

INDUCT ON A AND C.

```
THE THEOREM TO bE PROVED IS NOW:
[AND
    [AND
    - [EQUAL [CDRN C [APPEND NIL B]] [APPEND [CDRN C NIL] [CDRN [CDRN NIL C] B]]]
    - [EQUAL [CDRN NIL [APPEND A B]] [APPEND [CDRN NIL A] [CDRN [CDRN A NIL] B]]]]
    [IMPLIES [EQUAL [CDRN C [APPEND A B]] [APPEND [CDRN C A] [CDRN [CDRN A C] B]]]
        [EQUAL [CDRN [CONS C1 C] [APPENU [CONS A1 A] B]]
                                [APPEND [CDRN [CONS C1 C] [CONS A1 A]]
                        [CDRN [CDRN [CONS A1 A] [CONS C1 C]] B]]]]]
```

WHICH IS EOUIVALENT TO:
1

FUNCTIUN DEFINITIONS:
[APPENO [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]] [CDRN [LAMBDA [X Y] [COND Y [COND X [CDRN [CDR X] [CDR Y]] Y] NIL]]] [AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

## [IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

## PROFILE: [/ [A C], / ENR/ENR.]

TIME: 6.563 SECS.

```
[T 4 6]
[ 16.15 18 JULY
1973]
```

THEOREM TO BE PROVED:
[EQJAL [CDRN [APPEND B C] A] [CDRN C [CDRV B A]]]

MUST TRY INDUCTION.

INDUCT ON B AND A.

```
IHE THEOREM TO BE PROVED IS NOW:
[AIND [AND [EQUAL [CDRN [APPEND NIL C] A] [CDRN C [CDRN NIL A]]]
    [EQUAL [CDRN [APPEND B C] VIL] [CDRN C [CDRN B NIL]]I]
    [IMPLIES [EQUAL [CDRN [APPEND B C] A] [CURN C [CDRN B A]]]
        [EQUAL [CDRN [APPEND [CONS 31 B] C] [CONS A1 A]]
                        [CDRN C [CDRV [CONS 31 B] [CONS A1 A]]]]]]
```

WHICH IS EQUIVALENT TO:
1

FUNCTIUN DEFINITIONS:
[APPEND [LAMBDA [X Y] [COND $X$ [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[CDRN [LAMBDA [ $X$ Y] [COND Y [COND $X[C D R i N[C D R X][C D R Y]] Y$ NILJ]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
LIMPLItS [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

## PROFILE: $[/[B A], / E N R / E N R \cdot]$

TIME: 4.625 SECS.

```
[T 4 7
[ 16.15 18 JULY 1973]
```

```
THEOREM TO BEPPROVED:
[EQUAL [EQUAL A B] [EQUAL B A]]
WHICH IS EQUIVALENT TO:
[CUND [EQUAL A B] [EQUAL B A] [COND [EQUAL B A] NIL T]]
```

HERTILIZE WITH [EQUAL A B].
THE THEOREM TO BE PROVED IS NOW:
[CUND [COND [EQUAL A A] T [*1]]
[COND [COND [EQUAL B A] NIL T] T [EQUAL A B]]
NIL]
WHICH IS EQUIVALENT TO:
LCOND [EQUAL B A] [EQUAL A B] T]
FERTILIZE WITH [EQUAL B A].
1HE THEOREM TO BE PROVED IS NOW:
[COND [EQUAL. B B] T [*2]]

## WHICH IS EQUIVALENT TO:

T

FERTILIZERS:
*1 = [COND [EQUAL A B] NIL [COND [EQUAL B A] NIL T]]
*2 $=$ [COND [EQUAL B A] NIL T]

PROFILE: [/ NR/ENRF, /ENR/ENRF,/ENR.]

TIME: 2.063 SECS.

```
[T 4 8]
[ 16.15 18 JULY 1973]
```

```
THEOREM TO BE PROVED:
[IMPLIES [AND [EQUAL A B] [EQUAL B C]] [EQUAL A C]]
```

WHICH IS EQUIVALENT TO:
[CUND [EQUAL A B] [COND [EQUAL B C] [EQUAL A C] T] T]
FERTILIzE WITH [EQUAL A B].
IHE THEOREM TO BE PROVED IS NOW:
[COND [COND [EQUAL A C] [EQUAL A C] T] T [*1]]
WHICH IS EQUIVALENT TO:
I
FUNCTION DEFINITIONS:
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] Nilu]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
FERTILIZERS:
*1 = [COND [EQUAL A B] NIL T]

## PROFILE: [/ENR/ENRF;/R/ENR.]

TIME: 1.938 SECS.

```
[T 4 9] [ 16.15 18 JULY 1973]
```

THEOREM TO BE PROVED:
[IMPLIES [AND [BOOLEAN A] [AND [BOOLEAN B] [BOOLEAN C]]] [EQUAL [EQUAL A [EQUAL B C]] [EQJAL [EQUAL A B] C]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:
[buOLEAN [LAMBDA. [X] [COND X [EQUAL X T] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
profile: [/ENR/ENR.]

TIME: 13.69 SECS.

```
[T 5 5.1]
    [lllll}10.15 18 JULY 1973]
```

IHETREM TO BE PROVED:
[EQUAL [PLUS N M] [PLUS M N]]
must thy induction.
induct on m.

THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [PLUS N NIL] [PLUS NIL N]]
[IMPLIES [EQUAL [PLUS N M] [PLUS M N]] [EQUAL [PLUS N [CONS NIL M]] [PLUS [CONS NIL M] N]J]]

```
WHICH IS EQUIVALENT TO:
[COND [EQUAL [PLUS N NIL] N]
    [COND [EQUAL [PLUS N M] [PLUS M N]]
        [EQUAL [PLUS N [CONS NIL M]] [CONS NIL [PLUS M N]]]
        T]
    NILJ
```

FEHTilize With [equal [plus $N$ m] [plus m v]].
the theorem to be proved is now:
[CUND [EQUAL [PLUS N NIL] N]
[COND [EQUAL [PLUS $N$ [CONS VIL M]] [CONS NIL [PLUS N M]]] T [*1]] NIL]
(WORK UN FIRST CONJUNCT ONLY)

MUST TKY INDUCTION.

INDUCT ON N.

```
THE THEOREM TO BE PROVED IS NOW:
[CUND [AND [EQUAL [PLUS NIL NIL] NIL]
    [IMPLIES [EQUAL [PLUS V NIL] N]
                            [EQUAL [PLUS [CONS NIL N] NIL] [CONS NIL N]]]]
    [COND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]
    NILJ
```

WHICH IS EQUIVALENT TO:
[CUND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON N1.

THE THEOREM TO bE PROVED IS NOW:

```
[AND
    [COND [EQUAL [PLUS NIL [CONS NIL M]] [CONS NIL [PLUS NIL M]]] T [*1]]
    [IMPLIES
        [CONi [EQUAL [PLUS N1 [CONS NIL M]] [COVS NIL [PLUS N1 M]]] T [*1]]
        [COND
            [EQUAL [PLUS [CONS NIL N1] [CONS NIL M]] [CONS NIL [PLUS [CONS NIL N1] M]]]
            T
            [*1]]]]
```

```
WHICH IS EQUIVALENT TO:
```

1

FUNCTIUN DEFINITIONS:
[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLJS [CDR X] Y]] Y]]] [IMPLItS [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

LAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

FERTILIZERS:
*1 = [COND [EQUAL [PLUS N M] [PLUS M N]] VIL T]


``` /ENR/ENR.]
```

```
[T 5 2]
[. 16.16 18 JULY
1973]
```

THEOREM TO BE PROVED:
[EQUAL [PLUS N [PLUS M K]] [PLUS [PLUS N M] K]]

MUST TRY InDUCTION.

INDUCT ON N.

```
IHE THEOREM TO BE PROVED IS NOW:
[AND
    [EQUAL [PLUS NIL [PLUS M K]] [PLUS [PLUS NIL M] K]]
    [IMPLIES
            [EQUAL [PLUS N [PLUS M K]] [PLUS [PLUS N M] K]]
                            [EQUAL [PLUS [CONS NIL N] [PLUS M K]] [PLUS [PLUS [CONS NIL N] M] K]]]]
```

WHICH IS EQUIVALENT TO:
T

```
FUNCTION DEFINITIONS:
[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLJS [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T vil] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```


## PROFILE: [/ [N], /ENR/ENR.]

TIME: 4.688 SECS.

```
[T 5 3
[ 16.16 18 JuLY 1973]
```

IHEOREM TO BE PROVED:
[EUUAL [TIMES N M] [TIMES M N]]

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO bE PROVED IS NOW:
[AND [tQUAL [TIMES N NIL] [TIMES NIL N]]
[IMPLIES [EOUAL [TIMES N M] [TIMES M N]]
[EOUAL [TIMES $N$ [CONS VIL M]] [TIMES [CONS NIL M] N]]]]

WHICH IS EQUIVALENT TO:
[CUND [TIMES N NIL.]
NIL
[COND [EQUAL [TIMES N M] [TIMES M N]]
[EQUAL [TIMES N [CONS NIL M]] [PLUS N [TIMES M N]]] T] ]

FERTILIzE WITH [EQUAL [TIMES N M] [TIMES Y N]].

IHE THEOREM TO BE PROVED IS NOW:
[COND [TIMES N NIL]

NIL
[COND [EQUAL [TIMES N [CONS NIL M]] [PLUS N [TIMES N M]]] T [*1]]]

## (WURK ON FIRST CONJUNCT ONLY)

```
MUST TRY INDUCTION.
```

INDUCT ON N.

```
THE THEOREM TO BE PROVED IS NOW:
[CUND [A:ND [NOT [TIMES NIL NIL]]
    [IMPLIES [NOT [TIMES N NIL]] [VOT [TIMES [CONS NIL N] NIL]]]]
    [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]
    NILJ
```

WHICH IS EQUIVALENT TO:
[COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]

MUST TRY INDUCTION.

The theorem to be proved is now:
LAIND
[CONU [EQUAL [TIMES NIL [CONS NIL M]] [?LUS NIL [TIMES NIL M]]] T [*1]]
[IMPLIES [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]
[COND [EQUAL [TIMES [CONS NIL V1] [CONS NIL M]]
[PLUS [CONS VIL N1] [TIMES [CONS NIL N1] M]]]
T [*1]]]

WHICH IS EQUIVALENT TO:
[Cung
[EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]]
[COND [EQUAL [PLUS M [TIMES M1 [CONS NIL M]]] [PLUS N1 [PLUS M [TIMES N1 M]]]] . $T$

- [\#1]]

T」

FERTILIZE WITH [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 MI]].

IHE THEOREM TO BE PROVED IS NOW:
[Cund
[CUND [EQUAL [PLUS M [PLUS N1 [TIMES N1 M]]] [PLUS N1 [PLUS M [T]MES N1 M]]]] - $T$

- [*1]]

T
[*2]]
geveralize common subterms by replacing [times n1 m] by genrli.

IHE GENERALIZED TERM IS:
[CUND [COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1. [PLUS M GENRL1]]] T [*1]]

MUST TRY INDUCTION.

INDIJCT ON N1.

IHE THEOREM TO BE PROVED IS NOW:
[AND
[COND

- [COND [EQUAL [PLUS M [PLUS NIL GENRL1]] [PLUS NIL [PLUS M GENRL1]]] T [*1]]
- T
- [¥2]]
[IMPLIES
[COND
- [COND [EQUAL [PLUS : $\operatorname{a}$ [PLUS N1 GENRLi]] [PLUS N1 [PLUS M. GENRL1]]] T [*1]]
- T
- [*2]]
[CUND [COND [EOUAL [PLUS M [PLUS [CONS VIL N1] GENRL1]]
[PLUS [CONS VIL N1] [PLUS M GENRL1]]]
T
[*1]]
T
[*23]]]

WHICH IS EQUIVALENT TO:
[CUND [EQUAL [PLUS M [PLUS N1 GENRLI]] [PLUS N1 [PLUS M GENRL1]]] ECOND [COND [EQUAL [PLUS M [CONS NIL [PLUS N1 GENRL1]]] [CONS NIL [PLUS V1 [PLUS M GENRLi]]]]
T
[*1]]

## T

[*2]]
T]

FERTILIZE WITH [EQUAL [PLUS M [PLUS VI GEVRLI]] [PLUS N1 [PLUS M GENRLI]]].

```
THE THEOREM TO bE PROVED IS NOW:
[COND [COND [COND [EQUAL [PLUS M [CONS NIL [PLUS N1 GENRLI]]]
                                    [CONS NIL [PLUS y [PLUS N1 GENRL1]]]]
                                    T
                                    [*1]]
        T
        [*2]]
    T
    [*3]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS N1 GENRL1] BY GENRLZ.

THE GENERALIZED TERM IS:
LCOND
C.COND

- [COND [EQUAL [PLUS M [CONS NIL GENRL2]] [CONS NIL [PLUS M GENRL2]]] T [*1]]
- T
- [\#2]]

T
[*3]]

MUST TRY INDUCTION。

INDUCT ON M.
lHE THEOREM TO BE PROVED IS NOW:
[AND

```
[COND
-[COND [COND [EQUAL [PLUS NIL [CONS NIL GENRL2]] [CONS NIL [PLUS NIL GENRL2]]]
- T
[*1]]
- T
- [*2]]
.T
[ [*3]]
[IMPLIES
    [COND
    - COND
    - [CUNO [EQUAL [PLUS M [CONS NIL GENRL2]] [CONS NIL [PLUS M GENRL2]]] T [*1]]
    -1
    - [*2]]
    -T
    - [*3]]
    [CONO [COND [COND [EQUAL [PLUS [CONS NIL M] [CONS NIL GENRL2]]
                                    [CONS VIL [PLUS [CONS NIL M] GENRL2]]]
                                    T
                                    [*1]]
                \top
                        [*2]]
                T
                [*3]]]]
```

WHICH IS EQUIVALENT TO:
1

FUNCTION DEFINITIONS:
[TIUES [LAMBDA [X Y] [COND $X$ [PLUS Y [TIMES [CDR X] Y]] 0]]] [PLUS [LAMBDA [X Y] [COND $X$ [CONS NIL [PLJS [CDR X] Y]] Y]]] [IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[NOT [LAMBDA [X] [COND X VIL T]]]

FERTILIZERS:
*1 $=$ [COND [EQUAL [TIMES $N$ M] [TIMES M N]] NIL T.]
*2 $=$ [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]J] NIL T]

## generalizations:

GENRL1 = [TIMES N1 M]
GENRL2 $=$ [PLUS N1 GENRL1]
 $N R / E N R$.J

IIME: 32.75 SECS.

```
[T 5 4] [ 16.17 1.8 JULY 1973]
```

IHEDREM TO BE PROVED:
[EUUAL [TIMES N [PLUS M K]] [PLUS [TIMES $V$ M] [TIMES N K]]]

MUST TRY INDUCTION.

Induct on N .

# THE THEOREM TO BE PROVED IS NOW: <br> [AND [EQUAL [TIMES NIL [PLUS M K]] [PLUS [TIMES NIL M] [TIMES NIL K]]] [IMPLIES [EOUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]] [EQUAL [TIMES [CONS NIL N] [PLUS M K]] [PLUS [TIMES [CONS NIL N] M] [TIMES [CONS NIL N] K]J]]] 

WHICH IS EQUIVALENT TO:
[COND [EOUAL [TIMES $N$ [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]]
[EQUAL [PLUS [PLUS M K] [TIMES N [PLUS M K]]] [PLUS [PLUS M [TIMES N M]] [PLUS K [TIMES N K]].]
T]

FERTILIZE WITH [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]].

THE THEOREM TO BE PROVED IS NOW:
LCOND [EQUAL. [PLUS [PLUS M K] [PLUS [TIMES N M] [TIMES N K]]]
［PLUS［PLUS M［TIMES N Y］］［PLUS K［TIMES N K］］］］ T ［＊1］］

GENERALIZE COMMON SURTERMS BY REPLACING［TIMES N K］BY GENRLI AND［TIMES N M］BY GENRLZ．

The generalized term is：

```
[CUND [EQUAL. [PLUS [PLUS M K] [PLUS GENRL2 GENRL1]]
    [PLUS [PLUS M GENRL2] [PLUS < GENRL1]]]
    T
    [*1]]
```

MUST TRY INDUCTION．

INBIJCT ON M．

THE THEOREM TO BE PROVED IS NOW：
［AND［COND［EQUAL［PLUS［PLUS NIL K］［PLUS GENRL2 GENRLI］］ ［PLUS［PLUS NIL GENRL2］［PLUS K GENRL1］］］

T
［＊1］］
［IMPLIES［COND［EQUAL［PLUS．［PLUS M＜］［PLUS GENRL2 GENRLI］］
［PLUS［PLUS M GENRL2］［PLUS K GENRLI］］］
T
［＊1］］
［COND［EQUAL［PLUS［PLUS［CJNS NIL M］K］［PLUS GENRL2 GENRLI］］ ［PLUS［PLUS［CJNS NIL M］GENRL2］［PLUS K GENRL1］］］
$\uparrow$
［＊1コココ］

```
WHICH IS EQUIVALENT TO:
        T
        [*1]]
MUST TRY INDUCTION.
```

[CONO [EQUAI [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]
Induct on genrlz.
THE THEOREM TO BE PROVED IS NOW:
[AND
[COND [EQUAL [PLUS K [PLUS NIL GENRL1]] [PLUS VIL [PLUS K GENRL1]]] T [*1]]
[IMPLIES
[COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL? [PLUS K GENRL1]]]
- T
- [\%1]]
[COND [EQUAL [PLUS K [PLUS [CONS GEVRL21 GENRL2] GENRL1]]
[PLUS [CONS GENRL21 GEVRL2] [PLUS K GENRL1]]]
T
[*1]]]
WHICH IS EQUIVALENT TO:
[COND [EOUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]
[COND [EQUAL [PLUS K [CONS NIL [PLUS GENRL2 GENRLI]]]
[CONS NIL [PLUS GENRL2 [PLUS K GENRL1]]]]
T
[*1]]
T]
FERTILIZE WITH [EQUAL [PIUS K [PLUS GENRL? GENRL1]]
[PLUS GENRL? [PLUS $<~ G E N R L 1]]]$.

IHE THEOREM TO bE PROVED IS NOW:
[COND [COND [EQUAL [PLUS K [CONS VIL [PLUS GENRL2 GENRLI]]] [CONS NIL [PLUS K [PLUS GENRL2 GENRL1]]]]

```
T
    [*1]]
    T
    [*2]]
```

GENERALIZE COMMON SURTERMS BY REPLACING [?LUS GENRL2 GENRL1] BY GENRL3.

The general.ized term is:
[COND
[CUND [EQUAL [PLUS K [CONS NIL GENRL3]] [CONS NIL [PLUS K GENRL3]]] T [*1]]「 [*2]]

## MUST TRY INDUCTION.

INDUCT ON K.

```
THE THEOREM TO bE PROVED IS NOW:
```


## [AND

```
    [COND [CONII [EQUAL [PLUS NIL [CONS NIL GENRL3]] [CONS NIL [PLUS NIL GENRLZ]]]
        . T
                                    [*1]]
    - T
    - [*2]]
    [IMPLIES
        CCOND
```

- [CUND [EOUAL [PLUS K [CONS NIL GENRL3]] [CONS NIL [PLUS K GENRL3]]] T [*1]]
- 1
- [*2]]

LCONO [COND [EOUAL [PLUS [CONS NIL K] [JONS NIL GENRL3]] [CONS NIL [PLUS [CONS NIL K] GENRL3]]]
T
[:1]]
T
[*2]].]

WHICH IS EOUIVALENT TO:
|

FUNCTIUN DEFINITIONS:
[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLJS [CDR X] Y]] Y]]]
[TIMES [LAMRDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

FERTILIZERS:

```
*1 = [COND [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]] NIL T]
*2 = [COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]] NIL \(T]\)
```


## GENERALIZATIONS:

```
GENRL2 = [TIMES N M]
GENRL1 = [TIMES N K]
GENRL3 = [PIUS GENRL2 GENRL1]
```

PROFILE: [/ [N], / ENR/ENRX, /G[M], /ENR/ENR/ENR[GENRL2
], /ENR/ENR/ENRF, /G[K], /ENR/ENR.]
$\left[\begin{array}{lll}T & 5 & 4\end{array}\right]$

TIME: 31.19 SECS.

```
[T 5 5]
[ 16.18 18 JULY
1973]
```

THEOREM TO EE PROVED:
[EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]]

MUST TRY INDUCTION.

Induct on $N$.

THE THEOREM TO BE PROVED IS NOW:
[AND]
[EQUAL [TIMES NIL [TIMES M K]] [TIMES [TIMES NIL M] K]]
[IMPLIES
[EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N•M] K]]
[EQUAL 「TIMES [CONS NIL N] [TIMES M K]] [TIMES [TIMES [CONS NIL. N] M] K]]]]

WHICH IS EQUIVALENT TO:
[CUND [EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]] [EQUAL [PLUS [TIMES M K] [TIMES N [TIMES M K]]] [TIMES [PLUS M [TIMES N M]] K]]
T]
fertilize with [equal [times n [times m k]] [times [times N m] k]].

```
[COND [EQUAL [PLUS [TIMES M K] [TIMES [TIMES N M] K]]
            [TIMES [PLUS M [TIMES N M]] <]]
    T
    [*1]]
```

GENERALIZE COMMON SURTERMS BY REPLACING [TIMES N M] BY GENRL1.
The Generalized term is:
[COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
T
[*1]]

```
MUST TRY INDUCTION.
```

INDUCT DN M.

```
TME THLOREM TO BE FROVED IS NOW:
[AND
    [COND
    . [EQUAL [PLUS [TIMES NIL K] [TIMES GENRL1 K]] [TIMES [PLUS NIL GENRL1] K].]
    - T
    - [*1]]
    [IMPLIES
    [COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
    - T
    - [*1]]
    [COND [EQUAL [PLUS [TIMES [CONS NIL M] K] [TIMES GENRL1 K]]
                            [TIMES [PLUS [COVS VIL M] GENRLI] K]]
                T
                [*1]]]]
```

```
WHICH IS EQUIVALENT TO:
[CUND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
        [COND [EQUAL [PLUS [PLUS K [TIMES M K]] [TIMES GENRL1 K]]
                        [PLUS K [TIMES [PLUS M GENRL1] K]]]
            T
        [:1.]]
    T]
FERTILIZE WITH [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]]
    [TIMES [PLUS M GENRL1] K]].
THE THEOREM TO BE PROVED IS NOW:
[COND [COND [EQUAL [PLUS [PLUS K [TIMES M K]] [TIMES GENRL1 K]]
                            [PLUS K [PLUS [TIMES M K] [TIMES GENRLI K]]]]
        T
        [%1]]
    T
    [*2]]
gENERALIZE COMMON SUBTERMS BY REPLACING [TIMES GENRLI K] bY GENRL2 AND [TIMES M
K] BY GENRL3.
IHE GENERALIZED TERM IS:
```

```
[COND [COND [EQUAL [PLUS [PLUS K GENRL3] GENRL2] [PLUS K [PLUS GENRL3 GENRL2]]]
```

[COND [COND [EQUAL [PLUS [PLUS K GENRL3] GENRL2] [PLUS K [PLUS GENRL3 GENRL2]]]
T
T
[*1]]
[*1]]
T
T
[*2]]

```
    [*2]]
```

MUST THY INDUCTION。

INDUCT ON K.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [CONO
    -[CUND [EQUAL [PLUS [PLUS NIL GEVRL3] GEVRL2] [PLUS NIL [PLUS GENRL3 GENRL2]]]
    - T
    [*1]]
    .T
    .[*2]]
    [IMHLIES
        [COND
            - [COND [EQUAL [PLUS [PLUS K GENRL3] GENRL2] [PLUS K [PLUS GENRL3 GENRL2]]]
            . . T
            - [%1]]
            - T
            - [*2]]
    [COND [COND [EQUAL [PLUS [PLUS [CONS NIL K] GENRL3] GENRL2]
                                    [PLUS [CONS VIL K] [?LUS GENRL3 GENRL2]]]
                T
                [*1]]
            T
            [*21]]]
```

WHICH IS EQUIVALENT TO:
1

FUNCIION DEFINITIONS:
[TIUES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] O]]] [PLUS [LAMBDA [X Y] 「COND X [CONS NIL [PLJS [CDR X] Y]] Y]]] [IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

```
*1 = [COND [EQUAL [TINES N [TIMES M K]] [TIMES [TIMES N M] K]] NIL T]
*2 = [COND
    [FQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
        NIL
    T]
```


## generalizations:

## GENRL1 $=$ [TIMES $N \mathrm{M}]$

GENRLL $3=$ [TIMES M K]
GENRL2 $=[$ TIMES GENRL1 K]


TIME: 25.25 SECS.

```
[T 5 6
[ 16.18 18 JULY 1973]
```

THEOREM TO BE PROVED:
[EVEN1 [DOUBLE N]]

MUST TRy induction.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:
[AND [EVEN1 [DOUBLE NIL]]
[IMPLIES [EVEN1 [DOUBLE N]] [EVEN1 [DOUBLE [CONS NIL N]]]]]。

WHICH IS EQUIVALENT TO:
I

FUNCTIUN DEFINITIONS:
[DUUBLE [LAMRDA [X] [COND X [CONS NIL [COVS NIL [DOUBLE [CDR X]]]] 0]]] [EVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [EDR X]] NIL T] T].]
[IMPLIES [LAMBOA [X Y] [COND $X$ [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

```
[T 5 7]
[ 16.18 18 JULY 1973]
```

THEOREM TO BE PROVED:
[EQUAL [HALF [DOUBLE N]] N]

MUST TRY INDUCTIUN。

INDUCT ON N.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [HALF [DOUBLE NIL]] NIL]
    [IMPLIES [EQUAL [HALF [DOUBLE N]] N]
        [EQUAL [HALF [DOUBLE [CONS VIL N]]] [CONS NIL V]]]]
```

WHICH IS EQUIVALENT TO:
「

FUNCTIUN DEFINITIONS:
[DUUBLE [LAMBDA [X] [COND X [CONS NIL [COVS NIL [DOUBLE [CDR X]J]] 0]]]
[HALF [LAMBDA [X] [COND $X$ [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]] [IMPLIES [LAMBDA [X Y] [COND $X$ [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

## PROFILE: [/[N],/ENR/ENR.]

TIME: 2.625 SECS.

```
[T 5 8]
[ 16.18 18 JJLY 1973]
```

```
THEOKEM TO BE PROVED:
[IMPl_IES [EvEN1 N] [EQUAL [DOUBLE [HALF N]] N]]
```

WHICH IS EQUIVALENT TO:
[COND [EVEN1 N] [EQUAL [DOUBLE [HALF N]] V] T]
MUST TKY Induction.

```
(SPECIAL CASE REQUIRED)
```

INDUCT ON N.
THE THEOREM TO BE PROVED IS NOW:
[AND
[COND [EVEN1 NIL] [EQUAL [DOUbLE [HALF NIL]] NIL] T]
[AND
[COND [EVEN1 [CONS NIL NIL]]
- [EQUAL [DOUBLE [HALF [CONS NIL NIL]]] [CONS NIL NIL]]
- -T]
[IMPLIES
[COND [EVEN1 N] [EQUAL [DOUBLE [HALF N]] N] T]
[COND
[EVEN1 [CONS NIL [CONS NIL N]]]
[EQUAL [DOUBLE [HALF [CONS NIL [CONS NIL N]]]] [CONS NIL [CONS NIL N]]]
TJ]J]

```
WHICH IS EQUIVALENT TO:
|
FUNCTION DEFINITIONS:
[EVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]]
[HALF [LAMBDA [X] [COND X [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]] [DUUBLE [LAMBDA [X] [COND \(X\) [CONS NIL [COVS NIL [DOUBLE [CDR X]]]] 0] [IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND \(X\) [COND Y T NIL] NIL]]]
```

PROFILE: [/ENR/ENRS2[N], /ENR/ENR.]

TIME: 4.75 SECS.

```
[T 5 9
[ 16.19 18 JuLY 1973]
```

THEDREM TO BE PROVED:
[EUUAL [DOUBLE N] [TIMES 2 N]]

## WHICH IS EOUIVALENT TO:

[EQUAL [DOUBLE N] [PLUS N [PLUS N O]]]

```
MUST TRY INDUCTION.
```

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

## [AND

    [EQUAL [DOUble Nil] [PLUS Nil [PLUS NIL 0]]]
    [IMPLIES
        [EQUAL [DOUBLE \(N]\) [PLUS \(N\) [PLUS N O]]]
        [EOUAL [DOUBLE [CONS NIL N]] [PLUS [CONS NIL N] [PLUS [CONS NIL N] O]]]]]
    WHICH IS EQUIVALENT TO:
[CUND [EQUAL [DOUBLE N] [PLUS N [PLUS N O]]]
[EQUAL [CONS NIL [DOUBLE N]] [PLUS V [CONS NIL [PLUS N O]]]]
T]

THE THEOREM TO BE PROVED IS. NOW:
[COND LEQUAL [CONS NIL [PLUS N [PLUS N O]]] [PLUS N [CONS NIL [PLUS N 0]]]] T $[: * 1]$

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS N 0] BY GENRL1.

THE GENERALIZED TERM IS:
[CUND [EOUAL [CONS NIL [PLUS N GENRL1]] [PLUS N [CONS NIL GENRL1]]] T [\#1]]

MUST TRY INDUCTION.

INDUCT ON N.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [COND [EQUAL [CONS NIL [PLUS NIL GENRL1]] [PLUS NIL [CONS NIL GENRL1]]]
    . T
    - [*1]]
    [IMPLIES
        [COND [EOUAL [CONS NIL [PLUS N GENRL1]] [PLUS N [CONS NIL GENRL1]]] T [*1]]
        [CUND [EQUAL [CONS NIL [PLUS [CONS NIL N] GENRL1]]
                                [PLUS [CONS NIL N] [CONS NIL GENRL1]]]
            T
            [*1]]]]
```


## WHICH IS EQUIVALENT TO:

 1FUNCTIUN DEFINITIONS:
[DUUBLE [LAMBDA [X] [COND X [CONS NIL [COVS NIL [DOUBLE [CDR X]]]] 0]]] [TIMES [LAMBDA [X Y] [COND $X$ [PLUS Y [TIMES [CDR X] Y]] 0]]] [PLUS [LAMBDA [X Y] [COND $X$ [CONS NIL [PLUS [CDR X] Y]] Y]]] [IMPLIES [LAMBDA [X Y] [COND $X$ [COND Y T NIL] T]]]
[AND [LAMBDA $[X Y$ [COND $X$ [COND $Y$ T NIL] NIL]]]

FERTILIZERS:
*1 = [COND [EQUAL [DOUBLE N] [PLUS $N$ [PLUSN 0]]] NIL T]

GENERALIZATIONS:
GENRLI $=[$ PLUS $N 0]$

PROFILE: [/ENR/ENR[N], /ENR/ENRX, /G[N], /ENR/ENR。 ]

TIME: 9.188 SECS.

```
[T 5 10]
[ 16.19 18 JULY 1973]
```

```
IHEOREM TO BE PROVED:
[EQJAL [DOUBLE N] [TIMES v 2]]
```

MUST TRY INDUCTION.
INDUCT ON N.
THE THEOREM TO RE PROVED IS NOW:
[AND [EQUAL [DOUBLE NIL] [TIMES NIL 2]]
[IMPLIES [EQUAL [DOUBLE N] [TIMES N 2]]
[EQUAL [DOUBLE [CONS NIL N]] [TIMES [CONS NIL v] 2]]]]
WHICH IS EQUIVALENT TO:
T
rUNCTIUN DEFINITIONS:
[DUUBLE [LAMBDA [X] [COND X [CONS NIL [COVS NIL [DOUBLE [CDR X]]]] 0]]]
[TIMES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]
[PLJS [LAMBDA [X Y] [COND $X$ [CONS NIL [PLJS [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [ $X$ Y] [COND $X$ [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

PROFILE: [/[N],/ENR/ENR•]

```
[T 5 11] [ 16.2 18 JULY 1973]
```

THEDREM TO BE PROVED:
[EQUAL [EVEN1 N] [EVEN2 N]]
MUST TRY INDUCTION.
(Special Case required)

INUSCT ON N.

```
THE THEOREM TO BE PROVED IS NOW:
LAND
    [EJUAL [EVEN1 NIL] [EVEN2 NIL]]
    [A:ND
        [EQUAL [EVEN1 [CONS NIL NIL]] [EVEV2 [CONS NIL NIL]]]
        [IMPLIES
            [EQuAL [EVEN1 N] [EVEN2 N]]
            [EQUAL [EVEN1 [CONS NIL [CONS NIL N]]] [EVEN2 [CONS NIL [CONS NIL N]]]]]]]
```

WHICH IS EQUIVALENT TO:
|

# [EVEN1 [LAMBDA [X] [COND $X$ [COND [EVEN1 [CDR X]] NIL T] T]]] [EVEN2 [LAMBDA [X] [COND $X$ [COND [CDR X] [EVEN2 [CDR. [CDR X]]] NIL] T]]] [IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]] <br> [AND [LAMBDA $[X Y][C O N D X[C O N D Y T N I L] N I L]]]$ 

```
PRUFILE: [/ S2 [N], / ENR/ENR.]
```

flME: 2.933 SECS.

```
[T 6 1] [ 16.2 18 JULY 1973]
```

THEOREM TO BE PROVED:
[gT [LENGTH [CONS A B]] [LENGTH B]]

WHICH IS EQUIVALENT TO:
[GT [CONS NIL [LENGTH B]] [LENGTH B]]

MUST TRY INDUCTION.

INDUCT ON B.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [GT [CONS NIL [LENGTH vil]] [LEVGTH vil]]
    [IMPLIES [GT [CONS NIL [LENGTH 3]] [GENGTH B]]
    [GT [CONS NIL [LENGTH [CONS BI B]]] [LENGTH [CONS B1 BJ]]]]
```

WHICH IS EQUIVALENT TO:
T
[GT [LAMBDA [X Y] [COND $X$ [COND $Y$ [GT [CDZ $X][C D R ~ Y]] ~ T] ~ N I L]]] ~$ [IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA $[X Y$ y [COND $X$ [COND Y T NIL] NIL]]]

PROFILE: $[/ E N R / E N R[B]$, /ENR/ENR.]

TIME: 3.125 SECS.

```
[T 6.2] [ 16.2 18 JULY 1973]
```

PHEDREM TO BE PROVED:
[IMPLIES [AND [GT A B] [GT B C]] [GT A C]]

```
WHICH IS EQUIVALENT TO:
[COND [GT A B] [COND [GT B C] [GT A C] T] T]
```

MUST TRY INDUCTION.
INDUCT ON B, A AND C.

```
THE THEOREM TO BE PROVED IS NOW:
[Aiv)
    [AND [COND [GT A Nil] [COND [GT NIL C] [GT A C] T] T]
    - [AND [COND [GT NIL B] [COND [GT B C] [GT NIL C] T] T]
    - [COND [GT A B] [COND [GT B NIL] [GT A NIL] T] T]J]
    cIMPLIES
            [COND [GT A B] [COND [GT B C] [GT A C] T] T]
            [COND [GT [CONS A1 A] [CONS B1 B]]
                        [COND [GT [CONS B1 B] [CONS C1 C]] [GT [CONS A1 A] [CONS C1 C]] T]
                        T]J]
```

WHICH IS EQUIVALENT TO:
[COND [GT A B] [COND B [COND A T VIL] T] T]

MUST TKY INDUCTIDN.

INDUCT ON A AND R.

```
IHE THEOREM TO be PROVED IS NOW:
[ANB [AND [COND [GT NIL B] [COND 3 [COND VIL T NIL] T] T]
    [COND [GT A NIL] [COND NIL [COND A T NIL] T] T]]
    [IMPLIES [COND [GT A B] [COND B [COND A T NIL] T] T]
        [COND [GT [CONS A2 A] [CONS B2 B]]
    [COND [CONS B2 B] [COND [CONS AZ A] T NIL] T]
    T]]]
```

WHICH IS EQUIVALENT TO:
1

## FUNCTION DEFINITIONS:

```
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]
```

[AND [LAMBDA $[X Y$ ] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

PROFILE: [/ENR/ENR[RAC], /EVR/ENR/ENR[AB], /ENR D]

TIVE: 9.375 SECS.

```
[T6 3] [llllo.2 18 JULY 1973]
```

THEOREM TO BE PROVED:
[IMPLIES [GT A.B] [NOT [GT B A]J]
WHICH IS EQUIVALENT TO:
[CONO [GT A B] [COND [GT B A] NIL T] T]
MUST TRY INDUCTION.
INUJCT ON B AND A.
THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [GT A NIL] [COND [GT NIL A] NIL T] T]
        [COND [GT NIL B] [COND [GT B NIL] NIL T] T]]
    [IMPLIES [COND [GT A B] [CUND [GT B A] NIL T] T]
        [COND [GT [CONS A1 A] [CONS B1 B]]
                        [COND [GT [CONS 31 B] [CONS A1 A]] NIL T]
                        T]!]
```

WHICH IS EQUIVALENT TO:
1

```
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDZ X] [CDR Y]] T] NIL]]]
[NOT [LAMBDA [X] [COND X VIL T]]]
[ImPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ENR/ENR[BA], /ENR/ENR.]
TIME: 4.375 SECS.

```
LT 6 4] [ 16.21 18 JULY 1973]
```

```
THEJREM TO BE PROVED:
[LTE A [APPEND B A]]
```

MUST TRY INDUCTION.
INDUCT ON B.
THE THEOREM TO BE PROVED IS NOW:
[AŃd [LTE A [APPEND NIL A]]
[IMPLIES [LTE A [APPEND B A]] [LTE A [APPEND [CONS B1 B] A]]]]
WHICH IS EQUIVALENT TO:
[CUND [LTE A A]
[COND [LTE A [APPEND B A]] [LTE A [IONS B1 [APPEND B A]]] T]
NILJ
(WURK UN FIRST CONJUNCT ONLY)

```
MUST TRY INDUCTION.
```

INOUCT OVA.

```
lHE THEOREM TO bE PROVED IS NOW:
[COND [AND [LTE NIL NIL] [IMPLIES [LTE A A] [LTE [CONS A1 A] [CONS A1 A]]]]
    [COND [LTE A2 [APPEVD B A2]] [LTE A2 [CONS B.1 [APPEND 3 A2]]] T]
    NIL]
```

```
WHICH IS EQUTVALENT TO:
[COND [LTE AZ [APPEND 3 A2]] [LTE A2 [CONS B1 [APPEND B A2]]] T]
```

GENERALIZE COMMON SURTERMS BY REPLACING [APPEND B A2] BY GENRL1.
THE GEIVERALIZED TERM IS:
[CON! [LTE A2 GENRL1] [LTE A2 [COVS 31 GEVRL1]] T]
MUST TRY INDUCTION.
INDJCT ON GENRL1 AND A2.

THE THEOREM TO bE PROVED IS NOW:
[AND [AND [COND [LTE AZ NIL] [LTE A2 [CONS BI NIL]] T] [COND [LTE NIL GENRL1] [LTE NIL [CONS B1 GENRL1]] T]]

```
[IMPLIES [COND [LTE A2 GENRL1] [LTE A2 [CONS B1 GENRL1]] T]
    LCOND [LTE [CONS A21 A2] [CJNS GENRL11 GENRL1]]
    [LTE [CONS A21 A2] [CONS B1 [CONS GENRL11 GENRL1]]]
    T]]]
```

```
WHICH IS EQUIVALENT TO:
[CUND [LTE A2 GFNRL1]
    [COND [LTE A2 [CONS B1 GENRL1]] [LTE A2 [CONS GENRL11 GENRL1]] T]
    T]
```

MUST TKY INDUCTION。
INDUCT ON GENRL1 AND A?.

```
IHE IHEOREM TO BF PROVED IS NOW:
[AND
    [ANO [COND [LTE A2 NIL]
                                [COND [LTE A2 [CONS B1 NILJ] [LTE A2 [CONS GENRL11 NIL]] T]
                                T]
    [COND [LTE NIL GENRLL]
        [COND [LTE NIL [CONS 31 GENRL1]] [LTE NIL [CONS GENRL11 GENRL1]] T]
        T]]
    [IMPLIES
        [COND [LTE A2 GENRL1]
            . [COND [LTE A2 [CONS B1 GENRL1]] [LTE A2 [CONS GENRL11 GENRL1]] T]
            - T]
            [COND [LTE [CONS A22 A2] [CONS GEVRL12 GENRL1]]
            [COND [LTE [CONS A22 A2] [CONS B1 [CONS GENRL12 GENRL1]]]
                                    [LTE [CONS A22 A2] [CONS GENRL11 [CONS GENPL12 GENRL1]]]
                            T]
            T]]3
```

FUNCTIUN DEFINITIONS:
[APPEND [LAMBDA [X Y] [COND $X$ [CONS [CAR X] [APPEND [CDR X] Y]] Y]]] [LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]] [IMPLIES [LAMBDA [X Y] [COND X [COND Y T vIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

```
GENERALIZATIONS:
GENRL1 = [APPEND B AP]
```

PROFILE: [/ [B], /ENR/ENR \& [A] , /ENR/ENR/ENR G[GENRL1 A2]
, / ENR/ENR/ENR[GENRLIA2], /ENR/ENR.]

```
[T 6 5] [ 16.21 18 JULY 1973]
```

THEOREM TO BE PROVED:
[OK [LTE A B] [LTE B A]]

WHICH IS EQUIVALENT TO:
[CUND [LTE A B] T [LTE B A]]

## MUST TRY INDUCTION.

INDUCT OV B AND A.

```
THE THEOREM TO bE PROVED IS NOW:
[ANi)
    [AND [COND [LTE A NIL] T [LTE NIL A]] [COND [LTE NIL B] T [LTE B NIL]]]
    [IMPLIES
        [COND [LTE A B] T [LTE B A]]
        [COND [LTE [CONS A1 A] [CONS B1 3]] T [LTE [CONS B1 8] [CONS A1 A]]]]]
```

WHICH IS EQUIVALENT TO:
1
[LTE [LAMBDA [X Y] [COVD $X$ [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]] [OK [LAMBDA $[X Y$ [ $Y$ COND $X T$ [COND $Y T$ NIL]]]]
[AND [LAMBDA $[X Y$ [COND $X$ [COND $Y$ T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND $X$ [COND Y T NIL] T]]]

PROFILE: [/ENR/ENR[BA], /ENマ/ENR.]

TIME: 3.625 SECS:

```
[\begin{array}{lll}{1}&{6}&{6}\end{array}]
[ 16.21 18 JULY 1973]
```

THEDREM TO BE PROVED:
[OR [GI A B] [OR [GT B A] [EQJAL [LENGTH A] [LENGTH B]]]]
WHICH IS EQUIVALENT TO:
[CONA [GT A B] T [COND [GT B A] T [EOUAL [LENGTH A] [LENGTH B]]]]
MUST TRY INDUCTION.
INDIUCT ON A AND B.
THE THEOREM TO BE PROVED IS NOW:
[AND

```
    LAND [COND [GT NIL B] T [COVD [GT 3 NIL] T [EQUAL [LENGTH NIL] [LENGTH B]]]]
    - [COND [GT A NIL] T [COND [GT VIL A] T [EQUAL [LENGTH A] [LENGTH NIL]]]]]
    [IMPLIES [COND [GT A B] T [COND [GT B A] T [EQUAL [LENGTH A] [LENGTH B]]]]
            [COND [GT [CONS A1 A] [CONS B1 B]]
                T
                    [COND [GT [CONS 81 B] [CONS A1 A]]
                            T
                            [EQUAL [LENGTH [COVS A1 A]] [LENGTH [CONS B1 Bl]]]]]]
```

WHICH IS EQUIVALENT TO:
T

```
FUNCTIUN DEFINITIONS:
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]
[LEVGTH [LAMBDA [X] [COND X [CONS NIL [LEVGTH [CDR X]]] 0]]]
[OR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]
[AN!] [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
```

PROFILE: $[/ E N R / E N R[A B], / E N R / E N R$.
IIME: 7.875 SECS.

```
[T 5 7] [ [ 16.21 1.8 JULY 1.973]
```

THEOREM TO RE PROVED:
[EQUAL [MONOT2P A] [MOVOT1 A]]

```
WHICH IS EQUIVALENT TO:
[COND A [EQUAL [MONOT2 [CAR A] [CDR A]] [MONOT1 A]] T]
```

MUST TKY INDUCTION。
(SPECIAL CASE REOUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND
[COND NIL [EQUAL [MONOT2 [CAR NIL] [CDR VIL]] [MONOT1 NIL]] T] [AND [CUNU [CONS A1 NIL]

- [EQUAL [MONOT2 [CAR [CONS A1 NIL]] [CDR [CONS A1 NIL]]]
- T]

LIMPLIES
CDND

- [CONS A2 A]
- [EQUAL [MONOT2 [CAR [CONS A2 A]] [CDR [CONS A2 A]]] [MONOT1 [CONS A2 A]]] - 「] CCOND
[CONS A1 [CONS A? A]]
[EQUAL [MONOT2 [CAR [CONS A1 [CONS A2 A]]] [CDR [CONS A1 [CONS A2 A]]]]

```
* [MONOT1 [CONS A1 [CONS A2 A]J]]
```

TJコ]

```
WHICH IS EQUIVALENT TO:
[COND
    A
    CCOND
    - [EQUAL A2 [CAR A]]
    - [COND [EQUAL [MONOT2 A2 A] [MONOT1 A]]
    - [COND [EQUAL A1 A2] [EQUAL [MONOT2 A1 A] [MONOT1 A]] T]
    . . T]
    - LCOND [MONOT2 A2 A] T [COND [EQUAL
    []
```

FERTILIZE WITH [EQUAL A? [CAR A]].

```
THE THEOREM TO BE PROVED IS NOW:
[COND
    A
    [COND
    - [COND [COND [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]]
    - [COND [EQUAL A1 [CAR A]] [EQUAL [MONOT2 A1 A] [MONOT1 A]] T]
    - TI
. T
- [*1]]
- Cconb
- [COND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]
- T
[EQUAL A2 [CAR A]J]
-NILJ
T」
```

WHICH IS EQUIVALENT TO:
[COND
A
[COND [COND [COND [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]]
[COVD [EQUAL A1 [CAR AJ] [EQUAL [MONOT2 A1 A] [MONOT1 A]] T]
T]

```
- [COND [MONOT2 A2 A]
T
[COND [EQUAL A1 A2] [COND [MJNOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
NIL]
T]
```

FERTILIZE WITH [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]].

```
TME THEOREM TO BE PROVED IS NOW:
CCUND
    A
    [COND
    - CCONU
    - [COND [COND [EQUAL A1 [CAR A]] [EQUA [MONOT2 A1 A] [MONOT2 [CAR A] A]] T]
    - . T
    - • \(\because 2]\)
    - T
    [ H1] \(^{2}\)
    - [CUNU [MONOT2 A2 A]
    - T
    .NIL]
T]
```

FERTILIZE WITH [EQUAL AI [CAR A]].

```
THE THEOREM TO BE PROVED IS NOW:
[CONO
A
[COND
    .LCOND [COND [COND [EQUAL [MONOT2 A1 A] [MONOT2 A1 A]] T [*3]] T [*2]] T [*1]]
-[CUND [MONOT2 A2 A]
- T
- [COMD [ENUAL A1. A2] [COND [MONOT2 A1 A] [EOUAL A2 [CAR A]] T] T]]
.NILJ
T]
```

```
WHICH IS EQUIVALENT TO:
[COND A
    [COND [MONOT2 A2 A]
        T
        [COND [EOUAL A1 AZ] [COND [MOVOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
    T]
FERTILIZE WITH [EQUAL A1 A2].
```

```
THE THEOREM TO bE PROVED IS NOW:
```

THE THEOREM TO bE PROVED IS NOW:
ccovo
ccovo
A
A
[COND [MONOT2 A2 A] T [COND [COND [YONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]
[COND [MONOT2 A2 A] T [COND [COND [YONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]
T]

```
    T]
```

MUST TRY INDUCTION.
INUUCT ON A.
IHE THEOREM TO BE PROVED IS NOW:
LAND
[COND NIL
- [COND [MONOT2 A2 NIL]
T
[COND [COND [MONOT2 A1 VIL] [EQUAL A1 [CAR NIL]] T] T [*4]]]

- T]
[1YPLIES
[COND A
    - [COND [MONOT? A2 A]
- T
- [COND [COND [MONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]

```
    [conis
        [CONS AS A]
        [COND [MONOT2 A2 [CONS A3 A]]
        - T
        - [COND [COND [MONOT2 A1 [CONS AB A]] [EQUAL A.L [CAR [CONS AB A]]] T]
        . T
        @ [*4]]]
        T]]J
```

```
WHICH IS EQUIVALENT TO:
```

「
FUnction definitions:
[MUNOT2P [LAMBDA [X] [CONO X [MONOT2 [CAR X] [COR X]] T]]]
[ MUNOT?
[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] [MONOT2 X [CDR Y]] NIL] T]]]
[MONOT1
[LAMBDA
[ X ]
[CUND
$\times$
$[C O N D[C D R X][C O N D[E Q U A L[C A R X][C A R[C D R X]][M O N O T 1[C D R ~ X]] ~ N I L] T]$
TJ] J
[CARARG UNDEF]
[CDRARG UNDEF]
[IMPLIES [LAMBDA [X Y] [COND $X$ [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
FERTILIZERS:

* $1=[\operatorname{COND}$
[EQUAL A2 [CAF A]]
NIL
[COND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]]
*2 = [COND [EQUAL [MONOTZ [CAR A] A] [MONJT1 A]] NIL T]
*3 $=$ [COND [EQUAL A1 [CAR A]] NIL T]
$* 4=[C O N D[E Q U A L$ A1 A2] NIL T]

```
PROFILE: [/E NR/EENR/ENRRS1[A],/EENR/ENNR/ENRF/N/NR/
ENRF,/X,/ENR/ENRF,/[A],/ENR/ENR.]
```

TIME: 33.06 SECS.

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[T 5 8] [ [ 16.22 18 JULY 1973]
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THLDREM TO BE PROVED: [ORDERED [SORT A]]

MUST TRY INDUCTION.

INOJCT OV A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [URDERED [SORT NIL]]
    [IMPLIES [ORDERED [SORT A]] [ORDERED [SORT [CONS A1 A]]]]]
```

WHICH IS EQUIVALENT TO:
[COND [ORDERED [SORT A]] [ORDERED [ADDTOLIS A1 [SORT A]]] T]
gENERALIZE COMMON SURTERMS BY REPLACING [SORT A] BY GENRL1.
the generalized term is:
[CUND [ORDERED GENRL1] [ORDERED [ADDTOLIS A1 GENRL1]] T]

MUST TRY INDUCTION.

```
(SPECIAL CASE REQUIRED)
```

INDUCT ON GENRL1.

THE THEORE: TO BE PROVED IS NOW:

## [AND

[COND [ORDERED NIL] [ORDERED [ADDTOLIS A1 NIL]] T] [AND
[COND [ORDERED [CONS GENRL11 VIL]]

- [DRDERED [ADDTOLIS A1 [CONS GENRL11 NiL]]]
- T]
[.IAPLIES [COND [ORDERED [CONS GENRL12 GENRL1]]
[ORDERED [ADDTOLIS A1 [CONS gENRL12 GENRL1]]]
T]
[COND [ORDERED [CONS GEVRL11 [CONS GENRL12 GENRL1]]]
[ORDERED [ADDTOLIS A1 [CONS GENRL11 [CONS GENRL12 GENRL1]]]]
T]I]

WHICH IS EQUIVALENT TO:
[CUVD [LTE A1 GENRL11] T [LTE GENRL11 A1]]

MUST TRY INDUCTION.

```
THE THEOREM TO bE PROVED IS NOW:
[AND [AND [COND [LTE A1 NIL] T [LTE VIL A1]]
    [COND [LTE NIL GENRL11] T [LTE GENRL11 NIL]]]
    [IMPLIES [COND [LTE A1 GENRL11] T [LTE GENRL11 A1]]
        [COND [LTE [CONS A11 A1] [CONS GENRL111 GENRL11]]
        T
        [LTE [CONS GENRL111 GENRL11] [CONS A11 A1]]]]]
```

```
WHICH IS EQUIVALENT TO:
```

I

```
FUNCTIUN DEFINITIONS:
[SORT [LAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR XI]] NIL]]]
[OKMERED
    [LAMBDA
        [X]
        CCOND
            x
            [CUND [CUR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
            T]J
[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]
[ADDTOLIS
    [LAMBDA
        [X Y ]
        [COND Y
                    [COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]]
                    [cONS X NILJ]]]
[IMPLIES [LAMBUA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

GEINERALITATIONS:
GENPL. $1=[S O R T A]$

PROFILE: [/[A], /ENR/ENRGS1[GENRL1], /ENR/ENR/ENR/EN
$R / E N R[G E N R L 11 A 1], / E N R / E N R$, ]

## TIME: 57.06 SECS.

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[T 5 9] [ 16.24 1.8 JULY 1973]
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```
THEOREM TO RE PROVED:
[IMPLIES [AND [MONOT1 A] [MEMBER B A]] [EQUAL [CAR A] B]]
```

WHICH IS EQUIVAIENT TO:
[COND [MONOTA A] [COND [MEMBER B A] [EQUA- [CAR A] B] T] T]
MUST TRY INDUCTION.
(SPECIAL CASE REQUIRED)

INDJCT DA A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND]
    [CONi] [MONOTI NIL] [COND [MEMBER B NIL] [EQUAL [CAR NIL] B] T] T]
    [AND
            [COND [MONOT1 [CONS A1 NIL]]
            - [COND [MEMBER B [CONS A1 VIL]] [EQUAL [CAR [CONS A1 NIL]] B] T]
            - T]
            [IMPLIES [COND [MONOT1 [CONS A2 A]]
                            [COND [MEMBER B [CONS A2 A]] [EQUAL [CAR [CONS A2 A]] B] T]
                            T]
            [COND [MONOT. [CONS A1 [CONS A2 A]]]
                    [COND [MEMBER B [CONS A1 [CONS A2 A]]]
                                    [EQUAL [CAR [COVS A1 [CONS A2 A]]] B]
                                    T]
```

                                    T]J]
    ```
WHICH IS EQUIVALENT TO:
[COND
    [COND [EQUAL B A1] [EQUAL A1 B] T]
    CCOND
    .A
    .LCOVU
    - [EOUAL A2 [CAR A]]
    . [CUND [MONOT1 A]
        . [COND [EQUAL B A2]
    - [COND [EOUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
    . [COND [MEMBER B A]
    . [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                        T]]
    • - T]
- T]
.[CUNB [EQUAL B A2]
- [COND [EQUAL A? B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
- T]]
NIL]
```

FERTILIZE WITH [EQUAL B A1].

```
THE THEOREM TO BE PROVED IS NOW:
[CONO
    [COND [EQUAL B B] T [*1]]
    [COND
        - A
        . [CUNO
        - [EQUAL A2 [CAR A]]
        - [COND [MONOT1 A]
        . [COND [EQUAL B AZ]
                        [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                        [COND [MEMBER B A]
                                [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                                    T]]
            T]
        - i
        .[COND [EQUAL. B A2]
        - [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
        - T]J
NIL]
```

```
WHICH IS EQUIVALENT TO:
[Cund
    A
    CCOND
    - [EQual ar [car aj]
    - [COND [MONOT1 A]
    - [COND [EQUAL B Az]
    . [ [COND [EQUAL AZ B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
    . [COND [MEMBER B A]
                                    [COND [EQUAL A2 B] [CONO [EQUAL A1 A2] [EQUAL A1 B] T] T]
                                    T]]
        T]
    - i]
    [COND [EQUAL B A2]
        [COND [EQUAL A2 B].[COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
        T]]
```

FERTILIZE WITH [EQUAL AZ [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
CUND
```

A
CCOND
- [COND
- [MONOT1 A]
- CCOND
- . [EQUAL B [CAR a]]
. . [COND [EQUAL [CAR A] B] [COVD [EQUA: A1 [CAR A]] [EQUAL A1 B] T] T]
. . [COND [MEMBER B a]
. [COND [EQUAL [CAR A] 3] [COND [EQUAL A] [CAR A]] [EQUAL A1 B] T] T]

- . T]
    - TJ
. $T$
[ [ \% 2] ]
[COND [EQUAL B A2]
[COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 R] T] T]
T] ]
fertilize with [Equal b [CAR A]].

```
1HE THEOREM TO BE PROVED IS NOW:
[CUNO
    A
    [CON]
    - Lcona
    - [monotr a]
    - [Cond
    . .[COND [COMD [EQUAL B B] [COND [EQUAL A1 B] [EQUAL A1 B] T] T] T [*3]]
    . .[CONO
    . . [COND [MEmber b a]
    - . [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
    - . TJ
    . . 「
    . . [Equal b [car aj]]
    . .NIL]
    - T]
    .T
    .[*2]]
[COND [EQUAL B A2]
        [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 R] T] T]
        T]]
WHICH IS EQUIVALENT TO:
[CUND
    A
    CCOND
    - [cono
    - [MONOT1 A]
    - CCOND
    - [member b a]
    . . [COND [EQUAL [CAR A] b]
    . . [COND [EQUAL A1 [CAR A]] [COND [EQUAL A1 B] T [EQUAL B [CAR A]]] T]
    - . - T]
    . . 「]
    . TJ
    .T
    -[%2]]
    [COND [EQUAL B A2]
        [COND [EQUAL A2 B] [COND [EQUAL A1 A2].[EQUAL A1 B] T] T]
        T]]
```

FERTILIZE WITH [EQUAL [CAR A] B].

```
THE THEOREM TO bE PROVED IS NOW:
LCOND
    A
    CCOND
    - CCOND
    - [mONOT1 A]
    - [CUND [MEMBER B A]
    - [COND [COND [EQUAL A1 B] [COND [EQUAL A1 B] T [EQUAL B B]] T] T [*4]]
    - . T]
    - T]
    .T
    .[*2]]
    [COND [EQUAL B AZ]
        [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
        T]]
```

WHICH IS EQUIVALENT TO:
[COND A
T
[COND [EQUAL B A2]
[COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
T] ]
FERTILIZE WITH [EQUAL 8 AZ].
THE THEOREM TO BE PROVED IS NOW:
[COND A
T
[COND [COND [EQUAL B B] [COVD [EQUAL A1 B] [EQUAL A1 B] T] T] T [*5]]]
WHICH IS EQUIVALENT TO:
I

```
FUNCTIUN DEFINITIONS:
[MUNOT1
    [lAmBUA
        [X]
        [CONi]
            X
            [COND [CDR X] [COND [EQUAL [CAR X] [CAR [CDR X]]] [MONOT1 [CDR X]] NIL] T]
            T1]]
[MEMBER
                            [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[CARARG UNDEF]
```

FERTILIZERS:
*1 = [COND [EQUAL B A1] NIL T]
*2 $=$ [COND [EQUAL A2 [CAR A]] NIL T]
$* 3=[C O N D$
[EQUAL B [CAR A]]
NIL
[COND [MEMBER R A]
[COND [EQUAL [CAR A] 3] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
T1]
*4 $=[$ COND [EQUAL [CAR A] B] NIL 「]
*5 = [COND [EQUAL B A2] NIL T]

PROFILE: [/ENR/ENRS1[A], /ENR/ENR/ENRF, /ENR/ENR


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[T 6 10] [ 16.26 18 JULY 1973]
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THEOREM TO BE PROVED:
[LTE [CDRN A B] B]

MUST TRy induction.

INDUCT OV A AND B.

THE THEOREM TO BE PROVED IS NOW:
[AND
[AND [LTE [CDRN NIL B] B] [LTE [CDRV A NIL] NIL]]
[IMPLIES [LTE [CDRN A B] B] [LTE [CDRN [CONS A1 A] [CONS B1 B]] [CONS B1 B]]]]

WHICH IS EQUIVALENT TO:
[COND [LTE B B] [COND [LTE [CDRN A B] B] [LTE [CDRN A B] [CONS B1 B]] T] NIL]
(WORK UN FIRST CONJUNCT OVLY)

MUST TRY INDUCTION.

INIJCT ON R.

# THE THEOREM TO bE PROVED IS NOW: <br> [CUND [AND [LTE NIL NIL] [IMPLIES [LTE B 3] [LTE [CONS B2 B] [CONS B2 B]]]] [COND [LTE [CDRN A B3] B3] [LTE [CORN A B3] [CONS B1 B3]] T] NIL] 

## WHICH IS EQUIVALENT TO:

[COND [LTE [CDRN A B3] B3] [LTE [CDRN A B3] [CONS B1 B3]] T]

GENERALIZE COMMON SUBTERMS BY REPLACING [CDRN A B3] BY GENRLI.

The generalized term is:
[CUND LLTE GENRL1 B3] [LTE GENRL1 [CONS B1 B3]] T]

MUST TRY INDUCTION.

INDUCT ON B3 AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:
[AN] [AND [COND [LTE GENRL1 NIL] [LTE GENRL1 [CONS B1 NIL]] T] [COND [LTE NIL B3] [LTE NIL [COVS B1 B3]] T]] [IMPLIES [COND [LTE GENRL1 B3] [LTE GENRL1 [CONS B1 B3]] T]

```
[COND [LTE [CONS GENRL11 GEVRL1] [CONS B31 B3]]
    [LTE [CONS GEVRL11 GEVRL1] [CONS B1 [CONS B31. R3]]]
    T]J]
```

Which is equivalent to:
[CONO [LTE GENRL1 B3]
[COND [LTE GENRL1 [CONS B1 33]] [LTE GENRL1 [CONS B31 33]] T]
「]

MUST TRY INDUCTION.

INDJCT ON B3 AND GENRLI.

THE THEOREM TO BE PROVED IS. NOW:
[ANi]
[AND [COND [LTE GENRLI NIL]
[COND [LTE GENRL1 [CONS B1 VIL]] [LTE GENRL1 [CONS B31 NIL]] T]
.$\quad$ T]

- [COND [LTE NIL B3]
- [COND [LTE NIL [CUNS B1 B3]] [LTE NIL [CONS B31 B3]] T]

TJ]
[IMPLIES [COND [LTE GEVRL1 B3]
[COND [LTE GENRL1 [CONS B1 B3]] [LTE GENRL1 [CONS R31 B3]] T] T]
[COND [LTE [CONS GENRL12 GENR:1] [CONS B32 B3]]
[COND [LTE [CONS GENRL12 GENRLI] [CONS B1 [CONS B3? R3]]] [LTE [CONS GENRL12 GENRL1] [CONS B31 [CONS B32 B3]]] T]
T]J]

FUNCTION DEFINITIONS:
[CDRN [LAMBDA [X Y] [COND $Y$ [COND $X[C D R N ~[C D R ~ X] ~[C D R ~ Y]] ~ Y] ~ N I L]]] ~$
[LTE [Lambda [X Y] [COND $X$ [COND $y$ [ite [CDR X] [CDR Y]] Nil] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
generalizations:
GENRL1 $=[$ CDRN A B3]

PROFILE: $[/[A B], ~ / E N R / E V R \&[B], / E N R / E N R / E N R G[B 3 G E N R L$ 1], /ENR/ENR/EVR[B3GENRL1], /ENR/ENR.]

TIME: 13.06 SECS.

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[T 6 12] [ 16.26 18 JULY 1973]
```

THEOREM TO BE PROVED:
[EQUAL [LENGTH A] [LENGTH [SORT A]]]

MUST fRy induction.

Induct On A.

IHE THEOREM TO BE PROVED IS NOW:
[AND [EOUAL [lengTh Nil] [LENGTH [SORT NI_]]]
[IMPLIES [EQUAL [LENGTH A] [LENGTH [SORT A]]]
[EQUAL [LENGTH [CONS A1 A]] [LENGTH [SORT [CONS A1 A]J]]]]

```
WHICH IS EQUIVALENT TO:
[CuNi [EQUAL [LENGTH A] [LENGTH [SORT A]]]
    [EQUAL [CONS NIL [LENGTH A]] [LENGTA [ADDTOLIS A1 [SORT A]]]]
    T]
```

FERTILIZE WITH [EQUAL [LENGTH A] [LEVGTH [SORT A]]].

```
THE THEOREM TO bE PROVED IS NOW:
[CUND [EQUAL [CONS NIL [LENGTH [SORT A]]] [LENGTH [ADDTOLIS AI [SORT A]]]]
    T
    [*1]]
```

generalize common subterms by Replacing [sort a] by genrli.

The generalized Term Is:
[CUND [EQUAL [CONS NIL [LENGTH GENRL1]] [LENGTH [ADDTOLIS A1 GENRLi]]] T [*1]]

MUST TRY INDUCTION.

INDJCT ON GENRL1.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [COND [EQUAL [CONS NIL [LENGTH NIL]] [LEvGTH [ADDTOLIS A1 NIL]]] T [*1]]
    CLMPLIES
            [COND [EQUAL [CONS NIL [LENGTH GENRLI]] [LENGTH [ADDTOLIS A1 GENRL1]]]
        - T
        [COND [EQUAL. [CONS NIL [LENGTH [CONS GENRL11 GENRL1]]]
                                    [LENGTH [ADDTOLIS A1 [CONS GENRL11 GENRL1]]]]
            T
            [*1]J]]
```

FUNCTION DEFINITIONS:

```
[LENGTH [LAMBDA [X] [COND X [CONS NIL [LEVGTH [CDR X]]] 0]]]
[SORT [LAMBDA [X] [COND X [ADOTOLIS [CAR X] [SORT [CDR X]]] NIL]]]
[ADDTOLIS
    [LAMBDA
        [X Y]
        [CUND Y
                            [COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADOTOLIS X [CDR Y]]]]
                            [CONS X NIL]]]]
[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:
$* 1=[C O N D$ [EQUAL [LENGTH A] [LENGTH [SORT A]]] NIL T]

GENERALIZATIONS:
GENRL1 $=[\operatorname{SORTA}]$


YIME: 13.0 SECS.

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[T 6 14] [ 10́.26 18 JJLY 1973]
```

THEOREM TO BE PROVED:
[IMPLIES [ORDERED A] [EQUAL A [SORT A]]]

WHICH IS EQUIVALENT TO:
[CONO [ORDERED A] [EQUAL A [SORT A]] T]

MUST TRY INDUCTION.
(SPECIAL CASE REQUIRED)

INDUCT ON A.

```
THE THEOREM TO bE PROVED IS NOW:
[AN]
    [COND [ORDERED NIL] [EQUAL NIL [SORT NIL]] T]
    [AND
        [CONU [ORDERED [CONS A1 NIL]] [EQUAL [CONS A1 NIL] [SORT [CONS A1 NIL]]] T]
        [IMPLIES
            [COND [ORDERED [CONS A2 A]] [EQJAL [CONS A2 A] [SORT [CONS A2 A]J] T]
                        [COND [ORDERED [CONS A1 [CONS A2 A]]]
                                    [EQUAL [CONS A1 [CONS A2 A]] [SORT [CONS A1 [CONS A2 A]]]]
                                    T]J]J
```

```
[COND
    A
    [COND
    .[lTE A2 [CAR A]]
    .[CONU
    . [ORDERED A]
    - [COND
    . . [EQUAL [CONS ar a] [ADDTOLIS A2 [SJRT A]]]
. . [COND [LTE A1 A2]
. . . [EQUAL [CONS A1 [CONS A2 A]] [ADDTOLIS A1 [ADDTOLTS AZ [SORT A]]]]
. . . T]
. . T]
- T]
.T]
T]
```

```
FERTILIZE WITH [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]J].
```

THE THEOREM TO BE PROVED IS NDW:
CCOND
A
[COND
- [lte al [car a]]
- [COND [ORDERED A]
. . [COND [COND [LTE A1.A2]

T]
T
[*1]]
T]
T]
WHICH IS EQUIVALENT TO:
I

```
LORDERED
    [LAMBDA
        [X]
        [COND
            X
            [COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
            []]]
[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]
[SORT [LAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR X]]] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[ADDTOLIS
    [LAMBOA
        [X Y ]
        [COND Y
                        [COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]]
                    [CONS X NIL]]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]]] NIL T]
```

```
PROFILE: [/ENR/EN/ENS1[A], / ENR/R/ENR/ENR/RNR/F/R/ENR
/ ENR.]
```

TIME: 20.25 SECS.

```
[T ó 15]
[ 16.27 18 JULY 1973]
```

THEOREM TO BE PROVED:
[IMPLIES [ORDERED [APPEND A B]] [ORDERED A]]

WHICH IS EQUIVALENT TO:
[COND [DRDERED [APPEND A B]] [ORDERED A] 「]

MUST TRY INDUCTION.
(SPECIAL CASE REQUIRED)

INDUCT ON A.

1HE THEOREM TO BE PROVED IS NOW:
[AND
[COND [ORDERED [APPEND NIL B]] [ORDERED NIL] T]
[AND [COND [ORDERED [APPEND [CONS A1 NIL] 3]] [ORDERED [CONS A1 NIL]] T]
[IMPLIES [COND [ORDERED [APPEVD [CONS A2 A] B]] [ORDERED [CONS AZ A]] T] [COND [ORDERED [APPEND [CONS A1 [CONS A2 A]] B]] [ORDERFD [CONS A1 [CONS A? A]]] TJ]J]

```
FUNCTION DEFINITIONS:
[APPEND [LAMRDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
CORDERED
    [LAMBDA
        [X]
        [COND
            X
                [COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
            T]J]
[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ENR/ENRS1[A], /ENR/ENR.]
TIME: 14.69 SECS.

```
[T 5 16]
[. 16.27 18 JULY 1973]
```

```
THEOREM TO BE PROVED:
[IMPLIES [ORDERED [APPEND A B]] [ORDERED 3]]
WHICH IS EQUIVALENT TO:
[COND [ORDERED [APPEND A B]] [ORDERED B] T]
```

MUST TRY INDUCTION.
INDUCT ON A.
THE THEOREM TO BE PROVED IS NOW:
[AND [COND [ORDERED [APPEND NIL B]] [ORDERED B] T]
[IMPLIES [COND [ORDERED [APPEND A B]] [ORDERED B] T]
[COND [ORDERED [APPEND [CONS A1 A] B]] [ORDEREO Q] T]]]
WHICH IS EQUIVALENT TO:
[COND [ORDERED [APPEND A B]] T [COND [APPEND A B] T [ORDERED B]]]
MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND [COND [ORDERED [APPEND NIL B]] T [COVD [APPEN[ NIL B] T [ORDERED B]]] [IMPLIES [COND [ORDERED [APPEND A B]] T [COND [APPEND A B] T [ORDERED B]]] [COND [ORDERED [APPEND [CONS A2 A] B]]

T
[COND [APPEND [CONS A2 A] B] 1 [ORDERED B]]]]]

WHICH IS EQUIVALENT TO:
[COND [ORDERED B] T [COND 8 T NIL]]

MUST TRY INDUCTIDN.
(SPECIAL CASE REQUIRED)

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:
[AN] [COND [ORDERED NIL] T [COND VIL T NIL]]
[AND [COND [ORDERED [CONS B1 NIL]] T [COND [CONS B1 NIL] T NIL.] [IMPLIES [COND [ORDERED [CONS B2 B]] T [COND [CONS B2 B] T NIL]]
[COND [ORDERED [CONS B1 [CONS B2 B]]]
T
[COND [CONS B1 [CONS B2 8]] T NIL]]]]

```
WHICH IS EQUIVALENT TO:
```

$T$

FUNCTIUN DEFINITIONS:
[APPEND [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
CORDERED
[LAMBDA
[X]
[COND
X
[COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T] TJJ]
[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]] [IMPLIES [LAMBDA [X Y] [COND $X$ [COND $Y$ T VIL] T]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]

PROFILE: [/ENR/ENR[A], /ENR/ENR/ENR[A], /ENR/ENRS 1 [8], /ENR.]

IIME: 17.0 SECS.

```
[T 6 13] [ 16.29 18 JULY 1973]
```

THEJREM TO BE PROVED:
[LTE [HALF A] A]

MUST TRY INDUCTION.

## (SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NDW:
[AND [LTE [HALF NIL] NIL] [AND [LTE [HALF [CONS A1 NIL]] [CONS A1 NIL]] [IMPLIES [LTE [HALF A] A] [LTE [HALF [COVS A1 [CJNS A2 A]]] [CONS A1 [CONS A2 A]]]]]]

WHICH IS EQUIVALENT TO:
[COND [LTE [HALF A] A] [LTE [HALF A] [CONS A2 A].] T]

THE GENERALITED TERM IS:
[IMPLItS [NUMBERP GENRL1] [COND [LTE GENRL1 A] [LTE GENRL1 [CONS A2 A]] T]]

MUST TRY INDUCTION.

INDUCT ON A AND gENRLI.

THE THEOREM TO BE PROVED IS NOW:

## [AND

[AND [IMPLIES [HUMBERP GENRL1]
[COND [LTE GENRL1 VIL] [LTE GENRL1 [CONS A2 NIL7] T]]

- [IMPLIES [NUMBERP NIL] [COND [LTE NIL A] [LTE NIL [CONS A2 A]] T]]]

「.IMPLIES
[IMPLIES [NUMBERP GENRL1] [COND [LTE GENRL1 A] [LTE GENRL1 [CONS A2 A]] T]] [IMPLIES [NUMBERP [COVS GENRL11 GENRL1]]
[COND [LTE [CONS GENRL11 GENRL1] [CONS A3 A]]
[LTE [CONS GENRL11 GENRL1] [CONS A2 [CONS A3 A]]] TJJ]

WHICH IS EQUIVALENT TO:
CCOND
[HUMBERP GENRL1]
[COND

- [LTE GENRL1 A]
- [COND [LTE gENRL1 [CONS A2 A]] [COND GENRL11 T [LTE GENRL1 [CONS A3 A]]] T] - T$]$

T]

```
I VUUCT ON A AND GENRLI.
```

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [AVD]
    .[COND [NUMBERP GENRLI]
    - [CONO [LTE GENRL1 NIL]
    - [COND [LTE GENRL1 [CONS A2 vil]]
                                    [COND GENRL11 T [LTE GENRL1 [CONS A3 NIL]]]
                            T]
                            T]
. TJ
    . [cONi
    - [NUMBERP NIL]
- [CONil [LTE NIL A]
. [COND [LTE NIL [CONS A2 A]] [COVD GENRL11 T [LTE NIL [CONS A3 A]]] T]
    T]
        T]」
[IMPLIES
    [COND
        -[NUMbERP gENRLI]
        -[COND
        -[LIE GENRL1 A]
        - [CUND [LTE GENRL1 [CONS A2 A]] [COND GENRL11 T [LTE GENRL1 [CONS A3 A]]] T]
        - T]
        -T]
        [cund
        [NUMBERP [CONS GENRL12 GENRL1]]
        [coivD
        - [LTE [CONS GENRL12 GENRL1] [CONS A4 A]]
        - [COND [LTE [CONS GENRL12 GENRL1] [CONS A2 [CONS A4 A]]]
        . [COND GENRLI1 T [LTE [CONS GENRLI2 GENRLI] [CONS A3 [CONS A4 A]J]]
        - . T]
        . T]
        T]]]
```

WHICH IS EQUIVALENT TO:
I
[HALF [LAMBDA [X] [COND $X$ [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]] [LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [COR Y]] NiL] T]]] [IMPLIES [LAMBDA [X Y] [COND $X$ [COND $Y$ T VIL] T]]]
[AiND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[NUMBERP [LAMBDA [X] [COND X [COND [CAR X] NIL [NUMBERP [CDR X]]] T]]]
gENERALIZATIONS:
GENRL1 $=[$ HALF $A]$

PROFILE: [/ S2 [A], / E NR/ENR G[A GENRL1], /ENR/ENR/ENR[A [ENRLI], /ENR/EVR.J

TIME: 22.81 SECS.

```
[[\begin{array}{lll}{7}&{7}&{1}\end{array}]
[ 16.3 18 JULYY 1973]
```

THEOREM TO BE PROVED:
[EQUAL [COPY A] A]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [COPY NIL] NIL]
[IMPLIFS [AND [EQUAL [COPY A1] A1] [EQUAL [COPY A] A]] [EQUAL [COPY [CONS A1 A]] [CONS A1 A]J]]

## WHICH IS EQUIVALENT TO:

「

FUNCTION DEFINITIONS:
[CUPY [LAMBDA [X] [COND X [CONS [COPY [CAR X]] [COPY [CDR X]]] NIL]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

TIME: 2.438 SECS.

## 4

```
[T 7 2] [ 16.3 1.8 JULY 1973]
```

```
THEOREM TO BE PROVED:
[EQUAL [EQUALP A B] [EQUAL A B]]
```

WHICH IS EQUIVALENT TO:
[COND [EQUAL A B] [EQUALP A B] [COND [EQUALP A B] NIL T]]
FERTILIZE WITH [EQUAL A B].
IHE THEOREM TO BE PROVED IS NOW:
[COND [COND [EQUALP A A] T [*1]]
[COND [COND [EQUALP A B] NIL T] T [EQUAL A B]]
NILJ
WHICH IS EQUIVALENT TO:
[COND [COND [EQUALP A A] T [*1]] [COND [EQUALP A B] [EQUAL A B] T] NIL]
(WORK UN FIRST CONJUNCT ONLY)
MUST TRY INDUCTION.
INUUCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
[COND
    [AND [COND [EgUALP NIL NIL] T [*1]]
    - [IMPLIES [AND [COND [EQUALP A1 A1] T [*1]] [CON[ [EQUALP A A] T [*1]]]
    - [COND [EQUALP [CONS A1 A] [CONS A1 A]] T [*1]]]]
    [COND [EQUALP A2 B] [EOUAL A2 B] T]
    NIL]
```

WHICH IS EQUIVALENT TO:
[COND [EQUALP AZ B] [EQUAL A2 B] T]

MUST TRY INDUCTION.

INDUCT OV A2 AND B.


## FUNCTIUN DEFINITIONS:

[EQUALP
[lambia
[ X Y ]
[CUND
$x$
[COND Y [COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL] [COND Y NIL TJJJ]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

FERTILIZERS:

```
*1 = [COND [EQUAL A B] NIL [COND [EQUALP A B] NIL T]]
```

PROFILE: [//NR/ENRF//NR/ENR\&[A],/ENR/ENR/ENR[A2B ], /ENR/ENR.]

TIME: 14.87 SECS.

```
[T 7 3]
[. 16.31 18 JULY 1973]
```

THETREM TO BE PROVED:
[EQUAL [SUBST A A B] B]

MUST TRY induction。

INDUCT ON B.

THE THEOREM TO bE PROVED IS NOW:
[AND [EQUAL [SUBST A A NIL] NIL]
[IMPLIES [AND [EQUAL [SUBST A A B1] 31] [EQUAL [SUBST A A B] B]] [EOUAL [SUBST A A [CONS B1 3]] [CONS B1 B]]]]

WHICH IS EOUIVALENT TO:
T

FUNCTION DEFINITIONS:
[SUBST
[LAMBDA [X Y Z]
[COND [EQUAL Y 7.]
X
[COND $Z$ [CONS [SUBST $X$ Y [CAR Z]] [SUBST $X$ Y [CDR Z]]] NIL]]]]
[AND [LAMBDA $[X Y][C O N D X[C O N D Y T N I L]$ NIL]]]
[IMPLIES [LAMBDA [X Y] [COND $X$ [COND $Y$ T VIL] T]]]

PROFILE: [/ [B], / ENR/ENR/ENR.]

TIME: 6.125 SECS.

```
[T 7 4 ] [ [ 16.31 18 JULLY 1973]
THEOREM TO BE PROVED:
[IMPLIES [MEMBER A B] [OCCUR A B]]
WHICH IS EQUIVALENT TO:
[CONA [MEMBER A B] [OCCUR A B] T]
```

must try induction.

INDUCT DN $B$.

1HE THEOREM TO BE PROVED IS NOW:
[AND

```
    [COND [MEMBER A NIL] [OCCUR A NIL] T]
    [IMPLIES
        [AND [COND [MEMBER A B1] [OCCUR A B1] T] [COND [MEMBER A B] [OCCUR A B] T]]
        [CUND [MEMBER A [CONS B1 B]] [OCCUR A [CONS B1 B]] T]]]
```

```
WHICH IS EQUIVALENT TO:
[COND
    [MEMBER A R1]
    T
    [COND [MEMBER A B]
        T
        [COND [EQUAL A B1]
        [COND [EQUAL A [COVS 31 B]] T [COND [OCCUR A B1] T [OCCUR A B]]]
        T]J]
```

```
FERTILIzE WITH [EQUAL A B1].
```

THE THEOREM TO BE PROVED IS NOW:
[COVI)
[MEMBER A B1]
T
[COND [MEMBER A B]
T
[COND [COND [EQUAL A [CONS A B]] T [COND [OCCUR A A] T [OCCUR A B]]]
T
[\#1]」]
WHICH IS EQUIVALENT TO:
T

FUNCTION DEFINITIONS:
LME ABER
[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[OCCUR
[LAMBDA [. $X$ Y]
[COND [EQUAL $X Y]$
T

[IMPLIES [LAMBDA [ $X$ Y] [COND $X$ [COND $Y$ T VIL] T]]]
[AND [LAMBDA [ $X Y$ ] [COND $X$ [COND $Y T N I L] N I L]]$

FERTILIZERS:
*1 = [COND [EQUAL A B1] NIL T]

## PROFILE: [/ENR/ENR[B], /ENR/ENR/ENRF, /ENR/]

TIME: 12.44 SECS.

```
[T 7 5]
[ 16.31 18 JULY 1973]
```

THEOREM TO RE PROVED:
[IMPLIES [NOT [OCCUR A B]] [EQUAL [SUBST S A B] B]]

```
WHICH IS EQUIVALENT TO:
[COND [OCCUR A B] T [EQUAL [SUBST C A B] B]]
```

MUST TKY INDUCTION.
INDICT ON B.
THE THEOREM TO BE PROVED IS NOW:
[AND
[COND [OCCUR A NIL] T [EQUAL [SU3ST C A VIL] NIL]]
[IMPLIES
[ANU [COND [OCCUR A B1] T [EQUAL [SUBST C A B1] B1]]
- [COND [OCCUR A B] T [EQUAL [SUBST C A B] B]]]
[COND [OCCUR A [CONS B1 B]] T [EQUAL [SUBST C A [CONS BI B]] [CONS B1 B]]]]]
WHICH IS EQUIVALENT TO:
1
coccur

```
        [LAMBDA [X Y]
            [COND [EQUAL X Y]
        T
                            [COND Y [COND [OCCUR X [CAR Y]] T [OCCUR X [COR Y]]] NIL]]]]
[NUT [LAMBDA [X] [COND X NIL T]]]
[SUBST
    [LAMBDA [X Y Z]
        [COND [EOUAL Y Z]
        X
                            [COND Z [CONS [SUBST X Y [CAR Z]] [SUBST X Y [CDR Z]]] NIL]]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y. T VIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
PRUFILE: [//ENR/ENR[B],/ENR/EN/EN/ENR
```

TIME: 14.19 SECS.

```
[T 7 6] [ 16.31 18 JilLY 1973]
```

THE:JREM TO BE PROVED:

```
[EQJAL [EQUALP A B] [EQUALP B A]]
```

must try induction.
INDUCT ON B AND A.

```
THE THEOREM TO BE PROVED IS NOW:
[AN!
    [aNi] [EqUAL [EQUALP A NIL] [EQUALP NIL A]]
    - [EQUAL [EQUALP NIL 3] [EQUALP B NIL]]]
    CIMPLIES
        [AND [EQUAL [EQUALP A1 31] [EQUALP B1 A1]] [EQUAL [EQUALP A R] [EQUALP B A]]]
        [EQUAL [EQUALP [CONS A1 A] [CONS B1 B]] [EQUALP [CONS R1 B] [CONS A1 A]]]]]
```

```
WHICH IS EQUIVALENT TO:
[CUND [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]]
    [COND [EQUAL [EQUALP A B] [EQUALP B A]]
        [COND [EQUALP A1 B1]
        [COND [EQUALP B1 A1] T [COND [EQUALP A B] NIL. T]]
        [COND [EQUALP B1 A1] [CJNO [EQUALP B A] NIL T] T]]
        T]
    T]
```

```
THE THEOREM TO bE PROVED IS NOW:
[COND [COND [EOUAL [EQUALP A B] [EQUALP B A]]
    [COND [EQUALP A1 B1]
                        [COND [EOUALP A1 B1] T [COND [EQUALP A B] NIL T]]
                        [COND [EQUALP A1 B1] [CJND [EQUALP B A] NIL T] T]]
        T]
I
[*1]]
```

WHICH IS EQUIVALENT TO:
I
FUNCTION DEFINITIONS:
[EQJALH
[Lambua
$\left[\begin{array}{ll}\mathrm{X} & \mathrm{Y}\end{array}\right]$
[CONO
$x$
[COND Y [COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL]
[COND Y NIL TJJ]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
FERTILIZERS:
*1 = [COND [EQUAL [EQUALP A1 B1] [EQJALP 31 A1]] NIL T]
PRUFILE: [/ [RA], /ENR/ENRF, /R/ENR •]
FIME: 10.69 SECS.

```
[T 7 7]
[ 16.32 1.8 JULY 1973]
```

THEOREM TO BE PROVED:

```
[IMPLIES [AND [EQUALP A B] [EQUALP B C]] [EQUALP A C]]
```

WHICH IS EOUIVALENT TO:
[CUND [EQUALP A B] [COND [EQUALP B C] [EQJALP A C] T] T]
must try induction.
INDUCT ON R, A AND C.

THE THEOREM TO RE PROVED IS NOW:

## [AND]

```
[ANi] [COND [EQUALP A NIL] [COND [EQJALP NIL C] [EQUALP A C] T] T]
. [AND [COND [EQUALP NIL B] [COND [EQJALP B C] [EQUALP NIL C] T] T]
- [COND [EQUALP A B] [COND [EQUA_P B NIL] [EQUALP A NIL] T] T]]]
[IMPLIES
    [AND [COND [EQUALP A1 R1] [COND [EQUALP B1 C1] [EQUALP A1 C1] T] T]
    - [COND [EQUALP A B] [COND [EQUALP B C] [EQUALP A C] T] T]]
    ccond
[EQJALP [CONS A1 A] [CONS B1 B]]
[COND [EQUALP [CONS B1 B] [CONS C1 C]] [EQUALP [CONS A1 A] [CONS C1 C]] T]
T].]
```


## WHICH IS EQUIVALENT TO:

[COND [EQUALP A B] [COND a T [COND A NIL T]] T]

```
MUST TRY INDUCTION.
```

INDUCT ON A AND B.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [AND [COND [EQUALP NIL B] [COND B T [COND NIL NIL T]] T]
    [COND [EQUALP A NIL] [COND NIL T [COND A NIL T]] T]]
    [IMPLIES [AND [COND [EQUALP AZ 32] [COND B2 T [COND AZ. NIL T]] T]
            [COND [EQUALP A B] [COND B T [COND A NIL T]] T]]
            [COND [EQUALP [CONS A2 A] [CONS B2 B]]
                        [COND [CONS B2 B] T [COND [CONS A2 A] NIL T]]
                            T]J]
```

WHICH IS EQUIVALENT TO:
1
FUNCTIUN DEFINITIONS:
[EQUALP
「LAMBDA
[ X Y]
condo
X
[COND Y [CONI [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL]
[COND Y NIL TJ]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

## NR/ENR.J

TIME: 29.88 SECS.

```
[\begin{array}{lll}{7}&{7}\end{array}]
[ 16.33 18 JULY
1973]
```

```
IHEOREM TO BE PROVED:
[EQJAL [SWAPTREE [SWAPTREE A]] A]
```

MUST TRY INDUCTION.
(SPECIAL CASE REQUIRED)

IND:JCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND
    [g\UAL [SWAPTREE [SWAPTREE NIL]] NIL]
    [AND
        [EQUAL [SWAPTREE [SWAPTREE [CONS A1. NIL]]] [CONS A1 NIL]]
        [IMPLIES
            [AND [EQUAL [SWAPTREE [SWAPTREE A2]] A2] [EQUAL [SWAPTREE [SWAPTREE A]] A]]
            [EQUAL [SWAPTREE [SWAPTREE [COVS A1 [CJNS A2 A]]]] [CONS A1 [CONS A2 A]]]]]]
```

WHICH IS EQUIVALENT TO:
T

```
[SWAPTREE
    [LAMBUA
        [x]
        [COND
        X
        [COND
        - [CAR X]
        - X
        . [COND [CDR X]
        - [CONS NIL [CONS [SWAPTREE [CDz [CDR X]]] [SWAPTREE [CAR [CDR X]]]]]
        - X]]
        NILJ]J
[ANO [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
```

PROFILE: [/ S2 [A], /ENR/ENR/ENR.]

TIME: 7.875 SECS.

```
[T 7 9] [ 16.33 18 JULY 1973]
```

THEOREA TO BE PROVED:
[EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]
must thy induction.
(SPECIAL CASE REQUIRED)

INDUCT ON A.

```
THE IHEOREM TO bE PROVED IS NOW:
[AND
    [EQUAL [FLATTEN [SWAPTREE NIL]] [REVERSE [FLATTEN NIL]]]
    [AND
        [E,QUAL [FLATTEN [SWAPTREE [CONS A1 NIL]]] [REVERSE [FLATTEN [CONS A1 NIL]]]]
        [IMPLIES [AND [EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]]
                                    [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]]
                                    [EQUAL [FLATTEN [SWAPTREE [COVS A1 [CONS A2 A]]]]
                                    [REVERSE [FLATTEN [CONS A1 [CONS A2 A]]]]]]]]
```

WHICH IS EQUIVALENT TO:
[COND
[EQUAL [FLATTEN [SWAPTREE A2]] [REVEZSE [FLATTEN A2]]]
[COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]J]
- 「COND A1
- T
- [EQUAL [APPEND [FLATTEN [SWAPTREE A]] [FLATTEN [SWAPTREE A2]]]
. [REVERSE [APPEND [FLATTEN A2] [FLATTEN A]J]]]
- T]

T]

FERTILIZE WITH [EQUAL [FLATTEN [SNAPTREE A2]] [REVERSE [FLATTEN A2]]].

```
THE THEOREM TO BE PROVED IS NOW:
[CUND [COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]
    [COND A1
                            [EOUAL [APPEND [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A2]]]
                                    [REvERSE [APPEND [FLATTEN AZ] [FLATTEN A]]]]]
            T]
}
[*1]]
```

FEKTILIZE WITH [EQUAL [FLATTEN [SNAPTREE A]] [REVERSE [FLATTEN A]].].

```
IHE THEOREM TD BE PROVED IS NOW:
[CUNI [COND [COND A1
                            [EQUAL [APPEND [REVERSE [FLATTEN A]] [REVERSF [FLATTEN A2]]]
                                    [REVERSE [APPEND [FLATTEN A2] [FLATTEN A]]]]]
            T
            [:2]]
    T
    [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [FLATTEN A] BY GENRL1 AND [FLATTEN A2] B
Y GFNRL2.

```
            [EOUAL [APPENO [REVERSE GENRL1] [REVERSE GENRL2]]
                        [REVERSE [APPEND GENRL2 GENRL1]]]]
            T
            [%2]]
T
[*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL?。

```
IHE THEOREM TO BE PROVED IS NOW:
[AND
    [COND [COND [COND A1
    - T
                                    [EQUAL [APPEND [REVERSE GENRL1] [REVERSE NIL]]
                                    [REVERSE [APPEND NIL GENRL1]]]]
                T
                    [*2]]
            T
            [*1]]
    [IMPLIES
        [COND [COND [COND A1
            - T
            [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
                        [REVERSE [APPEVD GENRL2 GENRL1]]]]
                    T
                    [*2]]
            T
            [*1]]
    Lcunu
        CCOND [COND A1.
    . T
    . [EQUAL [APPEND [REVERSE GEVRL1] [REVERSE [CONS GENRL21 GENRL2]]]
                                [REVERSE [APPEVD [CONS GEVRL21 GENRL2] GENRL1]]]]
    . T
    . [*2]]
    T
    [*1]]]]
```

```
WHICH IS EQUIVALENT TO:
LCONO
    [COND [COND [COND A1 T [EQUAL [ADPEND [REVERSE GENRL1] NIL] [REVERSE GENRL1]]]
                        T
                    [*2]]
        T
        [*1]]
    COND
    -A1
    . '
    - ccund
    - [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
    . . [REVERSE [APPEND GENRL2 GENRL1]]]
    - [COND
    . .ccund
    . . [EgJal
    . . . [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
    . . . [APPEND [REVERSE [APPEND GEVRL2 GENRL1]] [CONS GENRL21 NILJ]]
    . . T
    . . [*2]]
    . . T
    . .[*1]]
    - T]]
    NIL.]
```

FFRTILIZF WITH [EQUAL [APPEND [REVERSE GEVRL1] [REVERSE GENRL2]] [REVERSE [APPEVD GEVRL2 GENRL.1]]].

```
THE THEOREM TO BE PROVED IS NOW:
[CUND
    [COND [COND [COND A1 T [EQUAL [APPEND [REVERSE GENRLI] NIL] [REVERSE GENRL1]]]
. T
                                    [*2]]
. T
- [%1]]
[COND
.A1
.1
. cconb
- CCOND
. .[COND
. . cegual
. . . [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
. . . [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
. . T
. . [*2]]
```

. . T
. . [ $\because 1]$ ]

- T
-[\#3」]] NIJ


## (WURK UN FIRST CONJUNCT OVLY)

generalize common subterms by replacing [zeverse genrli] by genrlu.

THE GENERALIZED TERM IS:
[COND [COND [COND A1 T [EQUAL [APPEND GENRL3 NIL] GENRL3]] T [*2]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON GENRLZ.

```
THE THEOREY TO BE PROVED IS NOW:
[CUND
    [AND
    -[CONO [COND [COND A1 T [EQUAL [APPEND N[L NIL] NIL]] T [*2]] T [*1]]
    .ETMPLIES
    . [COND [COND [COND A1 T [EQUAL [APPEND GENRL3 NIL] GENRL3]] T [*2]] T [*1]]
    - rcond
    - [CUND
    . . [COND A1
    . . . T
    . . . [EQUAL [APPEND [CONS GENRL31 GENRL3] NIL] [CONS GENRL31 GENRL3]]]
    . . T
    . . [%2]]
```

```
[ [*1]]]]
[COND
. A }1
.T
- LCOND
- CCDND
- [COND
- . egu等
. . [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
. . [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
. . T
- [ [*2]]
. . T
. . [*1]]
- T
-[*3]]]
N[1]
```

```
WHIGH IS EQUIVALENT TO:
CCUND
    A.1
T
[CDND
    [coNB
    -[COND
    - [E\UAL
    . . [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [.CONS GENRL21 NIL]]]
    - [APPENU [APPFND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
    - T
    -[.2]]
    .T
    [[*1]]
    I
    [:3]]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE GENRL2] BY GENRL 4 AND [REVERSE GENRLA] BY GENRLS.

THE GENERALIZED TERM IS:
[CuVD.
A1
T
CCOND [COND [COND [EQUAL [APPEND GENRLう [APPEND GENRL4 [CONS GENRL21 NIL]]]

```
                        [*2]]
                    T
        [*1]]
T
[*3]]]
```

must try induction.

INDUCT ON GENRL5.

THE THEOREM TO BE PROVED IS NOW:
[AND [COND

- A1
- $\quad \mathrm{T}$
- [COND [COND [COND [EQUAL [APPEND NIL [APPEND GENRL4 [CONS GENRL21 NIL]]] [APPEVD [APPEND NIL GENRL4] [CONS GENRL21 NIL]]] T
[*2]]


## T

[*1]]
T
]
[IMPLIES
Ccond

- A1
- $T$
- [COND [COND [COND [EQUAL [APPEVD GENRL; [APPEND GENRL4 [CONS GENRL21 NIL]]]
- T
[*2]]
- T
[*1]]
- $\quad$ T
- [*3] ]

LCone
A1
T
CCOND CCOND - CCOND

- [EQUAL [APPEND [CONS GENRL51 GENRL5] [APPEND GENRL4 [CONS GENRL21 NIL]]]
- [APPEND [APPEND [CONS GENRL51 GENRL5] GENRL4] [CONS GENRL21 NIL]]] $\left[\begin{array}{lll}T & 7 & 9\end{array}\right]$
- T
- $[* 2]]$
- $T$
.[*1]]
1
[*3]]]]


## WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

## [SWAPTREE

[LAMBUA
[×]
[COVD
X
CCOND

- [CAR X]
- $X$
- [COND [CDR X]
- [CONS NIL [CONS [SWAPTREE [CDR [CDR X]]] [SWAPTREE [CAR [CDR X]]]]] - X]J

N1LJJJ
[FLATTEN
[LAMBDA
[X]
[COND $X$
[COND [CAR X]
[CONS X NIL]
[COND [CDR X]
[APPEND [FLATTEN [CAR [CDR X]]] [FLATTEN [CDR [CDR X]]]] [CONS X NILJ]]
[CONS NIL NILJJ]]
[REVERSE
[LAMBDA [X] [COVD X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]
[APPENU [LAMBDA [X Y] [COND X [COVS [CAR X] [APPEND [CDR X] Y]] Y]]]
[AND [LAMBDA [X Y] [COND $X$ [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

FERTILIZERS:

```
*1 = [COND [EQUAL [FLATTEN [SWAPTREE AZ]] [REVERSE [FLATTEN A2]]] NIL T]
*2 = [COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]] NIL T]
*3 = [COND [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
    [REVERSE [APPEND GENRL2 GENRL1]]]
    NIL
```

```
[T 7 10] [. 10.36 18 JULY 1973]
```

```
THEOREM TO BE PROVED:
[EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]
```

MUST TRY INDUCTION.
(SPECIAL CASE REQUIRED)

INDUCT ON A.

```
THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [LENGTH [FLATTEN NIL]] [TIPCOJNT NIL]]
    [AND [EOUAL [LENGTH [FLATTEN [CONS A1 NIL]]] [TIPCOUNT [CONS A1 NIL]]]
        [IMPLIES [AND [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT AZ]]
                            [EgUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]]
                [FQUAL [LENGTH [FLATTEV [CONS A1 [CONS A2 A]]]]
                        [TIPCOUNT [CONS A1 [CONS A2 A]J]]].]]
```

WHICH IS EQUIVALENT TO:
[CONO [EQUAL [LENGTH [FLATTEN A2]] [TIPCOJNT A2]]
[COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]
[COND A1
T
[EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]]
[PLUS [TIPCOUNT AZ] [TIPCOUNT A]]]]
T]
T J

FERTILIZE WITH [EQUAL [LEVGTH [FLATTEN A2]] [TIPCOUNT A2]].

THE THEOREM TO BE PROVED IS NOW:
[COND [COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]] [COND A1
$\top$
[EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]] [PLUS [LENGTH [FLATTEN A2]] [TIPCOUNT A]]]]
T]
I
[*1]]

FERTILIZE WITH [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]].

The Theorem to be proved is now:
[COND [COND [COND $\begin{gathered}\text { A1 } \\ T\end{gathered}$
[EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]] [PLUS [LENGTH [FLATTEN A2]] [LENGTH [FLATTEN A]]]]]

```
T
    [%2]]
    T
    [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [FLATTEN A] BY GENRLI AND [FLATTEN A2] B y GENRL2.
the generalized term is:
[CONO [COND CCOND A1
T
[EQUAL [LENGTH [APPEND GENRL2 GENRL1]] [PLUS [LENGTH GEvRL?] [LENGTH GENRL1]]]]

T

```
    [*2]]
T
[*1]]
```

MUST TRY INDUCTION．

INDUCT ON GENRL2．

THE THEOREM TO bE PROVED IS NOW：

## ［AND

［COND
－［COND
－［COND
－ $\mathrm{A}^{\mathrm{A}}$
．．T
．．［EQUAL［LENGTH［APPEND NIL GENRL1］］［PLUS［LENGTH NIL］［LENGTH GENRLI］］］］
－T
－［＊2］］
－$「$
［：\％1］］
［IMPLIES ［COND［COND［COND A1

［EQUAL［LËNGTH［APPEND GENRL2 GENRL1］］ ［PLUS［LEVGTH GENRL2］［LENGTH GENRL1］］］］
－T
T
［＊2］］
－$\quad T$
－［\％17］
CCOND
［COND［COND A1

－［EQUAL［LENGTH［APPEND［CONS GENRL21 GENRL2］GENRL1］］
.$\quad$ T
－「＊2丁］
T
［＊1］」］

## WHICH IS EQUIVALENT TO:

T

FUNCTIUN DEFINITIONS:
cFlatten
[LAMBDA
[×]
[COND $X$
[COND [CAR X]
[CONS X VIL]
[COND [CDR X]
[APPEND [FLATTEN [CAR [CDR X]]] [FLATTEN [CDR [CDR X]]]]
[CONS X NIL]]
[CONS NIL NILJ]J]
[LENGTH [LAMBDA [X] [COND X [CONS NIL [LEVGTH [COR X]]] 0]]]
[tIPCOUNT
[LAMBDA
[×]
ccond
$x$
CCDND

- [CAR X]
- 1
. [COND [CDR X] [PLUS [TIPCOUNT [CAR [CDR X]]] [TIPCOUNT [CDR [CDR X]]]] 1]]
1〕]
[AND [LAMBDA $[X Y][C O N D X[C O N D Y$ T NIL] NIL]]]
[APPEND [LAMBDA $[X Y][C O N D X[C O N S ~[C A R ~ X] ~[A P P E N D ~[C D R ~ X] ~ Y]] ~ Y]]] ~$
[PLUS [LAMBDA [X Y] [COND $X$ [CONS NIL [PLUS [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA $[X Y$ [ $Y$ COND $X$ [COND $Y$ T VIL] T]]]

FERTILIZERS:

```
*1 = [COND [EQUAL [LENGTH [FLATTEV A2]] [TIPCOUNT A2]] NIL T]
*2 = [COND [EQUAL [LENGTH [FLATTEV A]] [TIPCOUNT A]] NIL T]
```


## GENRL2 $=[F L A T T E N$ A2]

## GENRLI $=[F L A T T E N A]$

PROFILE: [/ S2 [A], / ENR/ENR/ENRF, /F, /G[GENRL2] , /ENR/ ENR.]

TIME: 25.0 SECS.

```
[\begin{array}{lll}{1}&{3}&{2}\end{array}]
[ 16.37 1.8 JuLY 1973]
```

THEOREM TO BE PROVED:
[EQUAL [LINEAR [BINARYOF V]] N]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:
[AND [EQUAL [LINEAR [BINARYOF NIL]] NIL] [IMPLIES [EQUAL [LINEAR [AINARYOF N]] N] [EQUAL [LINEAR [RINARYOF [CONS NIL N]]] [CONS NIL N]]]]

## WHICH IS EQUIVALENT TO:

```
[CUND [EOUAL [LINEAR [BINARYOF N]] N]
    [EQUAL [LINEAR [RINADD [CONS 1 NIL] [BINARYOF N]]] [CONS NIL N]]
    T]
```

FERTILIZE WITH [EQUAL [linear [binaryof N]] N].

THE THEOREM TO BE PROVED IS NOW:
[COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] [BINARYOF N]]] [CONS NIL [LINEAR [BINARYOF V]J]]
generalize common surterms by replacing [binaryof n] by genrli.

```
IHE GENERALIZED TERM IS:
[CUND [EOUAL [LINEAR [BINADO [CONS 1 NIL] GENRLI]] [CONS NIL [LINEAR GENRL1]]]
    T
    [*1.]]
```

MUST TRY INDUCTION.

INDUCT ON GENRLI.

```
IHE THEOREM TO BE PROVED IS NOW:
[AN]
    [COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] NIL]] [CONS NIL [LINEAR NIL]]]
        - T
        - [*1]]
        [IMPLIES
        [cond
    - [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRLI]] [CONS NIL [LINFAR GENRL1]]]
        - T
    - [*1]]
    [CONU [EQUAL [LINEAR [BINADD [CONS 1 NIL] [CONS GENRL11 GENRL1]]]
        [CONS NIL [LINEAR [CONS GEVRL11 GENRL1]]]]
            T
            [*1]]!]
```

```
[CONO [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]
    [COND [COND GENRL11
                        [EQUAL [DOUSLE [LIVEAR [RINADD [CONS 1 NIL] GENRL1]]]
                        [CONS NIL [CONS VIL [DOUBLE [LINEAR GENRLI]]]]]
                    T]
        T
        [%1]]
    T]
```

FERTILIZE WITH [EQUAL [LINEAR [BINADD [COVS 1 NIL] GENRLI]]
[CONS NIL [LINEAR GENRLi]]].

```
IHE THEOREM TO BE PROVED IS NOW:
[CUND [COND [COND GENRL11
    [EQUAL [DOUBLE [COVS NIL [LINEAR GENRL1]]]
                                    [CONS NIL [CONS VIL [DOUBLE [LINEAR GENRL1]]]]]
            T]
        T
        [*1]]
    T
    [*2]]
```

WHICH IS EQUIVALENT TO:
T
FUNCTIUN DEFINITIDNS:
[BIVARYOF [LAMBDA [X] [COND X [BINADD [COVS 1 NIL] [BINARYOF [CDR X]]] NIL]]]
[lineak [lambda [X]
[COND $X$
[COND [CAR X]
[CONS NIL [DOJBLE [LINEAR [CDR X]]]]
[DOUBLE [LINEAR [CDR X]]]]
vilu]

```
[BINADO
    [lambi]a
        [X Y]
```

[CONO
$x$
LCOND $Y$

- [COND [CAR X]
- [COND [CAR Y]
[CONS 0 [BINADD [CONS 1 NIL] [BINADD [CDR $X][C D R ~ Y]]]]$
[CONS 1 [BINADD [CDR $X][C D R ~ Y]]]]$
$\therefore \quad[C O N S[C A R ~ Y][B I N A D D[C D R ~ X][C D R ~ Y]]]]$
- $X]$

Y] ] ]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[ANi] [LAABDA [X Y ] [COND $X$ [COND $Y T$ NIL] NIL]]]
[DOUBLE [LAMBDA [X] [COND $X$ [CONS NIL [COVS NIL [DOUBLE [CDR X]]]] 0]]]

FERTILIZERS:
$* 1=[C O N D$ [EQUAL [LINEAR [BINARYOF N]] iv NIL T]
$* 2=[C O N D$
[EQUAL [LINEAR [BINADD [COVS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]] NIL
T]

GENERALIZATIONS:
GENRL1 $=$ [BINARYOF N]

PRUFILE: [/ [N], /ENR/ENRX, / $\quad$ / [GENRL1], /ENR/ENR/ENRF , / ENR.]

TIME: 19.81 SECS.

