16.03HRS. 18 JULY 1973. ** RSB **

[THEOREMS PROVED]

```
[T 1 1]     [ 16.03 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [APPEND A [APPEND B C]] [APPEND [APPEND A B] C]]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [APPEND NIL [APPEND B C]] [APPEND [APPEND NIL B] C]]
 [IMPLIES
  [EQUAL [APPEND A [APPEND B C]] [APPEND [APPEND A B] C]]
  [EQUAL [APPEND [CONS A1 A] [APPEND B C]] [APPEND [APPEND [CONS A1 A] B] C]]]]


WHICH IS EQUIVALENT TO:

|


FUNCTION DEFINITIONS:

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [A] , / E N R / E N R .]

TIME: 4.813 SECS.

```
[T 1 2]     [ 16.03 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C] T]



MUST TRY INDUCTION.



INDUCT ON A.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [EQUAL [APPEND NIL B] [APPEND NIL C]] [EQUAL B C] T]
 [IMPLIES
   [COND [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C] T]
   [COND [EQUAL [APPEND [CONS A1 A] B] [APPEND [CONS A1 A] C]] [EQUAL B C] T]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:
```

```
                                                        [T 1 2]
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


PROFILE:  [/ E N R / E N R [A] , / E N R / E N R .]


TIME:  4.188 SECS.
```

```
[T 1 3]      [ 16.04 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]


MUST TRY INDUCTION.


INDUCT ON B.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [LENGTH [APPEND A NIL]] [LENGTH [APPEND NIL A]]]
 [IMPLIES
      [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]
      [EQUAL [LENGTH [APPEND A [CONS B1 B]]] [LENGTH [APPEND [CONS B1 B] A]]]]]


WHICH IS EQUIVALENT TO:

[COND
 [EQUAL [LENGTH [APPEND A NIL]] [LENGTH A]]
 [COND [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]
 .      [EQUAL [LENGTH [APPEND A [CONS B1 B]]] [CONS NIL [LENGTH [APPEND B A]]]]
 .      T]
 NIL]


FERTILIZE WITH [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]].
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 [EQUAL [LENGTH [APPEND A NIL]] [LENGTH A]]
 [COND [EQUAL [LENGTH [APPEND A [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A B]]]]
 .      T
 .      [*1]]
 NIL]
```

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 [AND [EQUAL [LENGTH [APPEND NIL NIL]] [LENGTH NIL]]
 .     [IMPLIES [EQUAL [LENGTH [APPEND A NIL]] [LENGTH A]]
 .              [EQUAL [LENGTH [APPEND [CONS A1 A] NIL]] [LENGTH [CONS A1 A]]]]]
 [COND
 .    [EQUAL [LENGTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]]
 .    T
 .    [*1]]
 NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND
     [EQUAL [LENGTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]]
     T
     [*1]]
```

MUST TRY INDUCTION.

INDUCT ON A2.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND
 . [EQUAL [LENGTH [APPEND NIL [CONS B1 B]]] [CONS NIL [LENGTH [APPEND NIL B]]]]
 . T
 . [*1]]
 [IMPLIES
  [COND
  .   [EQUAL [LENGTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]]
  .   T
  .   [*1]]
  [COND [EQUAL [LENGTH [APPEND [CONS A21 A2] [CONS B1 B]]]
                [CONS NIL [LENGTH [APPEND [CONS A21 A2] B]]]]
        T
        [*1]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]] NIL T]
```

```
PROFILE:  [/ [B] , / E N R / E N R X , / & [A] , / E N R / E N R / E N R [A2] ,
/ E N R / E N R .]
```

TIME: 16.12 SECS.

```
[T 1 4]      [ 16.04 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [REVERSE [APPEND NIL B]] [APPEND [REVERSE B] [REVERSE NIL]]]
     [IMPLIES [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]
              [EQUAL [REVERSE [APPEND [CONS A1 A] B]]
                     [APPEND [REVERSE B] [REVERSE [CONS A1 A]]]]]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL [REVERSE B] [APPEND [REVERSE B] NIL]]
      [COND [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]
            [EQUAL [APPEND [REVERSE [APPEND A B]] [CONS A1 NIL]]
                   [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]]
            T]
      NIL]


FERTILIZE WITH [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]].


THE THEOREM TO BE PROVED IS NOW:
```

```
[COND [EQUAL [REVERSE B] [APPEND [REVERSE B] NIL]]
      [COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]
                   [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]]
            T
            [*1]]
      NIL]
```

(WORK ON FIRST CONJUNCT ONLY)

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE B] BY GENRL1.

THE GENERALIZED TERM IS:

[EQUAL GENRL1 [APPEND GENRL1 NIL]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 [AND
 .  [EQUAL NIL [APPEND NIL NIL]]
 .  [IMPLIES [EQUAL GENRL1 [APPEND GENRL1 NIL]]
 .           [EQUAL [CONS GENRL11 GENRL1] [APPEND [CONS GENRL11 GENRL1] NIL]]]]
 [COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]
 .            [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]]
 .      T
 .      [*1]]
 NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]
             [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE A] BY GENRL2 AND [REVERSE B] BY GENRL3.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [APPEND [APPEND GENRL3 GENRL2] [CONS A1 NIL]]
             [APPEND GENRL3 [APPEND GENRL2 [CONS A1 NIL]]]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL3.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [APPEND [APPEND NIL GENRL2] [CONS A1 NIL]]
 .             [APPEND NIL [APPEND GENRL2 [CONS A1 NIL]]]]
 .      T
 .      [*1]]
 [IMPLIES
     [COND [EQUAL [APPEND [APPEND GENRL3 GENRL2] [CONS A1 NIL]]
     .             [APPEND GENRL3 [APPEND GENRL2 [CONS A1 NIL]]]]
```

```
.      T
                                                                      [T 1 4]

.      [*1]]
[COND [EQUAL [APPEND [APPEND [CONS GENRL31 GENRL3] GENRL2] [CONS A1 NIL]]
             [APPEND [CONS GENRL31 GENRL3] [APPEND GENRL2 [CONS A1 NIL]]]]
       T
       [*1]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[REVERSE
    [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]
           NIL
           T]
```

GENERALIZATIONS:

GENRL1 = [REVERSE B]

GENRL3 = [REVERSE B]

GENRL2 = [REVERSE A]

PROFILE: [/ [A] , / E N R / E N R X , / & G [GENRL1] , / E N R / E N R / E N R
G [GENRL3] , / E N R / E N R .]

TIME: 21.38 SECS.

```
[T 1 5]      [ 16.05 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [LENGTH [REVERSE D]] [LENGTH D]]



MUST TRY INDUCTION.



INDUCT ON D.




THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [LENGTH [REVERSE NIL]] [LENGTH NIL]]
     [IMPLIES [EQUAL [LENGTH [REVERSE D]] [LENGTH D]]
              [EQUAL [LENGTH [REVERSE [CONS D1 D]]] [LENGTH [CONS D1 D]]]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [LENGTH [REVERSE D]] [LENGTH D]]
      [EQUAL [LENGTH [APPEND [REVERSE D] [CONS D1 NIL]]] [CONS NIL [LENGTH D]]]
      T]



FERTILIZE WITH [EQUAL [LENGTH [REVERSE D]] [LENGTH D]].



THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [LENGTH [APPEND [REVERSE D] [CONS D1 NIL]]]
             [CONS NIL [LENGTH [REVERSE D]]]]
      T
```

```
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE D] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [LENGTH [APPEND GENRL1 [CONS D1 NIL]]] [CONS NIL [LENGTH GENRL1]]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [LENGTH [APPEND NIL [CONS D1 NIL]]] [CONS NIL [LENGTH NIL]]]
 .      T
 .      [*1]]
 [IMPLIES
  [COND
  .    [EQUAL [LENGTH [APPEND GENRL1 [CONS D1 NIL]]] [CONS NIL [LENGTH GENRL1]]]
  .    T
  .    [*1]]
  [COND [EQUAL [LENGTH [APPEND [CONS GENRL11 GENRL1] [CONS D1 NIL]]]
                [CONS NIL [LENGTH [CONS GENRL11 GENRL1]]]]
         T
         [*1]]]]
```

WHICH IS EQUIVALENT TO:

[T 1 5]

T

```
FUNCTION DEFINITIONS:

[REVERSE
      [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


FERTILIZERS:

*1 = [COND [EQUAL [LENGTH [REVERSE D]] [LENGTH D]] NIL T]


GENERALIZATIONS:

GENRL1 = [REVERSE D]


PROFILE:  [/ [D] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R .]


TIME:  9.438 SECS.
```

```
[T 1 6]      [ 16.05 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [REVERSE [REVERSE A]] A]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [REVERSE [REVERSE NIL]] NIL]
     [IMPLIES [EQUAL [REVERSE [REVERSE A]] A]
              [EQUAL [REVERSE [REVERSE [CONS A1 A]]] [CONS A1 A]]]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL [REVERSE [REVERSE A]] A]
      [EQUAL [REVERSE [APPEND [REVERSE A] [CONS A1 NIL]]] [CONS A1 A]]
      T]


FERTILIZE WITH [EQUAL [REVERSE [REVERSE A]] A].


THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [REVERSE [APPEND [REVERSE A] [CONS A1 NIL]]]
             [CONS A1 [REVERSE [REVERSE A]]]]
      T
```

```
     [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE A] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND
     [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]
     T
     [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [REVERSE [APPEND NIL [CONS A1 NIL]]] [CONS A1 [REVERSE NIL]]]
 .      T
 .      [*1]]
 [IMPLIES
  [COND
  .  [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]
  .  T
  .  [*1]]
  [COND [EQUAL [REVERSE [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]]]
               [CONS A1 [REVERSE [CONS GENRL11 GENRL1]]]]
        T
        [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]
 [COND
 .    [EQUAL [APPEND [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS GENRL11 NIL]]
 .    .       [CONS A1 [APPEND [REVERSE GENRL1] [CONS GENRL11 NIL]]]]
 .    T
 .    [*1]]
 T]
```

```
FERTILIZE WITH [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]]
                      [CONS A1 [REVERSE GENRL1]]].
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [EQUAL [APPEND [CONS A1 [REVERSE GENRL1]] [CONS GENRL11 NIL]]
                   [CONS A1 [APPEND [REVERSE GENRL1] [CONS GENRL11 NIL]]]]
            T
            [*1]]
      T
      [*2]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[REVERSE
       [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [REVERSE [REVERSE A]] A] NIL T]

*2 = [COND [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]]
                  [CONS A1 [REVERSE GENRL1]]]
           NIL
           T]
```

GENERALIZATIONS:

GENRL1 = [REVERSE A]

PROFILE: [/ [A] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R / E N R F , / E N R .]

TIME: 12.94 SECS.

```
[T 1 7]      [ 16.05 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES A [EQUAL [LAST [REVERSE A]] [CAR A]]]


WHICH IS EQUIVALENT TO:

[COND A [EQUAL [LAST [REVERSE A]] [CAR A]] T]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND NIL [EQUAL [LAST [REVERSE NIL]] [CAR NIL]] T]
 [IMPLIES
  [COND A [EQUAL [LAST [REVERSE A]] [CAR A]] T]
  [COND [CONS A1 A] [EQUAL [LAST [REVERSE [CONS A1 A]]] [CAR [CONS A1 A]]] T]]]


WHICH IS EQUIVALENT TO:

[COND A
      [COND [EQUAL [LAST [REVERSE A]] [CAR A]]
            [EQUAL [LAST [APPEND [REVERSE A] [CONS A1 NIL]]] A1]
            T]
      T]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE A] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND A
      [COND [EQUAL [LAST GENRL1] [CAR A]]
            [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1]
            T]
      T]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND A
 .     [COND [EQUAL [LAST NIL] [CAR A]]
 .           [EQUAL [LAST [APPEND NIL [CONS A1 NIL]]] A1]
 .           T]
 .     T]
 [IMPLIES
     [COND A
     .     [COND [EQUAL [LAST GENRL1] [CAR A]]
     .           [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1]
     .           T]
     .     T]
     [COND A
           [COND [EQUAL [LAST [CONS GENRL11 GENRL1]] [CAR A]]
                 [EQUAL [LAST [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]]] A1]
                 T]
           T]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 A
 [COND
 .[EQUAL [LAST GENRL1] [CAR A]]
 .[COND
 .     [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1]
 .     [COND GENRL1 [COND [APPEND GENRL1 [CONS A1 NIL]] T [EQUAL GENRL11 A1]] T]
 .     T]
 .T]
 T]
```

FERTILIZE WITH [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .[EQUAL [LAST GENRL1] [CAR A]]
 .[COND
 .  [COND GENRL1
 .  .     [COND [APPEND GENRL1 [CONS [LAST [APPEND GENRL1 [CONS A1 NIL]]] NIL]]
 .  .           T
 .  .           [EQUAL GENRL11 [LAST [APPEND GENRL1 [CONS A1 NIL]]]]]
 .  .     T]
 .  T
 .  [*1]]
 .T]
 T]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND
 .A
 .[COND
 .   [EQUAL [LAST NIL] [CAR A]]
 .   [COND [COND NIL
 .   .            [COND [APPEND NIL [CONS [LAST [APPEND NIL [CONS A1 NIL]]] NIL]]
 .   .                  T
 .   .                  [EQUAL GENRL11 [LAST [APPEND NIL [CONS A1 NIL]]]]]
 .   .            T]
 .   .      T
 .   .      [*1]]
 .   T]
 .T]
 [IMPLIES
  [COND
  .A
  .[COND
  . [EQUAL [LAST GENRL1] [CAR A]]
  . [COND
  . .[COND
  . .     GENRL1
  . .     [COND [APPEND GENRL1 [CONS [LAST [APPEND GENRL1 [CONS A1 NIL]]] NIL]]
  . .     .     T
  . .     .     [EQUAL GENRL11 [LAST [APPEND GENRL1 [CONS A1 NIL]]]]]
  . .     T]
  . .T
  . .[*1]]
  . T]
  .T]
  [COND
   A
   [COND
   .[EQUAL [LAST [CONS GENRL12 GENRL1]] [CAR A]]
   .[COND
   . [COND
   . .[CONS GENRL12 GENRL1]
   . .[COND
   . .  [APPEND [CONS GENRL12 GENRL1]
   . .  .       [CONS [LAST [APPEND [CONS GENRL12 GENRL1] [CONS A1 NIL]]] NIL]]
   . .  T
   . .  [EQUAL GENRL11 [LAST [APPEND [CONS GENRL12 GENRL1] [CONS A1 NIL]]]]]
   . .T]
   . T
   . [*1]]
   .T]
   T]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[REVERSE
       [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

[LAST [LAMBDA [X] [COND X [COND [CDR X] [LAST [CDR X]] [CAR X]] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[CARARG UNDEF]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1] NIL T]
```

GENERALIZATIONS:

```
GENRL1 = [REVERSE A]
```

PROFILE: [/ E N R / E N R [A] , / E N R / E N R / E N R G [GENRL1] , / E N R / E N R / E N R F , / [GENRL1] , / E N R .]

TIME: 23.13 SECS.

```
[T 2 1]     [ 16.06 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [MEMBER A B] [MEMBER A [APPEND B C]]]



WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] [MEMBER A [APPEND B C]] T]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND
   [COND [MEMBER A NIL] [MEMBER A [APPEND NIL C]] T]
   [IMPLIES [COND [MEMBER A B] [MEMBER A [APPEND B C]] T]
            [COND [MEMBER A [CONS B1 B]] [MEMBER A [APPEND [CONS B1 B] C]] T]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[MEMBER
```

[T 2 1]

```
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


PROFILE:  [/ E N R / E N R [B] , / E N R / E N R .]


TIME:  5.063 SECS.
```

```
[T 2 2]     [ 16.07 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [MEMBER A B] [MEMBER A [APPEND C B]]]


WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] [MEMBER A [APPEND C B]] T]


MUST TRY INDUCTION.


INDUCT ON C.


THE THEOREM TO BE PROVED IS NOW:

[AND [COND [MEMBER A B] [MEMBER A [APPEND NIL B]] T]
     [IMPLIES [COND [MEMBER A B] [MEMBER A [APPEND C B]] T]
              [COND [MEMBER A B] [MEMBER A [APPEND [CONS C1 C] B]] T]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]
```

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


PROFILE:  [/ E N R / E N R [C] , / E N R / E N R .]


TIME:  6.063 SECS.
```

```
[T 2 3]     [ 16.07 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [AND [NOT [EQUAL A [CAR B]]] [MEMBER A B]] [MEMBER A [CDR B]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL A [CAR B]] T [COND [MEMBER A B] [MEMBER A [CDR B]] T]]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [EQUAL A [CAR NIL]] T [COND [MEMBER A NIL] [MEMBER A [CDR NIL]] T]]
 [IMPLIES [COND [EQUAL A [CAR B]] T [COND [MEMBER A B] [MEMBER A [CDR B]] T]]
          [COND [EQUAL A [CAR [CONS B1 B]]]
                T
                [COND [MEMBER A [CONS B1 B]] [MEMBER A [CDR [CONS B1 B]]] T]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:
```

[T 2 3]

```
[NOT [LAMBDA [X] [COND X NIL T]]]

[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[CARARG UNDEF]

[CDRARG UNDEF]


PROFILE:  [/ E N R / E N R [B] , / E N R / E N R .]


TIME:  5.938 SECS.
```

```
[T 2 4]      [ 16.07 18 JULY 1973]



THEOREM TO BE PROVED:

[IMPLIES [OR [MEMBER A B] [MEMBER A C]] [MEMBER A [APPEND B C]]]



WHICH IS EQUIVALENT TO:

[COND [MEMBER A B]
      [MEMBER A [APPEND B C]]
      [COND [MEMBER A C] [MEMBER A [APPEND B C]] T]]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND [COND [MEMBER A NIL]
           [MEMBER A [APPEND NIL C]]
           [COND [MEMBER A C] [MEMBER A [APPEND NIL C]] T]]
     [IMPLIES [COND [MEMBER A B]
                    [MEMBER A [APPEND B C]]
                    [COND [MEMBER A C] [MEMBER A [APPEND B C]] T]]
              [COND [MEMBER A [CONS B1 B]]
                    [MEMBER A [APPEND [CONS B1 B] C]]
                    [COND [MEMBER A C] [MEMBER A [APPEND [CONS B1 B] C]] T]]]]



WHICH IS EQUIVALENT TO:

[
```

FUNCTION DEFINITIONS:

```
[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[OR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ E N R / E N R [B] , / E N R / E N R .]

TIME: 11.88 SECS.

```
[T 2 5]      [ 16.07 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [AND [MEMBER A B] [MEMBER A C]] [MEMBER A [INTERSEC B C]]]



WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] [COND [MEMBER A C] [MEMBER A [INTERSEC B C]] T] T]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [MEMBER A NIL] [COND [MEMBER A C] [MEMBER A [INTERSEC NIL C]] T] T]
 [IMPLIES [COND [MEMBER A B] [COND [MEMBER A C] [MEMBER A [INTERSEC B C]] T] T]
          [COND [MEMBER A [CONS B1 B]]
                [COND [MEMBER A C] [MEMBER A [INTERSEC [CONS B1 B] C]] T]
                T]]]



WHICH IS EQUIVALENT TO:

[COND
   [MEMBER A B]
   T
   [COND [EQUAL A B1]
         [COND [MEMBER A C] [COND [MEMBER B1 C] T [MEMBER A [INTERSEC B C]]] T]
         T]]
```

FERTILIZE WITH [EQUAL A B1].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
    [MEMBER A B]
    T
    [COND [COND [MEMBER A C] [COND [MEMBER A C] T [MEMBER A [INTERSEC B C]]] T]
          T
          [*1]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[INTERSEC [LAMBDA [X Y]
                  [COND X
                        [COND [MEMBER [CAR X] Y]
                              [CONS [CAR X] [INTERSEC [CDR X] Y]]
                              [INTERSEC [CDR X] Y]]
                        NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL A B1] NIL T]
```

PROFILE: [/ E N R / E N R [B] , / E N R / E N R / E N R / E N R F , / R / E N R

.]

TIME: 19.31 SECS.

```
[T 2 6]     [ 16.08 18 JULY 1973]



THEOREM TO BE PROVED:

[IMPLIES [OR [MEMBER A B] [MEMBER A C]] [MEMBER A [UNION B C]]]



WHICH IS EQUIVALENT TO:

[COND [MEMBER A B]
      [MEMBER A [UNION B C]]
      [COND [MEMBER A C] [MEMBER A [UNION B C]] T]]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND [COND [MEMBER A NIL]
           [MEMBER A [UNION NIL C]]
           [COND [MEMBER A C] [MEMBER A [UNION NIL C]] T]]
     [IMPLIES [COND [MEMBER A B]
                    [MEMBER A [UNION B C]]
                    [COND [MEMBER A C] [MEMBER A [UNION B C]] T]]
              [COND [MEMBER A [CONS B1 B]]
                    [MEMBER A [UNION [CONS B1 B] C]]
                    [COND [MEMBER A C] [MEMBER A [UNION [CONS B1 B] C]] T]]]]



WHICH IS EQUIVALENT TO:

[COND
    [MEMBER A B]
    T
```

```
[COND [MEMBER A C]
      T
      [COND [EQUAL A B1] [COND [MEMBER B1 C] [MEMBER A [UNION B C]] T] T]]]
```

FERTILIZE WITH [EQUAL A B1].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [MEMBER A B]
      T
      [COND [MEMBER A C]
            T
            [COND [COND [MEMBER A C] [MEMBER A [UNION B C]] T] T [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[MEMBER
    [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[OR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]

[UNION
 [LAMBDA
  [X Y]
  [COND
   X
   [COND [MEMBER [CAR X] Y] [UNION [CDR X] Y] [CONS [CAR X] [UNION [CDR X] Y]]]
   Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL A B1] NIL T]

PROFILE:  [/ E N R / E N R [B] , / E N R / E N R / E N R / E N R F , / R / E N R
 .]

TIME:  27.06 SECS.
```

```
[T 2 7]      [ 16.09 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [SUBSET A B] [EQUAL [UNION A B] B]]


WHICH IS EQUIVALENT TO:

[COND [SUBSET A B] [EQUAL [UNION A B] B] T]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND [COND [SUBSET NIL B] [EQUAL [UNION NIL B] B] T]
     [IMPLIES [COND [SUBSET A B] [EQUAL [UNION A B] B] T]
              [COND [SUBSET [CONS A1 A] B] [EQUAL [UNION [CONS A1 A] B] B] T]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[SUBSET
    [LAMBDA [X Y] [COND X [COND [MEMBER [CAR X] Y] [SUBSET [CDR X] Y] NIL] T]]]
```

```
[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[UNION
 [LAMBDA
  [X Y]
  [COND
   X
   [COND [MEMBER [CAR X] Y] [UNION [CDR X] Y] [CONS [CAR X] [UNION [CDR X] Y]]]
   Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]



PROFILE:  [/ E N R / E N R [A] , / E N R / E N R ,]



TIME:  6.625 SECS.
```

```
[T 2 8]      [ 16.09 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [SUBSET A B] [EQUAL [INTERSEC A B] A]]


WHICH IS EQUIVALENT TO:

[COND [SUBSET A B] [EQUAL [INTERSEC A B] A] T]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND [COND [SUBSET NIL B] [EQUAL [INTERSEC NIL B] NIL] T]
     [IMPLIES [COND [SUBSET A B] [EQUAL [INTERSEC A B] A] T]
              [COND [SUBSET [CONS A1 A] B]
                    [EQUAL [INTERSEC [CONS A1 A] B] [CONS A1 A]]
                    T]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:
```

```
[SUBSET
    [LAMBDA [X Y] [COND X [COND [MEMBER [CAR X] Y] [SUBSET [CDR X] Y] NIL] T]]]

[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[INTERSEC [LAMBDA [X Y]
                 [COND X
                       [COND [MEMBER [CAR X] Y]
                             [CONS [CAR X] [INTERSEC [CDR X] Y]]
                             [INTERSEC [CDR X] Y]]
                       NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ E N R / E N R [A] , / E N R / E N R / E N R ,]

TIME: 8.563 SECS.

[T 2 9]   [16.09 18 JULY 1973]

THEOREM TO BE PROVED:

```
[EQUAL [MEMBER A B] [NOT [EQUAL [ASSOC A [PAIRLIST B C]] NIL]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [ASSOC A [PAIRLIST B C]] [MEMBER A B] [COND [MEMBER A B] NIL T]]
```

MUST TRY INDUCTION.

INDUCT ON C AND B.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [AND
 .[COND [ASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MEMBER A B] NIL T]]
 .[COND [ASSOC A [PAIRLIST NIL C]] [MEMBER A NIL] [COND [MEMBER A NIL] NIL T]]]
 [IMPLIES
         [COND [ASSOC A [PAIRLIST B C]] [MEMBER A B] [COND [MEMBER A B] NIL T]]
         [COND [ASSOC A [PAIRLIST [CONS B1 B] [CONS C1 C]]]
               [MEMBER A [CONS B1 B]]
               [COND [MEMBER A [CONS B1 B]] NIL T]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [ASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MEMBER A B] NIL T]]
```

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [ASSOC A [PAIRLIST NIL NIL]] [MEMBER A NIL] [COND [MEMBER A NIL] NIL T]]
 [IMPLIES
       [COND [ASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MEMBER A B] NIL T]]
       [COND [ASSOC A [PAIRLIST [CONS B2 B] NIL]]
             [MEMBER A [CONS B2 B]]
             [COND [MEMBER A [CONS B2 B]] NIL T]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[MEMBER
    [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[PAIRLIST
   [LAMBDA [X Y]
           [COND X
                 [COND Y
                       [CONS [CONS [CAR X] [CAR Y]] [PAIRLIST [CDR X] [CDR Y]]]
                       [CONS [CONS [CAR X] NIL] [PAIRLIST [CDR X] NIL]]]
                 NIL]]]

[ASSOC
   [LAMBDA [X Y]
           [COND Y
                 [COND [CAR Y]
                       [COND [EQUAL X [CAR [CAR Y]]] [CAR Y] [ASSOC X [CDR Y]]]
                       [ASSOC X [CDR Y]]]
                 NIL]]]
```

```
                                                            [T 2 9]

[NOT [LAMBDA [X] [COND X NIL T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]


PROFILE:  [/ E N R / E N R [C B] , / E N R / E N R / E N R [B] , / E N R / E N R
 .]


TIME:  15.0 SECS.
```

```
[T 3 1]      [ 16.09 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [MAPLIST [APPEND A B] C] [APPEND [MAPLIST A C] [MAPLIST B C]]]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [MAPLIST [APPEND NIL B] C] [APPEND [MAPLIST NIL C] [MAPLIST B C]]]
 [IMPLIES [EQUAL [MAPLIST [APPEND A B] C] [APPEND [MAPLIST A C] [MAPLIST B C]]]
          [EQUAL [MAPLIST [APPEND [CONS A1 A] B] C]
                 [APPEND [MAPLIST [CONS A1 A] C] [MAPLIST B C]]]]]


WHICH IS EQUIVALENT TO:

[


FUNCTION DEFINITIONS:

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[MAPLIST
      [LAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]

[APPLY UNDEF]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

```
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [A] , / E N R / E N R .]

TIME: 6.25 SECS.

```
[T 3 2]     [ 16.1 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [LENGTH [MAPLIST A B]] [LENGTH A]]



MUST TRY INDUCTION.



INDUCT ON A.



THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [LENGTH [MAPLIST NIL B]] [LENGTH NIL]]
     [IMPLIES [EQUAL [LENGTH [MAPLIST A B]] [LENGTH A]]
              [EQUAL [LENGTH [MAPLIST [CONS A1 A] B]] [LENGTH [CONS A1 A]]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[MAPLIST
      [LAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]

[APPLY UNDEF]

[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [A] , / E N R / E N R .]

TIME: 3.438 SECS.

```
[T 3 3]      [ 16.13 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [REVERSE [MAPLIST NIL B]] [MAPLIST [REVERSE NIL] B]]
 [IMPLIES
  [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]]
  [EQUAL [REVERSE [MAPLIST [CONS A1 A] B]] [MAPLIST [REVERSE [CONS A1 A]] B]]]]

WHICH IS EQUIVALENT TO:

[COND [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]]
      [EQUAL [APPEND [REVERSE [MAPLIST A B]] [CONS [APPLY B A1] NIL]]
             [MAPLIST [APPEND [REVERSE A] [CONS A1 NIL]] B]]
      T]

FERTILIZE WITH [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]].

THE THEOREM TO BE PROVED IS NOW:
```

```
[COND [EQUAL [APPEND [MAPLIST [REVERSE A] B] [CONS [APPLY B A1] NIL]]
             [MAPLIST [APPEND [REVERSE A] [CONS A1 NIL]] B]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE A] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [APPEND [MAPLIST GENRL1 B] [CONS [APPLY B A1] NIL]]
             [MAPLIST [APPEND GENRL1 [CONS A1 NIL]] B]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
LAND
 [COND [EQUAL [APPEND [MAPLIST NIL B] [CONS [APPLY B A1] NIL]]
 .            [MAPLIST [APPEND NIL [CONS A1 NIL]] B]]
 .      T
 .      [*1]]
 [IMPLIES
  [COND [EQUAL [APPEND [MAPLIST GENRL1 B] [CONS [APPLY B A1] NIL]]
  .            [MAPLIST [APPEND GENRL1 [CONS A1 NIL]] B]]
  .      T
  .      [*1]]
  [COND
      [EQUAL [APPEND [MAPLIST [CONS GENRL11 GENRL1] B] [CONS [APPLY B A1] NIL]]
      .      [MAPLIST [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]] B]]
      T
      [*1]]]]
```

```
WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[MAPLIST
      [LAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]

[APPLY UNDEF]

[REVERSE
       [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


FERTILIZERS:

*1 = [COND [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]] NIL T]


GENERALIZATIONS:

GENRL1 = [REVERSE A]


PROFILE:  [/ [A] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R .]


TIME:  12.19 SECS.
```

```
[T 4 1]      [ 16.13 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]



MUST TRY INDUCTION.



INDUCT ON A.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [LIT [APPEND NIL B] C D] [LIT NIL [LIT B C D] D]]
 [IMPLIES
     [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]
     [EQUAL [LIT [APPEND [CONS A1 A] B] C D] [LIT [CONS A1 A] [LIT B C D] D]]]]



WHICH IS EQUIVALENT TO:

[COND
 [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]
 [EQUAL [APPLY D A1 [LIT [APPEND A B] C D]] [APPLY D A1 [LIT A [LIT B C D] D]]]
 T]



FERTILIZE WITH [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]].



THE THEOREM TO BE PROVED IS NOW:
```

```
[COND [EQUAL [APPLY D A1 [LIT [APPEND A B] C D]]
             [APPLY D A1 [LIT [APPEND A B] C D]]]
      T
      [*1]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[LIT [LAMBDA [X Y Z] [COND X [APPLY Z [CAR X] [LIT [CDR X] Y Z]] Y]]]

[APPLY UNDEF]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]] NIL T]
```

PROFILE: [/ [A] , / E N R / E N R X , / E N R .]

TIME: 8.313 SECS.

[T 4 2]   [16.14 18 JULY 1973]

THEOREM TO BE PROVED:

```
[IMPLIES [AND [BOOLEAN A] [BOOLEAN B]]
         [EQUAL [AND [IMPLIES A B] [IMPLIES B A]] [EQUAL A B]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[BOOLEAN [LAMBDA [X] [COND X [EQUAL X T] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ E N R / E N R .]

TIME:, 4.313 SECS.

```
[T 4 3]      [ 16.14 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [ELEMENT B A] [ELEMENT [APPEND C B] [APPEND C A]]]


MUST TRY INDUCTION.


INDUCT ON C.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [ELEMENT B A] [ELEMENT [APPEND NIL B] [APPEND NIL A]]]
     [IMPLIES [EQUAL [ELEMENT B A] [ELEMENT [APPEND C B] [APPEND C A]]]
              [EQUAL [ELEMENT B A]
                     [ELEMENT [APPEND [CONS C1 C] B] [APPEND [CONS C1 C] A]]]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[ELEMENT
        [LAMBDA [X Y] [COND Y [COND X [ELEMENT [CDR X] [CDR Y]] [CAR Y]] NIL]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [C] ; / E N R / E N R .]

TIME: 5.25 SECS.

[T 4 4]   [16.14 18 JULY 1973]

THEOREM TO BE PROVED:

```
[IMPLIES [ELEMENT B A] [MEMBER [ELEMENT B A] A]]
```

WHICH IS EQUIVALENT TO:

```
[COND [ELEMENT B A] [MEMBER [ELEMENT B A] A] T]
```

MUST TRY INDUCTION.

INDUCT ON B AND A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [ELEMENT NIL A] [MEMBER [ELEMENT NIL A] A] T]
          [COND [ELEMENT B NIL] [MEMBER [ELEMENT B NIL] NIL] T]]
     [IMPLIES [COND [ELEMENT B A] [MEMBER [ELEMENT B A] A] T]
              [COND [ELEMENT [CONS B1 B] [CONS A1 A]]
                    [MEMBER [ELEMENT [CONS B1 B] [CONS A1 A]] [CONS A1 A]]
                    T]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[ELEMENT
       [LAMBDA [X Y] [COND Y [COND X [ELEMENT [CDR X] [CDR Y]] [CAR Y]] NIL]]]

[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


PROFILE:  [/ E N R / E N R [B A] , / E N R / E N R / E N R .]


TIME:  6.188 SECS.
```

```
[T 4 5]     [ 16.14 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [CDRN C [APPEND A B]] [APPEND [CDRN C A] [CDRN [CDRN A C] B]]]



MUST TRY INDUCTION.



INDUCT ON A AND C.




THE THEOREM TO BE PROVED IS NOW:

[AND
 [AND
 . [EQUAL [CDRN C [APPEND NIL B]] [APPEND [CDRN C NIL] [CDRN [CDRN NIL C] B]]]
 . [EQUAL [CDRN NIL [APPEND A B]] [APPEND [CDRN NIL A] [CDRN [CDRN A NIL] B]]]]
 [IMPLIES [EQUAL [CDRN C [APPEND A B]] [APPEND [CDRN C A] [CDRN [CDRN A C] B]]]
          [EQUAL [CDRN [CONS C1 C] [APPEND [CONS A1 A] B]]
                 [APPEND [CDRN [CONS C1 C] [CONS A1 A]]
                         [CDRN [CDRN [CONS A1 A] [CONS C1 C]] B]]]]]



WHICH IS EQUIVALENT TO:

|



FUNCTION DEFINITIONS:

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[CDRN [LAMBDA [X Y] [COND Y [COND X [CDRN [CDR X] [CDR Y]] Y] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

```
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ [A C] , / E N R / E N R .]

TIME: 6.563 SECS.

```
[T 4 6]     [ 16.15 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [CDRN [APPEND B C] A] [CDRN C [CDRN B A]]]



MUST TRY INDUCTION.



INDUCT ON B AND A.




THE THEOREM TO BE PROVED IS NOW:

[AND [AND [EQUAL [CDRN [APPEND NIL C] A] [CDRN C [CDRN NIL A]]]
          [EQUAL [CDRN [APPEND B C] NIL] [CDRN C [CDRN B NIL]]]]
     [IMPLIES [EQUAL [CDRN [APPEND B C] A] [CDRN C [CDRN B A]]]
              [EQUAL [CDRN [APPEND [CONS B1 B] C] [CONS A1 A]]
                     [CDRN C [CDRN [CONS B1 B] [CONS A1 A]]]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[CDRN [LAMBDA [X Y] [COND Y [COND X [CDRN [CDR X] [CDR Y]] Y] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ [B A] , / E N R / E N R .]

TIME: 4.625 SECS.

```
[T 4 7]     [ 16.15 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [EQUAL A B] [EQUAL B A]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL A B] [EQUAL B A] [COND [EQUAL B A] NIL T]]


FERTILIZE WITH [EQUAL A B].


THE THEOREM TO BE PROVED IS NOW:

[COND [COND [EQUAL A A] T [*1]]
      [COND [COND [EQUAL B A] NIL T] T [EQUAL A B]]
      NIL]


WHICH IS EQUIVALENT TO:

[COND [EQUAL B A] [EQUAL A B] T]


FERTILIZE WITH [EQUAL B A].


THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL B B] T [*2]]
```

WHICH IS EQUIVALENT TO:

T

FERTILIZERS:

```
*1 = [COND [EQUAL A B] NIL [COND [EQUAL B A] NIL T]]

*2 = [COND [EQUAL B A] NIL T]
```

PROFILE: [/ N R / E N R F , / E N R / E N R F , / E N R .]

TIME: 2.063 SECS.

```
[T 4 8]     [ 16.15 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [AND [EQUAL A B] [EQUAL B C]] [EQUAL A C]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL A B] [COND [EQUAL B C] [EQUAL A C] T] T]


FERTILIZE WITH [EQUAL A B].


THE THEOREM TO BE PROVED IS NOW:

[COND [COND [EQUAL A C] [EQUAL A C] T] T [*1]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]


FERTILIZERS:

*1 = [COND [EQUAL A B] NIL T]
```

PROFILE: [/ E N R / E N R F , / R / E N R .]

TIME: 1.938 SECS.

```
[T 4 9]     [ 16.15 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [AND [BOOLEAN A] [AND [BOOLEAN B] [BOOLEAN C]]]
         [EQUAL [EQUAL A [EQUAL B C]] [EQUAL [EQUAL A B] C]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[BOOLEAN [LAMBDA [X] [COND X [EQUAL X T] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]


PROFILE:  [/ E N R / E N R .]


TIME:  13.69 SECS.
```

[T 5.1]   [16.15 18 JULY 1973]

```
THEOREM TO BE PROVED:

[EQUAL [PLUS N M] [PLUS M N]]



MUST TRY INDUCTION.



INDUCT ON M.




THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [PLUS N NIL] [PLUS NIL N]]
     [IMPLIES [EQUAL [PLUS N M] [PLUS M N]]
              [EQUAL [PLUS N [CONS NIL M]] [PLUS [CONS NIL M] N]]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [PLUS N NIL] N]
      [COND [EQUAL [PLUS N M] [PLUS M N]]
            [EQUAL [PLUS N [CONS NIL M]] [CONS NIL [PLUS M N]]]
            T]
      NIL]



FERTILIZE WITH [EQUAL [PLUS N M] [PLUS M N]].



THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [PLUS N NIL] N]
```

```
      [COND [EQUAL [PLUS N [CONS NIL M]] [CONS NIL [PLUS N M]]] T [*1]]
      NIL]
```

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

```
[COND [AND [EQUAL [PLUS NIL NIL] NIL]
           [IMPLIES [EQUAL [PLUS N NIL] N]
                    [EQUAL [PLUS [CONS NIL N] NIL] [CONS NIL N]]]]
      [COND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]
      NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]
```

MUST TRY INDUCTION.

INDUCT ON N1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [PLUS NIL [CONS NIL M]] [CONS NIL [PLUS NIL M]]] T [*1]]
 [IMPLIES
  [COND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]
  [COND
    [EQUAL [PLUS [CONS NIL N1] [CONS NIL M]] [CONS NIL [PLUS [CONS NIL N1] M]]]
    T
    [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

LAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [PLUS N M] [PLUS M N]] NIL T]
```

PROFILE:

```
PROFILE:  [/ [M] , / E N R / E N R X , / & [N] , / E N R / E N R / E N R [N1] ,
/ E N R / E N R ,]
```

TIME: 11.31 SECS.

```
[T 5 2]      [ 16.16 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [PLUS N [PLUS M K]] [PLUS [PLUS N M] K]]


MUST TRY INDUCTION.


INDUCT ON N.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [PLUS NIL [PLUS M K]] [PLUS [PLUS NIL M] K]]
 [IMPLIES
        [EQUAL [PLUS N [PLUS M K]] [PLUS [PLUS N M] K]]
        [EQUAL [PLUS [CONS NIL N] [PLUS M K]] [PLUS [PLUS [CONS NIL N] M] K]]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [N] , / E N R / E N R .]

TIME: 4.688 SECS.

```
[T 5 3]      [ 16.16 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [TIMES N M] [TIMES M N]]


MUST TRY INDUCTION.


INDUCT ON M.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [TIMES N NIL] [TIMES NIL N]]
     [IMPLIES [EQUAL [TIMES N M] [TIMES M N]]
              [EQUAL [TIMES N [CONS NIL M]] [TIMES [CONS NIL M] N]]]]


WHICH IS EQUIVALENT TO:

[COND [TIMES N NIL]
      NIL
      [COND [EQUAL [TIMES N M] [TIMES M N]]
            [EQUAL [TIMES N [CONS NIL M]] [PLUS N [TIMES M N]]]
            T]]


FERTILIZE WITH [EQUAL [TIMES N M] [TIMES M N]].


THE THEOREM TO BE PROVED IS NOW:

[COND [TIMES N NIL]
```

```
      NIL
      [COND [EQUAL [TIMES N [CONS NIL M]] [PLUS N [TIMES N M]]] T [*1]]]
```

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

```
[COND [AND [NOT [TIMES NIL NIL]]
           [IMPLIES [NOT [TIMES N NIL]] [NOT [TIMES [CONS NIL N] NIL]]]]
      [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]
      NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]
```

MUST TRY INDUCTION.

INDUCT ON N1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
  [COND [EQUAL [TIMES NIL [CONS NIL M]] [PLUS NIL [TIMES NIL M]]] T [*1]]
  [IMPLIES [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]
           [COND [EQUAL [TIMES [CONS NIL N1] [CONS NIL M]]
                        [PLUS [CONS NIL N1] [TIMES [CONS NIL N1] M]]]
                 T
                 [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]]
 [COND [EQUAL [PLUS M [TIMES N1 [CONS NIL M]]] [PLUS N1 [PLUS M [TIMES N1 M]]]]
 .     T
 .     [*1]]
 T]
```

FERTILIZE WITH [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
  [COND [EQUAL [PLUS M [PLUS N1 [TIMES N1 M]]] [PLUS N1 [PLUS M [TIMES N1 M]]]]
  .     T
  .     [*1]]
  T
  [*2]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES N1 M] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]] T [*1]]
      T
```

```
    [*2]]



MUST TRY INDUCTION.



INDUCT ON N1.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND
 .  [COND [EQUAL [PLUS M [PLUS NIL GENRL1]] [PLUS NIL [PLUS M GENRL1]]] T [*1]]
 .  T
 .  [*2]]
 [IMPLIES
  [COND
  .   [COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]] T [*1]]
  .   T
  .   [*2]]
  [COND [COND [EQUAL [PLUS M [PLUS [CONS NIL N1] GENRL1]]
                     [PLUS [CONS NIL N1] [PLUS M GENRL1]]]
              T
              [*1]]
        T
        [*2]]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]]
      [COND [COND [EQUAL [PLUS M [CONS NIL [PLUS N1 GENRL1]]]
                         [CONS NIL [PLUS N1 [PLUS M GENRL1]]]]
                  T
                  [*1]]
            T
            [*2]]
      T]
```

FERTILIZE WITH [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [COND [EQUAL [PLUS M [CONS NIL [PLUS N1 GENRL1]]]
                         [CONS NIL [PLUS M [PLUS N1 GENRL1]]]]
                  T
                  [*1]]
            T
            [*2]]
      T
      [*3]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS N1 GENRL1] BY GENRL2.

THE GENERALIZED TERM IS:

```
[COND
 [COND
 .  [COND [EQUAL [PLUS M [CONS NIL GENRL2]] [CONS NIL [PLUS M GENRL2]]] T [*1]]
 .  T
 .  [*2]]
 T
 [*3]]
```

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
```

```
[COND
.[COND [COND [EQUAL [PLUS NIL [CONS NIL GENRL2]] [CONS NIL [PLUS NIL GENRL2]]]
.             T
.             [*1]]
.        T
.        [*2]]
.T
.[*3]]
[IMPLIES
 [COND
 .[COND
 . [COND [EQUAL [PLUS M [CONS NIL GENRL2]] [CONS NIL [PLUS M GENRL2]]] T [*1]]
 . T
 . [*2]]
 .T
 .[*3]]
 [COND [COND [COND [EQUAL [PLUS [CONS NIL M] [CONS NIL GENRL2]]
                          [CONS NIL [PLUS [CONS NIL M] GENRL2]]]
                  T
                  [*1]]
             T
             [*2]]
        T
        [*3]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[TIMES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[NOT [LAMBDA [X] [COND X NIL T]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [TIMES N M] [TIMES M N]] NIL T]

*2 = [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] NIL T]
```

```
*3 = [COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]] NIL T]


GENERALIZATIONS:

GENRL1 = [TIMES N1 M]

GENRL2 = [PLUS N1 GENRL1]


PROFILE:  [/ [M] , / E N R / E N R X , / & [N] , / E N R / E N R / E N R [N1] ,
/ E N R / E N R / E N R F , / G [N1] , / E N R / E N R / E N R F , / G [M] , / E
 N R / E N R .]


TIME:  32.75 SECS.
```

```
[T 5 4]      [ 16.17 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]]



MUST TRY INDUCTION.



INDUCT ON N.




THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [TIMES NIL [PLUS M K]] [PLUS [TIMES NIL M] [TIMES NIL K]]]
     [IMPLIES [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]]
              [EQUAL [TIMES [CONS NIL N] [PLUS M K]]
                     [PLUS [TIMES [CONS NIL N] M] [TIMES [CONS NIL N] K]]]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]]
      [EQUAL [PLUS [PLUS M K] [TIMES N [PLUS M K]]]
             [PLUS [PLUS M [TIMES N M]] [PLUS K [TIMES N K]]]]
      T]



FERTILIZE WITH [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]].



THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [PLUS [PLUS M K] [PLUS [TIMES N M] [TIMES N K]]]
```

```
            [PLUS [PLUS M [TIMES N M]] [PLUS K [TIMES N K]]]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES N K] BY GENRL1 AND [TIMES N M] BY GENRL2.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [PLUS [PLUS M K] [PLUS GENRL2 GENRL1]]
             [PLUS [PLUS M GENRL2] [PLUS K GENRL1]]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [COND [EQUAL [PLUS [PLUS NIL K] [PLUS GENRL2 GENRL1]]
                  [PLUS [PLUS NIL GENRL2] [PLUS K GENRL1]]]
           T
           [*1]]
     [IMPLIES [COND [EQUAL [PLUS [PLUS M K] [PLUS GENRL2 GENRL1]]
                           [PLUS [PLUS M GENRL2] [PLUS K GENRL1]]]
                    T
                    [*1]]
              [COND [EQUAL [PLUS [PLUS [CONS NIL M] K] [PLUS GENRL2 GENRL1]]
                           [PLUS [PLUS [CONS NIL M] GENRL2] [PLUS K GENRL1]]]
                    T
                    [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL2.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [PLUS K [PLUS NIL GENRL1]] [PLUS NIL [PLUS K GENRL1]]] T [*1]]
 [IMPLIES
      [COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]
      .     T
      .     [*1]]
      [COND [EQUAL [PLUS K [PLUS [CONS GENRL21 GENRL2] GENRL1]]
                   [PLUS [CONS GENRL21 GENRL2] [PLUS K GENRL1]]]
            T
            [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]
      [COND [EQUAL [PLUS K [CONS NIL [PLUS GENRL2 GENRL1]]]
                   [CONS NIL [PLUS GENRL2 [PLUS K GENRL1]]]]
            T
            [*1]]
      T]
```

```
FERTILIZE WITH [EQUAL [PLUS K [PLUS GENRL2 GENRL1]]
                      [PLUS GENRL2 [PLUS K GENRL1]]].
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [EQUAL [PLUS K [CONS NIL [PLUS GENRL2 GENRL1]]]
                   [CONS NIL [PLUS K [PLUS GENRL2 GENRL1]]]]
            T
            [*1]]
      T
      [*2]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS GENRL2 GENRL1] BY GENRL3.

THE GENERALIZED TERM IS:

```
[COND
    [COND [EQUAL [PLUS K [CONS NIL GENRL3]] [CONS NIL [PLUS K GENRL3]]] T [*1]]
    T
    [*2]]
```

MUST TRY INDUCTION.

INDUCT ON K.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [COND [EQUAL [PLUS NIL [CONS NIL GENRL3]] [CONS NIL [PLUS NIL GENRL3]]]
 .           T
 .           [*1]]
 .     T
 .     [*2]]
 [IMPLIES
  [COND
```

```
 . [COND [EQUAL [PLUS K [CONS NIL GENRL3]] [CONS NIL [PLUS K GENRL3]]] T [*1]]
                                                                [T 5 4]

 . T
 . [*2]]
[COND [COND [EQUAL [PLUS [CONS NIL K] [CONS NIL GENRL3]]
                   [CONS NIL [PLUS [CONS NIL K] GENRL3]]]
            T
            [*1]]
      T
      [*2]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[TIMES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]] NIL T]

*2 = [COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]
           NIL
           T]
```

GENERALIZATIONS:

```
GENRL2 = [TIMES N M]

GENRL1 = [TIMES N K]

GENRL3 = [PLUS GENRL2 GENRL1]
```

PROFILE: [/ [N] , / E N R / E N R X , / G [M] , / E N R / E N R / E N R [GENRL2
] , / E N R / E N R / E N R F , / G [K] , / E N R / E N R .]
[T 5 4]

TIME: 31.19 SECS.

```
[T 5 5]      [ 16.18 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]]



MUST TRY INDUCTION.



INDUCT ON N.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [TIMES NIL [TIMES M K]] [TIMES [TIMES NIL M] K]]
 [IMPLIES
    [EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]]
    [EQUAL [TIMES [CONS NIL N] [TIMES M K]] [TIMES [TIMES [CONS NIL N] M] K]]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]]
      [EQUAL [PLUS [TIMES M K] [TIMES N [TIMES M K]]]
             [TIMES [PLUS M [TIMES N M]] K]]
      T]



FERTILIZE WITH [EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]].



THE THEOREM TO BE PROVED IS NOW:
```

```
[COND [EQUAL [PLUS [TIMES M K] [TIMES [TIMES N M] K]]
             [TIMES [PLUS M [TIMES N M]] K]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES N M] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND
 .    [EQUAL [PLUS [TIMES NIL K] [TIMES GENRL1 K]] [TIMES [PLUS NIL GENRL1] K]]
 .    T
 .    [*1]]
 [IMPLIES
    [COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
    .     T
    .     [*1]]
    [COND [EQUAL [PLUS [TIMES [CONS NIL M] K] [TIMES GENRL1 K]]
                 [TIMES [PLUS [CONS NIL M] GENRL1] K]]
          T
          [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
      [COND [EQUAL [PLUS [PLUS K [TIMES M K]] [TIMES GENRL1 K]]
                   [PLUS K [TIMES [PLUS M GENRL1] K]]]
            T
            [*1]]
      T]
```

FERTILIZE WITH [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]]
[TIMES [PLUS M GENRL1] K]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [EQUAL [PLUS [PLUS K [TIMES M K]] [TIMES GENRL1 K]]
                   [PLUS K [PLUS [TIMES M K] [TIMES GENRL1 K]]]]
            T
            [*1]]
      T
      [*2]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES GENRL1 K] BY GENRL2 AND [TIMES M K] BY GENRL3.

THE GENERALIZED TERM IS:

```
[COND [COND [EQUAL [PLUS [PLUS K GENRL3] GENRL2] [PLUS K [PLUS GENRL3 GENRL2]]]
            T
            [*1]]
      T
      [*2]]
```

MUST TRY INDUCTION.

INDUCT ON K.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND
 .[COND [EQUAL [PLUS [PLUS NIL GENRL3] GENRL2] [PLUS NIL [PLUS GENRL3 GENRL2]]]
 .       T
 .       [*1]]
 .T
 .[*2]]
 [IMPLIES
  [COND
  .    [COND [EQUAL [PLUS [PLUS K GENRL3] GENRL2] [PLUS K [PLUS GENRL3 GENRL2]]]
  .    .      T
  .    .      [*1]]
  .    T
  .    [*2]]
  [COND [COND [EQUAL [PLUS [PLUS [CONS NIL K] GENRL3] GENRL2]
                     [PLUS [CONS NIL K] [PLUS GENRL3 GENRL2]]]
              T
              [*1]]
        T
        [*2]]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[TIMES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

```
FERTILIZERS:
                                                    [T 5 5]

*1 = [COND [EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]] NIL T]

*2 = [COND
          [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
          NIL
          T]


GENERALIZATIONS:

GENRL1 = [TIMES N M]

GENRL3 = [TIMES M K]

GENRL2 = [TIMES GENRL1 K]


PROFILE:  [/ [N] , / E N R / E N R X , / G [M] , / E N R / E N R / E N R F , / G
 [K] , / E N R / E N R .]


TIME:  25.25 SECS.
```

```
[T 5 6]      [ 16.18 18 JULY 1973]


THEOREM TO BE PROVED:

[EVEN1 [DOUBLE N]]



MUST TRY INDUCTION.



INDUCT ON N.




THE THEOREM TO BE PROVED IS NOW:

[AND [EVEN1 [DOUBLE NIL]]
     [IMPLIES [EVEN1 [DOUBLE N]] [EVEN1 [DOUBLE [CONS NIL N]]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[DOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

[EVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

[T 5 6]

PROFILE: [/ [N] , / E N R / E N R .]

TIME: 2.25 SECS.

```
[T 5 7]      [ 16.18 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [HALF [DOUBLE N]] N]



MUST TRY INDUCTION.



INDUCT ON N.




THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [HALF [DOUBLE NIL]] NIL]
     [IMPLIES [EQUAL [HALF [DOUBLE N]] N]
              [EQUAL [HALF [DOUBLE [CONS NIL N]]] [CONS NIL N]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[DOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

[HALF [LAMBDA [X] [COND X [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [N] , / E N R / E N R .]

TIME: 2.625 SECS.

[T 5 8]   [16.18 18 JULY 1973]

THEOREM TO BE PROVED:

```
[IMPLIES [EVEN1 N] [EQUAL [DOUBLE [HALF N]] N]]
```

WHICH IS EQUIVALENT TO:

```
[COND [EVEN1 N] [EQUAL [DOUBLE [HALF N]] N] T]
```

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EVEN1 NIL] [EQUAL [DOUBLE [HALF NIL]] NIL] T]
 [AND
  [COND [EVEN1 [CONS NIL NIL]]
  .      [EQUAL [DOUBLE [HALF [CONS NIL NIL]]] [CONS NIL NIL]]
  .      T]
  [IMPLIES
   [COND [EVEN1 N] [EQUAL [DOUBLE [HALF N]] N] T]
   [COND
        [EVEN1 [CONS NIL [CONS NIL N]]]
        [EQUAL [DOUBLE [HALF [CONS NIL [CONS NIL N]]]] [CONS NIL [CONS NIL N]]]
        T]]]]
```

WHICH IS EQUIVALENT TO:

|

FUNCTION DEFINITIONS:

```
[EVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]]
[HALF [LAMBDA [X] [COND X [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]]
[DOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ E N R / E N R S2 [N] , / E N R / E N R .]

TIME: 4.75 SECS.

```
[T 5 9]     [ 16.19 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [DOUBLE N] [TIMES 2 N]]



WHICH IS EQUIVALENT TO:

[EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]]



MUST TRY INDUCTION.



INDUCT ON N.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [DOUBLE NIL] [PLUS NIL [PLUS NIL 0]]]
 [IMPLIES
      [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]]
      [EQUAL [DOUBLE [CONS NIL N]] [PLUS [CONS NIL N] [PLUS [CONS NIL N] 0]]]]]



WHICH IS EQUIVALENT TO:

[COND [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]]
      [EQUAL [CONS NIL [DOUBLE N]] [PLUS N [CONS NIL [PLUS N 0]]]]
      T]



FERTILIZE WITH [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]].
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND [EQUAL [CONS NIL [PLUS N [PLUS N 0]]] [PLUS N [CONS NIL [PLUS N 0]]]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS N 0] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [CONS NIL [PLUS N GENRL1]] [PLUS N [CONS NIL GENRL1]]] T [*1]]
```

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [CONS NIL [PLUS NIL GENRL1]] [PLUS NIL [CONS NIL GENRL1]]]
 .      T
 .      [*1]]
 [IMPLIES
    [COND [EQUAL [CONS NIL [PLUS N GENRL1]] [PLUS N [CONS NIL GENRL1]]] T [*1]]
    [COND [EQUAL [CONS NIL [PLUS [CONS NIL N] GENRL1]]
                 [PLUS [CONS NIL N] [CONS NIL GENRL1]]]
          T
          [*1]]]]
```

[T 5 9]

```
WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[DOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

[TIMES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


FERTILIZERS:

*1 = [COND [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]] NIL T]


GENERALIZATIONS:

GENRL1 = [PLUS N 0]


PROFILE:  [/ E N R / E N R [N] , / E N R / E N R X , / G [N] , / E N R / E N R .
]


TIME:  9.188 SECS.
```

```
[T 5 10]      [ 16.19 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [DOUBLE N] [TIMES N 2]]



MUST TRY INDUCTION.



INDUCT ON N.



THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [DOUBLE NIL] [TIMES NIL 2]]
     [IMPLIES [EQUAL [DOUBLE N] [TIMES N 2]]
              [EQUAL [DOUBLE [CONS NIL N]] [TIMES [CONS NIL N] 2]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:

[DOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

[TIMES [LAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ [N] , / E N R / E N R .]

```
[T 5 11]      [ 16.2 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [EVEN1 N] [EVEN2 N]]


MUST TRY INDUCTION.


(SPECIAL CASE REQUIRED)


INDUCT ON N.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [EVEN1 NIL] [EVEN2 NIL]]
 [AND
  [EQUAL [EVEN1 [CONS NIL NIL]] [EVEN2 [CONS NIL NIL]]]
  [IMPLIES
     [EQUAL [EVEN1 N] [EVEN2 N]]
     [EQUAL [EVEN1 [CONS NIL [CONS NIL N]]] [EVEN2 [CONS NIL [CONS NIL N]]]]]]]


WHICH IS EQUIVALENT TO:

[


FUNCTION DEFINITIONS:
```

```
[EVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]]

[EVEN2 [LAMBDA [X] [COND X [COND [CDR X] [EVEN2 [CDR [CDR X]]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ S2 [N] , / E N R / E N R .]

TIME: 2.938 SECS.

```
[T 6 1]      [ 16.2 18 JULY 1973]


THEOREM TO BE PROVED:

[GT [LENGTH [CONS A B]] [LENGTH B]]


WHICH IS EQUIVALENT TO:

[GT [CONS NIL [LENGTH B]] [LENGTH B]]


MUST TRY INDUCTION.


INDUCT ON B.


THE THEOREM TO BE PROVED IS NOW:

[AND [GT [CONS NIL [LENGTH NIL]] [LENGTH NIL]]
     [IMPLIES [GT [CONS NIL [LENGTH B]] [LENGTH B]]
              [GT [CONS NIL [LENGTH [CONS B1 B]]] [LENGTH [CONS B1 B]]]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]
```

```
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ E N R / E N R [B] , / E N R / E N R .]

TIME: 3.125 SECS.

```
[T 6.2]      [ 16.2 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [AND [GT A B] [GT B C]] [GT A C]]



WHICH IS EQUIVALENT TO:

[COND [GT A B] [COND [GT B C] [GT A C] T] T]



MUST TRY INDUCTION.



INDUCT ON B, A AND C.




THE THEOREM TO BE PROVED IS NOW:

[AND
 [AND [COND [GT A NIL] [COND [GT NIL C] [GT A C] T] T]
 .    [AND [COND [GT NIL B] [COND [GT B C] [GT NIL C] T] T]
 .         [COND [GT A B] [COND [GT B NIL] [GT A NIL] T] T]]]
 [IMPLIES
      [COND [GT A B] [COND [GT B C] [GT A C] T] T]
      [COND [GT [CONS A1 A] [CONS B1 B]]
            [COND [GT [CONS B1 B] [CONS C1 C]] [GT [CONS A1 A] [CONS C1 C]] T]
            T]]]



WHICH IS EQUIVALENT TO:

[COND [GT A B] [COND B [COND A T NIL] T] T]
```

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [GT NIL B] [COND B [COND NIL T NIL] T] T]
          [COND [GT A NIL] [COND NIL [COND A T NIL] T] T]]
     [IMPLIES [COND [GT A B] [COND B [COND A T NIL] T] T]
              [COND [GT [CONS A2 A] [CONS B2 B]]
                    [COND [CONS B2 B] [COND [CONS A2 A] T NIL] T]
                    T]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ E N R / E N R [R A C] , / E N R / E N R / E N R [A B] , / E N R .]

TIME: 9.375 SECS.

```
[T 6 3]      [ 16.2 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [GT A B] [NOT [GT B A]]]



WHICH IS EQUIVALENT TO:

[COND [GT A B] [COND [GT B A] NIL T] T]



MUST TRY INDUCTION.



INDUCT ON B AND A.



THE THEOREM TO BE PROVED IS NOW:

[AND [AND [COND [GT A NIL] [COND [GT NIL A] NIL T] T]
          [COND [GT NIL B] [COND [GT B NIL] NIL T] T]]
     [IMPLIES [COND [GT A B] [COND [GT B A] NIL T] T]
              [COND [GT [CONS A1 A] [CONS B1 B]]
                    [COND [GT [CONS B1 B] [CONS A1 A]] NIL T]
                    T]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:
```

```
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]
[NOT [LAMBDA [X] [COND X NIL T]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ E N R / E N R [B A] , / E N R / E N R .]

TIME: 4.375 SECS.

```
[T 6 4]      [ 16.21 18 JULY 1973]


THEOREM TO BE PROVED:

[LTE A [APPEND B A]]


MUST TRY INDUCTION.


INDUCT ON B.


THE THEOREM TO BE PROVED IS NOW:

[AND [LTE A [APPEND NIL A]]
     [IMPLIES [LTE A [APPEND B A]] [LTE A [APPEND [CONS B1 B] A]]]]


WHICH IS EQUIVALENT TO:

[COND [LTE A A]
      [COND [LTE A [APPEND B A]] [LTE A [CONS B1 [APPEND B A]]] T]
      NIL]


(WORK ON FIRST CONJUNCT ONLY)


MUST TRY INDUCTION.
```

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[COND [AND [LTE NIL NIL] [IMPLIES [LTE A A] [LTE [CONS A1 A] [CONS A1 A]]]]
      [COND [LTE A2 [APPEND B A2]] [LTE A2 [CONS B1 [APPEND B A2]]] T]
      NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND [LTE A2 [APPEND B A2]] [LTE A2 [CONS B1 [APPEND B A2]]] T]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [APPEND B A2] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [LTE A2 GENRL1] [LTE A2 [CONS B1 GENRL1]] T]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1 AND A2.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [LTE A2 NIL] [LTE A2 [CONS B1 NIL]] T]
          [COND [LTE NIL GENRL1] [LTE NIL [CONS B1 GENRL1]] T]]
```

```
[IMPLIES [COND [LTE A2 GENRL1] [LTE A2 [CONS B1 GENRL1]] T]
         [COND [LTE [CONS A21 A2] [CONS GENRL11 GENRL1]]
               [LTE [CONS A21 A2] [CONS B1 [CONS GENRL11 GENRL1]]]
               T]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [LTE A2 GENRL1]
      [COND [LTE A2 [CONS B1 GENRL1]] [LTE A2 [CONS GENRL11 GENRL1]] T]
      T]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1 AND A2.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [AND [COND [LTE A2 NIL]
 .          [COND [LTE A2 [CONS B1 NIL]] [LTE A2 [CONS GENRL11 NIL]] T]
 .          T]
 .    [COND [LTE NIL GENRL1]
 .          [COND [LTE NIL [CONS B1 GENRL1]] [LTE NIL [CONS GENRL11 GENRL1]] T]
 .          T]]
 [IMPLIES
      [COND [LTE A2 GENRL1]
      .     [COND [LTE A2 [CONS B1 GENRL1]] [LTE A2 [CONS GENRL11 GENRL1]] T]
      .     T]
      [COND [LTE [CONS A22 A2] [CONS GENRL12 GENRL1]]
            [COND [LTE [CONS A22 A2] [CONS B1 [CONS GENRL12 GENRL1]]]
                  [LTE [CONS A22 A2] [CONS GENRL11 [CONS GENRL12 GENRL1]]]
                  T]
            T]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

GENERALIZATIONS:

GENRL1 = [APPEND B A2]

PROFILE: [/ [B] , / E N R / E N R & [A] , / E N R / E N R / E N R G [GENRL1 A2] , / E N R / E N R / E N R [GENRL1 A2] , / E N R / E N R .]

TIME: 17.75 SECS.

```
[T 6 5]      [ 16.21 18 JULY 1973]


THEOREM TO BE PROVED:

[OR [LTE A B] [LTE B A]]



WHICH IS EQUIVALENT TO:

[COND [LTE A B] T [LTE B A]]



MUST TRY INDUCTION.



INDUCT ON B AND A.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [AND [COND [LTE A NIL] T [LTE NIL A]] [COND [LTE NIL B] T [LTE B NIL]]]
 [IMPLIES
         [COND [LTE A B] T [LTE B A]]
         [COND [LTE [CONS A1 A] [CONS B1 B]] T [LTE [CONS B1 B] [CONS A1 A]]]]]



WHICH IS EQUIVALENT TO:

[



FUNCTION DEFINITIONS:
```

```
[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]
[OR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]


PROFILE:  [/ E N R / E N R [B A] , / E N R / E N R .]


TIME:  3.625 SECS.
```

```
[T 6 6]      [ 16.21 18 JULY 1973]


THEOREM TO BE PROVED:

[OR [GT A B] [OR [GT B A] [EQUAL [LENGTH A] [LENGTH B]]]]


WHICH IS EQUIVALENT TO:

[COND [GT A B] T [COND [GT B A] T [EQUAL [LENGTH A] [LENGTH B]]]]


MUST TRY INDUCTION.


INDUCT ON A AND B.


THE THEOREM TO BE PROVED IS NOW:

[AND
  [AND [COND [GT NIL B] T [COND [GT B NIL] T [EQUAL [LENGTH NIL] [LENGTH B]]]]
       [COND [GT A NIL] T [COND [GT NIL A] T [EQUAL [LENGTH A] [LENGTH NIL]]]]]
  [IMPLIES [COND [GT A B] T [COND [GT B A] T [EQUAL [LENGTH A] [LENGTH B]]]]
           [COND [GT [CONS A1 A] [CONS B1 B]]
                 T
                 [COND [GT [CONS B1 B] [CONS A1 A]]
                       T
                       [EQUAL [LENGTH [CONS A1 A]] [LENGTH [CONS B1 B]]]]]]]


WHICH IS EQUIVALENT TO:

T
```

FUNCTION DEFINITIONS:

```
[GT [LAMBDA [X Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]
[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]
[OR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ E N R / E N R [A B] , / E N R / E N R .]

TIME: 7.875 SECS.

[T 6 7]   [16.21 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [MONOT2P A] [MONOT1 A]]

WHICH IS EQUIVALENT TO:

[COND A [EQUAL [MONOT2 [CAR A] [CDR A]] [MONOT1 A]] T]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND NIL [EQUAL [MONOT2 [CAR NIL] [CDR NIL]] [MONOT1 NIL]] T]
 [AND
  [COND [CONS A1 NIL]
  .      [EQUAL [MONOT2 [CAR [CONS A1 NIL]] [CDR [CONS A1 NIL]]]
  .             [MONOT1 [CONS A1 NIL]]]
  .      T]
  [IMPLIES
   [COND
   .  [CONS A2 A]
   .  [EQUAL [MONOT2 [CAR [CONS A2 A]] [CDR [CONS A2 A]]] [MONOT1 [CONS A2 A]]]
   .  T]
   [COND
        [CONS A1 [CONS A2 A]]
        [EQUAL [MONOT2 [CAR [CONS A1 [CONS A2 A]]] [CDR [CONS A1 [CONS A2 A]]]]
```

```
.       [MONOT1 [CONS A1 [CONS A2 A]]]]
T]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 A
 [COND
 .     [EQUAL A2 [CAR A]]
 .     [COND [EQUAL [MONOT2 A2 A] [MONOT1 A]]
 .     .     [COND [EQUAL A1 A2] [EQUAL [MONOT2 A1 A] [MONOT1 A]] T]
 .     .     T]
 .     [COND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]]
 T]
```

FERTILIZE WITH [EQUAL A2 [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .[COND [COND [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]]
 .            [COND [EQUAL A1 [CAR A]] [EQUAL [MONOT2 A1 A] [MONOT1 A]] T]
 .            T]
 .      T
 .      [*1]]
 .[COND
 .      [COND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]
 .      T
 .      [EQUAL A2 [CAR A]]]
 .NIL]
 T]
```

WHICH IS EQUIVALENT TO:

```
[COND
 A
 [COND [COND [COND [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]]
 .                 [COND [EQUAL A1 [CAR A]] [EQUAL [MONOT2 A1 A] [MONOT1 A]] T]
 .                 T]
 .           T
```

```
.            [*1]]
.      [COND [MONOT2 A2 A]
.            T
.            [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
.      NIL]
 T]
```

FERTILIZE WITH [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .[COND
 .   [COND [COND [EQUAL A1 [CAR A]] [EQUAL [MONOT2 A1 A] [MONOT2 [CAR A] A]] T]
 .   .     T
 .   .     [*2]]
 .   T
 .   [*1]]
 .[COND [MONOT2 A2 A]
 .      T
 .      [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
 .NIL]
 T]
```

FERTILIZE WITH [EQUAL A1 [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .[COND [COND [COND [EQUAL [MONOT2 A1 A] [MONOT2 A1 A]] T [*3]] T [*2]] T [*1]]
 .[COND [MONOT2 A2 A]
 .      T
 .      [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
 .NIL]
 T]
```

```
WHICH IS EQUIVALENT TO:

[COND A
      [COND [MONOT2 A2 A]
            T
            [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
      T]



FERTILIZE WITH [EQUAL A1 A2].



THE THEOREM TO BE PROVED IS NOW:

[COND
 A
 [COND [MONOT2 A2 A] T [COND [COND [MONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]
 T]



MUST TRY INDUCTION.



INDUCT ON A.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND NIL
 .      [COND [MONOT2 A2 NIL]
 .            T
 .            [COND [COND [MONOT2 A1 NIL] [EQUAL A1 [CAR NIL]] T] T [*4]]]
 .      T]
 [IMPLIES
  [COND A
  .      [COND [MONOT2 A2 A]
  .            T
  .            [COND [COND [MONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]
```

```
  .      T]
 [COND
     [CONS A3 A]
     [COND [MONOT2 A2 [CONS A3 A]]
     .      T
     .      [COND [COND [MONOT2 A1 [CONS A3 A]] [EQUAL A1 [CAR [CONS A3 A]]] T]
     .            T
     .            [*4]]]
     T]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[MONOT2P [LAMBDA [X] [COND X [MONOT2 [CAR X] [CDR X]] T]]]

[MONOT2
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] [MONOT2 X [CDR Y]] NIL] T]]]

[MONOT1
 [LAMBDA
  [X]
  [COND
     X
     [COND [CDR X] [COND [EQUAL [CAR X] [CAR [CDR X]]] [MONOT1 [CDR X]] NIL] T]
     T]]]

[CARARG UNDEF]

[CDRARG UNDEF]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND
      [EQUAL A2 [CAR A]]
      NIL
      [COND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]]

*2 = [COND [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]] NIL T]
```

```
*3 = [COND [EQUAL A1 [CAR A]] NIL T]

*4 = [COND [EQUAL A1 A2] NIL T]


PROFILE:  [/ E N R / E N R / E N R S1 [A] , / E N R / E N R / E N R F , / N R /
E N R F , / X , / E N R / E N R F , / [A] , / E N R / E N R .]


TIME:  33.06 SECS.
```

[T 6 8]   [16.22 18 JULY 1973]

THEOREM TO BE PROVED:

[ORDERED [SORT A]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [ORDERED [SORT NIL]]
     [IMPLIES [ORDERED [SORT A]] [ORDERED [SORT [CONS A1 A]]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [ORDERED [SORT A]] [ORDERED [ADDTOLIS A1 [SORT A]]] T]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [SORT A] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [ORDERED GENRL1] [ORDERED [ADDTOLIS A1 GENRL1]] T]
```

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [ORDERED NIL] [ORDERED [ADDTOLIS A1 NIL]] T]
 [AND
    [COND [ORDERED [CONS GENRL11 NIL]]
    .      [ORDERED [ADDTOLIS A1 [CONS GENRL11 NIL]]]
    .      T]
    [IMPLIES [COND [ORDERED [CONS GENRL12 GENRL1]]
                   [ORDERED [ADDTOLIS A1 [CONS GENRL12 GENRL1]]]
                   T]
             [COND [ORDERED [CONS GENRL11 [CONS GENRL12 GENRL1]]]
                   [ORDERED [ADDTOLIS A1 [CONS GENRL11 [CONS GENRL12 GENRL1]]]]
                   T]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [LTE A1 GENRL11] T [LTE GENRL11 A1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL11 AND A1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [LTE A1 NIL] T [LTE NIL A1]]
          [COND [LTE NIL GENRL11] T [LTE GENRL11 NIL]]]
     [IMPLIES [COND [LTE A1 GENRL11] T [LTE GENRL11 A1]]
              [COND [LTE [CONS A11 A1] [CONS GENRL111 GENRL11]]
                    T
                    [LTE [CONS GENRL111 GENRL11] [CONS A11 A1]]]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[SORT [LAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR X]]] NIL]]]

[ORDERED
 [LAMBDA
  [X]
  [COND
      X
      [COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
      T]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[ADDTOLIS
 [LAMBDA
     [X Y]
     [COND Y
           [COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]]
           [CONS X NIL]]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

GENERALIZATIONS:

```
GENRL1 = [SORT A]
```

PROFILE: [/ [A] , / E N R / E N R G S1 [GENRL1] , / E N R / E N R / E N R / E N
R / E N R [GENRL11 A1] , / E N R / E N R .]

TIME: 57.06 SECS.

[T 6 9]   [16.24 18 JULY 1973]

THEOREM TO BE PROVED:

[IMPLIES [AND [MONOT1 A] [MEMBER B A]] [EQUAL [CAR A] B]]

WHICH IS EQUIVALENT TO:

[COND [MONOT1 A] [COND [MEMBER B A] [EQUAL [CAR A] B] T] T]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [MONOT1 NIL] [COND [MEMBER B NIL] [EQUAL [CAR NIL] B] T] T]
 [AND
     [COND [MONOT1 [CONS A1 NIL]]
     .      [COND [MEMBER B [CONS A1 NIL]] [EQUAL [CAR [CONS A1 NIL]] B] T]
     .      T]
     [IMPLIES [COND [MONOT1 [CONS A2 A]]
                    [COND [MEMBER B [CONS A2 A]] [EQUAL [CAR [CONS A2 A]] B] T]
                    T]
              [COND [MONOT1 [CONS A1 [CONS A2 A]]]
                    [COND [MEMBER B [CONS A1 [CONS A2 A]]]
                          [EQUAL [CAR [CONS A1 [CONS A2 A]]] B]
                          T]
                    T]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 [COND [EQUAL B A1] [EQUAL A1 B] T]
 [COND
 .A
 .[COND
 .   [EQUAL A2 [CAR A]]
 .   [COND [MONOT1 A]
 .   .      [COND [EQUAL B A2]
 .   .             [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .   .             [COND [MEMBER B A]
 .   .                    [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .   .                    T]]
 .   .      T]
 .   T]
 .[COND [EQUAL B A2]
 .       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .       T]]
 NIL]
```

FERTILIZE WITH [EQUAL B A1].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 [COND [EQUAL B B] T [*1]]
 [COND
 .A
 .[COND
 .   [EQUAL A2 [CAR A]]
 .   [COND [MONOT1 A]
 .   .      [COND [EQUAL B A2]
 .   .             [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .   .             [COND [MEMBER B A]
 .   .                    [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .   .                    T]]
 .   .      T]
 .   T]
 .[COND [EQUAL B A2]
 .       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .       T]]
 NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND
 A
 [COND
 .   [EQUAL A2 [CAR A]]
 .   [COND [MONOT1 A]
 .   .     [COND [EQUAL B A2]
 .   .           [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .   .           [COND [MEMBER B A]
 .   .                 [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
 .   .                 T]]
 .   .     T]
 .   T]
 [COND [EQUAL B A2]
       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T]]
```

FERTILIZE WITH [EQUAL A2 [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .[COND
 . [MONOT1 A]
 . [COND
 . .  [EQUAL B [CAR A]]
 . .  [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
 . .  [COND [MEMBER B A]
 . .        [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
 . .        T]]
 . T]
 .T
 .[*2]]
 [COND [EQUAL B A2]
       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T]]
```

FERTILIZE WITH [EQUAL B [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .[COND
 . [MONOT1 A]
 . [COND
 . .[COND [COND [EQUAL B B] [COND [EQUAL A1 B] [EQUAL A1 B] T] T] T [*3]]
 . .[COND
 . .   [COND [MEMBER B A]
 . . .       [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
 . . .       T]
 . .   T
 . .   [EQUAL B [CAR A]]]
 . .NIL]
 . T]
 .T
 .[*2]]
 [COND [EQUAL B A2]
       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 A
 [COND
 .[COND
 . [MONOT1 A]
 . [COND
 . .   [MEMBER B A]
 . .   [COND [EQUAL [CAR A] B]
 . . .       [COND [EQUAL A1 [CAR A]] [COND [EQUAL A1 B] T [EQUAL B [CAR A]]] T]
 . . .       T]
 . .   T]
 . T]
 .T
 .[*2]]
 [COND [EQUAL B A2]
       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T]]
```

FERTILIZE WITH [EQUAL [CAR A] B].

```
THE THEOREM TO BE PROVED IS NOW:

[COND
 A
 [COND
 .[COND
 .   [MONOT1 A]
 .   [COND [MEMBER B A]
 .   .      [COND [COND [EQUAL A1 B] [COND [EQUAL A1 B] T [EQUAL B B]] T] T [*4]]
 .   .      T]
 .   T]
 .T
 .[*2]]
 [COND [EQUAL B A2]
       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T]]



WHICH IS EQUIVALENT TO:

[COND A
      T
      [COND [EQUAL B A2]
            [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
            T]]



FERTILIZE WITH [EQUAL B A2].



THE THEOREM TO BE PROVED IS NOW:

[COND A
      T
      [COND [COND [EQUAL B B] [COND [EQUAL A1 B] [EQUAL A1 B] T] T] T [*5]]]



WHICH IS EQUIVALENT TO:

T
```

FUNCTION DEFINITIONS:

```
[MONOT1
 [LAMBDA
  [X]
  [COND
     X
     [COND [CDR X] [COND [EQUAL [CAR X] [CAR [CDR X]]] [MONOT1 [CDR X]] NIL] T]
     T]]]

[MEMBER
     [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[CARARG UNDEF]
```

FERTILIZERS:

```
*1 = [COND [EQUAL B A1] NIL T]

*2 = [COND [EQUAL A2 [CAR A]] NIL T]

*3 = [COND
      [EQUAL B [CAR A]]
      NIL
      [COND [MEMBER B A]
            [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
            T]]

*4 = [COND [EQUAL [CAR A] B] NIL T]

*5 = [COND [EQUAL B A2] NIL T]
```

PROFILE: [/ E N R / E N R S1 [A] , / E N R / E N R / E N R F , / E N R / E N R F , / F , / E N R / E N R / E N R F , / E N R / E N R F , / E N R / E N R .]

TIME: 49.63 SECS.

```
[T 6 10]     [ 16.26 18 JULY 1973]

THEOREM TO BE PROVED:

[LTE [CDRN A B] B]

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

[AND
 [AND [LTE [CDRN NIL B] B] [LTE [CDRN A NIL] NIL]]
 [IMPLIES [LTE [CDRN A B] B] [LTE [CDRN [CONS A1 A] [CONS B1 B]] [CONS B1 B]]]]

WHICH IS EQUIVALENT TO:

[COND [LTE B B] [COND [LTE [CDRN A B] B] [LTE [CDRN A B] [CONS B1 B]] T] NIL]

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON B.
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND [AND [LTE NIL NIL] [IMPLIES [LTE B B] [LTE [CONS B2 B] [CONS B2 B]]]]
      [COND [LTE [CDRN A B3] B3] [LTE [CDRN A B3] [CONS B1 B3]] T]
      NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND [LTE [CDRN A B3] B3] [LTE [CDRN A B3] [CONS B1 B3]] T]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [CDRN A B3] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [LTE GENRL1 B3] [LTE GENRL1 [CONS B1 B3]] T]
```

MUST TRY INDUCTION.

INDUCT ON B3 AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [LTE GENRL1 NIL] [LTE GENRL1 [CONS B1 NIL]] T]
          [COND [LTE NIL B3] [LTE NIL [CONS B1 B3]] T]]
     [IMPLIES [COND [LTE GENRL1 B3] [LTE GENRL1 [CONS B1 B3]] T]
```

```
            [COND [LTE [CONS GENRL11 GENRL1] [CONS B31 B3]]
                  [LTE [CONS GENRL11 GENRL1] [CONS B1 [CONS B31 B3]]]
                  T]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [LTE GENRL1 B3]
      [COND [LTE GENRL1 [CONS B1 B3]] [LTE GENRL1 [CONS B31 B3]] T]
      T]
```

MUST TRY INDUCTION.

INDUCT ON B3 AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
   [AND [COND [LTE GENRL1 NIL]
   .          [COND [LTE GENRL1 [CONS B1 NIL]] [LTE GENRL1 [CONS B31 NIL]] T]
   .          T]
   .     [COND [LTE NIL B3]
   .          [COND [LTE NIL [CONS B1 B3]] [LTE NIL [CONS B31 B3]] T]
   .          T]]
   [IMPLIES [COND [LTE GENRL1 B3]
                  [COND [LTE GENRL1 [CONS B1 B3]] [LTE GENRL1 [CONS B31 B3]] T]
                  T]
            [COND [LTE [CONS GENRL12 GENRL1] [CONS B32 B3]]
                  [COND [LTE [CONS GENRL12 GENRL1] [CONS B1 [CONS B32 B3]]]
                        [LTE [CONS GENRL12 GENRL1] [CONS B31 [CONS B32 B3]]]
                        T]
                  T]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[CDRN [LAMBDA [X Y] [COND Y [COND X [CDRN [CDR X] [CDR Y]] Y] NIL]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

GENERALIZATIONS:

```
GENRL1 = [CDRN A B3]
```

```
PROFILE:  [/ [A B] , / E N R / E N R & [B] , / E N R / E N R / E N R G [B3 GENRL
1] , / E N R / E N R / E N R [B3 GENRL1] , / E N R / E N R .]
```

TIME: 18.06 SECS.

```
[T 6 12]      [ 16.26 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [LENGTH A] [LENGTH [SORT A]]]


MUST TRY INDUCTION.


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [LENGTH NIL] [LENGTH [SORT NIL]]]
     [IMPLIES [EQUAL [LENGTH A] [LENGTH [SORT A]]]
              [EQUAL [LENGTH [CONS A1 A]] [LENGTH [SORT [CONS A1 A]]]]]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL [LENGTH A] [LENGTH [SORT A]]]
      [EQUAL [CONS NIL [LENGTH A]] [LENGTH [ADDTOLIS A1 [SORT A]]]]
      T]


FERTILIZE WITH [EQUAL [LENGTH A] [LENGTH [SORT A]]].


THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [CONS NIL [LENGTH [SORT A]]] [LENGTH [ADDTOLIS A1 [SORT A]]]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [SORT A] BY GENRL1.

THE GENERALIZED TERM IS:

[COND [EQUAL [CONS NIL [LENGTH GENRL1]] [LENGTH [ADDTOLIS A1 GENRL1]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [CONS NIL [LENGTH NIL]] [LENGTH [ADDTOLIS A1 NIL]]] T [*1]]
 [IMPLIES
          [COND [EQUAL [CONS NIL [LENGTH GENRL1]] [LENGTH [ADDTOLIS A1 GENRL1]]]
          .     T
          .     [*1]]
          [COND [EQUAL [CONS NIL [LENGTH [CONS GENRL11 GENRL1]]]
                       [LENGTH [ADDTOLIS A1 [CONS GENRL11 GENRL1]]]]
                T
                [*1]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

[SORT [LAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR X]]] NIL]]]

[ADDTOLIS
 [LAMBDA
    [X Y]
    [COND Y
          [COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]]
          [CONS X NIL]]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [LENGTH A] [LENGTH [SORT A]]] NIL T]
```

GENERALIZATIONS:

```
GENRL1 = [SORT A]
```

```
PROFILE:  [/ [A] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R / E N R .]
```

TIME: 13.0 SECS.

```
[T 6 14]      [ 16.26 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [ORDERED A] [EQUAL A [SORT A]]]



WHICH IS EQUIVALENT TO:

[COND [ORDERED A] [EQUAL A [SORT A]] T]



MUST TRY INDUCTION.



(SPECIAL CASE REQUIRED)



INDUCT ON A.




THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [ORDERED NIL] [EQUAL NIL [SORT NIL]] T]
 [AND
  [COND [ORDERED [CONS A1 NIL]] [EQUAL [CONS A1 NIL] [SORT [CONS A1 NIL]]] T]
  [IMPLIES
          [COND [ORDERED [CONS A2 A]] [EQUAL [CONS A2 A] [SORT [CONS A2 A]]] T]
          [COND [ORDERED [CONS A1 [CONS A2 A]]]
                [EQUAL [CONS A1 [CONS A2 A]] [SORT [CONS A1 [CONS A2 A]]]]
                T]]]]



WHICH IS EQUIVALENT TO:
```

```
[COND
 A
 [COND
 .[LTE A2 [CAR A]]
 .[COND
 . [ORDERED A]
 . [COND
 . .   [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]]]
 . .   [COND [LTE A1 A2]
 . .   .     [EQUAL [CONS A1 [CONS A2 A]] [ADDTOLIS A1 [ADDTOLIS A2 [SORT A]]]]
 . .   .     T]
 . .   T]
 . T]
 .T]
 T]
```

FERTILIZE WITH [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 A
 [COND
 .    [LTE A2 [CAR A]]
 .    [COND [ORDERED A]
 .    .     [COND [COND [LTE A1 A2]
 .    .                 [EQUAL [CONS A1 [CONS A2 A]] [ADDTOLIS A1 [CONS A2 A]]]
 .    .                 T]
 .    .           T
 .    .           [*1]]
 .    .     T]
 .    T]
 T]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[ORDERED
 [LAMBDA
  [X]
  [COND
      X
      [COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
      T]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[SORT [LAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR X]]] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[ADDTOLIS
 [LAMBDA
    [X Y]
    [COND Y
          [COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]]
          [CONS X NIL]]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]]] NIL T]
```

PROFILE: [/ E N R / E N R S1 [A] , / E N R / E N R / E N R / E N R F , / E N R / E N R .]

TIME: 20.25 SECS.

```
[T 6 15]      [ 16.27 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [ORDERED [APPEND A B]] [ORDERED A]]



WHICH IS EQUIVALENT TO:

[COND [ORDERED [APPEND A B]] [ORDERED A] T]



MUST TRY INDUCTION.



(SPECIAL CASE REQUIRED)



INDUCT ON A.



THE THEOREM TO BE PROVED IS NOW:

[AND
  [COND [ORDERED [APPEND NIL B]] [ORDERED NIL] T]
  [AND [COND [ORDERED [APPEND [CONS A1 NIL] B]] [ORDERED [CONS A1 NIL]] T]
       [IMPLIES [COND [ORDERED [APPEND [CONS A2 A] B]] [ORDERED [CONS A2 A]] T]
                [COND [ORDERED [APPEND [CONS A1 [CONS A2 A]] B]]
                      [ORDERED [CONS A1 [CONS A2 A]]]
                      T]]]]



WHICH IS EQUIVALENT TO:

T
```

FUNCTION DEFINITIONS:

```
[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[ORDERED
 [LAMBDA
  [X]
  [COND
      X
      [COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
      T]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

PROFILE: [/ E N R / E N R S_1 [A] , / E N R / E N R .]

TIME: 14.69 SECS.

```
[T 6 16]      [ 16.27 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [ORDERED [APPEND A B]] [ORDERED B]]



WHICH IS EQUIVALENT TO:

[COND [ORDERED [APPEND A B]] [ORDERED B] T]



MUST TRY INDUCTION.



INDUCT ON A.



THE THEOREM TO BE PROVED IS NOW:

[AND [COND [ORDERED [APPEND NIL B]] [ORDERED B] T]
     [IMPLIES [COND [ORDERED [APPEND A B]] [ORDERED B] T]
              [COND [ORDERED [APPEND [CONS A1 A] B]] [ORDERED B] T]]]



WHICH IS EQUIVALENT TO:

[COND [ORDERED [APPEND A B]] T [COND [APPEND A B] T [ORDERED B]]]



MUST TRY INDUCTION.
```

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [COND [ORDERED [APPEND NIL B]] T [COND [APPEND NIL B] T [ORDERED B]]]
     [IMPLIES [COND [ORDERED [APPEND A B]] T [COND [APPEND A B] T [ORDERED B]]]
              [COND [ORDERED [APPEND [CONS A2 A] B]]
                    T
                    [COND [APPEND [CONS A2 A] B] T [ORDERED B]]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [ORDERED B] T [COND B T NIL]]
```

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [COND [ORDERED NIL] T [COND NIL T NIL]]
     [AND [COND [ORDERED [CONS B1 NIL]] T [COND [CONS B1 NIL] T NIL]]
          [IMPLIES [COND [ORDERED [CONS B2 B]] T [COND [CONS B2 B] T NIL]]
                   [COND [ORDERED [CONS B1 [CONS B2 B]]]
                         T
                         [COND [CONS B1 [CONS B2 B]] T NIL]]]]]
```

```
WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[ORDERED
 [LAMBDA
  [X]
  [COND
      X
      [COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
      T]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


PROFILE:  [/ E N R / E N R [A] , / E N R / E N R / E N R [A] , / E N R / E N R S
1 [B] , / E N R .]


TIME:  17.0 SECS.
```

```
[T 6 18]      [ 16.29 18 JULY 1973]
```

THEOREM TO BE PROVED:

```
[LTE [HALF A] A]
```

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [LTE [HALF NIL] NIL]
     [AND [LTE [HALF [CONS A1 NIL]] [CONS A1 NIL]]
          [IMPLIES [LTE [HALF A] A]
                   [LTE [HALF [CONS A1 [CONS A2 A]]] [CONS A1 [CONS A2 A]]]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [LTE [HALF A] A] [LTE [HALF A] [CONS A2 A]] T]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [HALF A] BY GENRL1.

THE GENERALIZED TERM IS:

```
[IMPLIES [NUMBERP GENRL1] [COND [LTE GENRL1 A] [LTE GENRL1 [CONS A2 A]] T]]
```

MUST TRY INDUCTION.

INDUCT ON A AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [AND [IMPLIES [NUMBERP GENRL1]
 .             [COND [LTE GENRL1 NIL] [LTE GENRL1 [CONS A2 NIL]] T]]
 .    [IMPLIES [NUMBERP NIL] [COND [LTE NIL A] [LTE NIL [CONS A2 A]] T]]]
 [IMPLIES
    [IMPLIES [NUMBERP GENRL1] [COND [LTE GENRL1 A] [LTE GENRL1 [CONS A2 A]] T]]
    [IMPLIES [NUMBERP [CONS GENRL11 GENRL1]]
             [COND [LTE [CONS GENRL11 GENRL1] [CONS A3 A]]
                   [LTE [CONS GENRL11 GENRL1] [CONS A2 [CONS A3 A]]]
                   T]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 [NUMBERP GENRL1]
 [COND
 .  [LTE GENRL1 A]
 .  [COND [LTE GENRL1 [CONS A2 A]] [COND GENRL11 T [LTE GENRL1 [CONS A3 A]]] T]
 .  T]
 T]
```

MUST TRY INDUCTION.

INDUCT ON A AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [AND
 .[COND [NUMBERP GENRL1]
 .      [COND [LTE GENRL1 NIL]
 .            [COND [LTE GENRL1 [CONS A2 NIL]]
 .                  [COND GENRL11 T [LTE GENRL1 [CONS A3 NIL]]]
 .                  T]
 .            T]
 .      T]
 .[COND
 .  [NUMBERP NIL]
 .  [COND [LTE NIL A]
 .  .     [COND [LTE NIL [CONS A2 A]] [COND GENRL11 T [LTE NIL [CONS A3 A]]] T]
 .  .     T]
 .  T]]
 [IMPLIES
  [COND
  .[NUMBERP GENRL1]
  .[COND
  . [LTE GENRL1 A]
  . [COND [LTE GENRL1 [CONS A2 A]] [COND GENRL11 T [LTE GENRL1 [CONS A3 A]]] T]
  . T]
  .T]
  [COND
   [NUMBERP [CONS GENRL12 GENRL1]]
   [COND
   .   [LTE [CONS GENRL12 GENRL1] [CONS A4 A]]
   .   [COND [LTE [CONS GENRL12 GENRL1] [CONS A2 [CONS A4 A]]]
   .   .     [COND GENRL11 T [LTE [CONS GENRL12 GENRL1] [CONS A3 [CONS A4 A]]]]
   .   .     T]
   .   T]
   T]]]
```

WHICH IS EQUIVALENT TO:

```
|
```

FUNCTION DEFINITIONS:

```
[HALF [LAMBDA [X] [COND X [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]]

[LTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[NUMBERP [LAMBDA [X] [COND X [COND [CAR X] NIL [NUMBERP [CDR X]]] T]]]


GENERALIZATIONS:

GENRL1 = [HALF A]


PROFILE:  [/ S2 [A] , / E N R / E N R G [A GENRL1] , / E N R / E N R / E N R [A
GENRL1] , / E N R / E N R .]


TIME:  22.81 SECS.
```

```
[T 7 1]      [ 16.3 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [COPY A] A]



MUST TRY INDUCTION.



INDUCT ON A.




THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [COPY NIL] NIL]
     [IMPLIES [AND [EQUAL [COPY A1] A1] [EQUAL [COPY A] A]]
              [EQUAL [COPY [CONS A1 A]] [CONS A1 A]]]]



WHICH IS EQUIVALENT TO:

T




FUNCTION DEFINITIONS:

[COPY [LAMBDA [X] [COND X [CONS [COPY [CAR X]] [COPY [CDR X]]] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]



PROFILE:  [/ [A] , / E N R / E N R .]
```

TIME: 2.438 SECS.

```
[T 7 2]      [ 16.3 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [EQUALP A B] [EQUAL A B]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL A B] [EQUALP A B] [COND [EQUALP A B] NIL T]]


FERTILIZE WITH [EQUAL A B].


THE THEOREM TO BE PROVED IS NOW:

[COND [COND [EQUALP A A] T [*1]]
      [COND [COND [EQUALP A B] NIL T] T [EQUAL A B]]
      NIL]


WHICH IS EQUIVALENT TO:

[COND [COND [EQUALP A A] T [*1]] [COND [EQUALP A B] [EQUAL A B] T] NIL]


(WORK ON FIRST CONJUNCT ONLY)


MUST TRY INDUCTION.


INDUCT ON A.
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND
    [AND [COND [EQUALP NIL NIL] T [*1]]
    .    [IMPLIES [AND [COND [EQUALP A1 A1] T [*1]] [COND [EQUALP A A] T [*1]]]
    .                  [COND [EQUALP [CONS A1 A] [CONS A1 A]] T [*1]]]]
    [COND [EQUALP A2 B] [EQUAL A2 B] T]
    NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUALP A2 B] [EQUAL A2 B] T]
```

MUST TRY INDUCTION.

INDUCT ON A2 AND B.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [EQUALP NIL B] [EQUAL NIL B] T]
          [COND [EQUALP A2 NIL] [EQUAL A2 NIL] T]]
     [IMPLIES [AND [COND [EQUALP A21 B1] [EQUAL A21 B1] T]
                   [COND [EQUALP A2 B] [EQUAL A2 B] T]]
              [COND [EQUALP [CONS A21 A2] [CONS B1 B]]
                    [EQUAL [CONS A21 A2] [CONS B1 B]]
                    T]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[EQUALP
 [LAMBDA
  [X Y]
  [COND
      X
      [COND Y [COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL]
      [COND Y NIL T]]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL A B] NIL [COND [EQUALP A B] NIL T]]
```

PROFILE: [/ N R / E N R F , / N R / E N R & [A] , / E N R / E N R / E N R [A2 B
] , / E N R / E N R .]

TIME: 14.87 SECS.

```
[T 7 3]      [ 16.31 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [SUBST A A B] B]


MUST TRY INDUCTION.


INDUCT ON B.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [SUBST A A NIL] NIL]
     [IMPLIES [AND [EQUAL [SUBST A A B1] B1] [EQUAL [SUBST A A B] B]]
              [EQUAL [SUBST A A [CONS B1 B]] [CONS B1 B]]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:

[SUBST
   [LAMBDA [X Y Z]
           [COND [EQUAL Y Z]
                 X
                 [COND Z [CONS [SUBST X Y [CAR Z]] [SUBST X Y [CDR Z]]] NIL]]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ [B] , / E N R / E N R / E N R .]

TIME: 6.125 SECS.

```
[T 7 4]      [ 16.31 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [MEMBER A B] [OCCUR A B]]



WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] [OCCUR A B] T]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [MEMBER A NIL] [OCCUR A NIL] T]
 [IMPLIES
    [AND [COND [MEMBER A B1] [OCCUR A B1] T] [COND [MEMBER A B] [OCCUR A B] T]]
    [COND [MEMBER A [CONS B1 B]] [OCCUR A [CONS B1 B]] T]]]



WHICH IS EQUIVALENT TO:

[COND
   [MEMBER A B1]
   T
   [COND [MEMBER A B]
         T
         [COND [EQUAL A B1]
               [COND [EQUAL A [CONS B1 B]] T [COND [OCCUR A B1] T [OCCUR A B]]]
               T]]]
```

FERTILIZE WITH [EQUAL A B1].

THE THEOREM TO BE PROVED IS NOW:

```
[COND
      [MEMBER A B1]
      T
      [COND [MEMBER A B]
            T
            [COND [COND [EQUAL A [CONS A B]] T [COND [OCCUR A A] T [OCCUR A B]]]
                  T
                  [*1]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[MEMBER
      [LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[OCCUR
      [LAMBDA [X Y]
              [COND [EQUAL X Y]
                    T
                    [COND Y [COND [OCCUR X [CAR Y]] T [OCCUR X [CDR Y]]] NIL]]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL A B1] NIL T]
```

PROFILE: [/ E N R / E N R [B] , / E N R / E N R / E N R F , / E N R .]

TIME: 12.44 SECS.

```
[T 7 5]      [ 16.31 18 JULY 1973]


THEOREM TO BE PROVED:

[IMPLIES [NOT [OCCUR A B]] [EQUAL [SUBST C A B] B]]



WHICH IS EQUIVALENT TO:

[COND [OCCUR A B] T [EQUAL [SUBST C A B] B]]



MUST TRY INDUCTION.



INDUCT ON B.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [OCCUR A NIL] T [EQUAL [SUBST C A NIL] NIL]]
 [IMPLIES
   [AND [COND [OCCUR A B1] T [EQUAL [SUBST C A B1] B1]]
   .    [COND [OCCUR A B] T [EQUAL [SUBST C A B] B]]]
   [COND [OCCUR A [CONS B1 B]] T [EQUAL [SUBST C A [CONS B1 B]] [CONS B1 B]]]]]



WHICH IS EQUIVALENT TO:

T



FUNCTION DEFINITIONS:
```

```
[OCCUR
     [LAMBDA [X Y]
             [COND [EQUAL X Y]
                   T
                   [COND Y [COND [OCCUR X [CAR Y]] T [OCCUR X [CDR Y]]] NIL]]]]

[NOT [LAMBDA [X] [COND X NIL T]]]

[SUBST
   [LAMBDA [X Y Z]
           [COND [EQUAL Y Z]
                 X
                 [COND Z [CONS [SUBST X Y [CAR Z]] [SUBST X Y [CDR Z]]] NIL]]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]


PROFILE:  [/ E N R / E N R [B] , / E N R / E N R / E N R .]


TIME:  14.19 SECS.
```

```
[T 7 6]      [ 16.31 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [EQUALP A B] [EQUALP B A]]


MUST TRY INDUCTION.


INDUCT ON B AND A.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [AND [EQUAL [EQUALP A NIL] [EQUALP NIL A]]
 .    [EQUAL [EQUALP NIL B] [EQUALP B NIL]]]
 [IMPLIES
  [AND [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]] [EQUAL [EQUALP A B] [EQUALP B A]]]
  [EQUAL [EQUALP [CONS A1 A] [CONS B1 B]] [EQUALP [CONS B1 B] [CONS A1 A]]]]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]]
      [COND [EQUAL [EQUALP A B] [EQUALP B A]]
            [COND [EQUALP A1 B1]
                  [COND [EQUALP B1 A1] T [COND [EQUALP A B] NIL T]]
                  [COND [EQUALP B1 A1] [COND [EQUALP B A] NIL T] T]]
            T]
      T]


FERTILIZE WITH [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]].
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [EQUAL [EQUALP A B] [EQUALP B A]]
            [COND [EQUALP A1 B1]
                  [COND [EQUALP A1 B1] T [COND [EQUALP A B] NIL T]]
                  [COND [EQUALP A1 B1] [COND [EQUALP B A] NIL T] T]]
            T]
      T
      [*1]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[EQUALP
 [LAMBDA
  [X Y]
  [COND
      X
      [COND Y [COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL]
      [COND Y NIL T]]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

FERTILIZERS:

*1 = [COND [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]] NIL T]

PROFILE: [/ [B A] , / E N R / E N R F , / R / E N R .]

TIME: 10.69 SECS.

```
[T 7 7]      [ 16.32 18 JULY 1973]



THEOREM TO BE PROVED:

[IMPLIES [AND [EQUALP A B] [EQUALP B C]] [EQUALP A C]]



WHICH IS EQUIVALENT TO:

[COND [EQUALP A B] [COND [EQUALP B C] [EQUALP A C] T] T]



MUST TRY INDUCTION.



INDUCT ON B, A AND C.



THE THEOREM TO BE PROVED IS NOW:

[AND
 [AND [COND [EQUALP A NIL] [COND [EQUALP NIL C] [EQUALP A C] T] T]
 .    [AND [COND [EQUALP NIL B] [COND [EQUALP B C] [EQUALP NIL C] T] T]
 .         [COND [EQUALP A B] [COND [EQUALP B NIL] [EQUALP A NIL] T] T]]]
 [IMPLIES
  [AND [COND [EQUALP A1 B1] [COND [EQUALP B1 C1] [EQUALP A1 C1] T] T]
  .    [COND [EQUALP A B] [COND [EQUALP B C] [EQUALP A C] T] T]]
  [COND
     [EQUALP [CONS A1 A] [CONS B1 B]]
     [COND [EQUALP [CONS B1 B] [CONS C1 C]] [EQUALP [CONS A1 A] [CONS C1 C]] T]
     T]]]



WHICH IS EQUIVALENT TO:

[COND [EQUALP A B] [COND B T [COND A NIL T]] T]
```

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [AND [COND [EQUALP NIL B] [COND B T [COND NIL NIL T]] T]
          [COND [EQUALP A NIL] [COND NIL T [COND A NIL T]] T]]
     [IMPLIES [AND [COND [EQUALP A2 B2] [COND B2 T [COND A2 NIL T]] T]
                   [COND [EQUALP A B] [COND B T [COND A NIL T]] T]]
              [COND [EQUALP [CONS A2 A] [CONS B2 B]]
                    [COND [CONS B2 B] T [COND [CONS A2 A] NIL T]]
                    T]]]
```

WHICH IS EQUIVALENT TO:

```
T
```

FUNCTION DEFINITIONS:

```
[EQUALP
 [LAMBDA
  [X Y]
  [COND
      X
      [COND Y [COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL]
      [COND Y NIL T]]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

PROFILE: [/ E N R / E N R [B A C] , / E N R / E N R / E N R / E N R [A B] , / E

N R / E N R .]

[T 7 7]

TIME: 29.88 SECS.

```
[T 7 8]      [ 16.33 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [SWAPTREE [SWAPTREE A]] A]


MUST TRY INDUCTION.


(SPECIAL CASE REQUIRED)


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [SWAPTREE [SWAPTREE NIL]] NIL]
 [AND
  [EQUAL [SWAPTREE [SWAPTREE [CONS A1 NIL]]] [CONS A1 NIL]]
  [IMPLIES
   [AND [EQUAL [SWAPTREE [SWAPTREE A2]] A2] [EQUAL [SWAPTREE [SWAPTREE A]] A]]
   [EQUAL [SWAPTREE [SWAPTREE [CONS A1 [CONS A2 A]]]] [CONS A1 [CONS A2 A]]]]]]


WHICH IS EQUIVALENT TO:

T


FUNCTION DEFINITIONS:
```

```
[SWAPTREE
 [LAMBDA
  [X]
  [COND
   X
   [COND
   .   [CAR X]
   .   X
   .   [COND [CDR X]
   .         [CONS NIL [CONS [SWAPTREE [CDR [CDR X]]] [SWAPTREE [CAR [CDR X]]]]]
   .         X]]
   NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]


PROFILE:  [/ S2 [A] , / E N R / E N R / E N R .]


TIME:  7.875 SECS.
```

```
[T 7 9]      [ 16.33 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]


MUST TRY INDUCTION.


(SPECIAL CASE REQUIRED)


INDUCT ON A.


THE THEOREM TO BE PROVED IS NOW:

[AND
 [EQUAL [FLATTEN [SWAPTREE NIL]] [REVERSE [FLATTEN NIL]]]
 [AND
   [EQUAL [FLATTEN [SWAPTREE [CONS A1 NIL]]] [REVERSE [FLATTEN [CONS A1 NIL]]]]
   [IMPLIES [AND [EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]]
                 [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]]
            [EQUAL [FLATTEN [SWAPTREE [CONS A1 [CONS A2 A]]]]
                   [REVERSE [FLATTEN [CONS A1 [CONS A2 A]]]]]]]]


WHICH IS EQUIVALENT TO:

[COND
     [EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]]
     [COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]
     .     [COND A1
     .           T
     .           [EQUAL [APPEND [FLATTEN [SWAPTREE A]] [FLATTEN [SWAPTREE A2]]]
     .                  [REVERSE [APPEND [FLATTEN A2] [FLATTEN A]]]]]
     .     T]
```

```
                                                                      [T 7 9]

      T]


FERTILIZE WITH [EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]].


THE THEOREM TO BE PROVED IS NOW:

[COND [COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]
            [COND A1
                  T
                  [EQUAL [APPEND [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A2]]]
                         [REVERSE [APPEND [FLATTEN A2] [FLATTEN A]]]]]
            T]
      T
      [*1]]


FERTILIZE WITH [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]].


THE THEOREM TO BE PROVED IS NOW:

[COND [COND [COND A1
                  T
                  [EQUAL [APPEND [REVERSE [FLATTEN A]] [REVERSE [FLATTEN A2]]]
                         [REVERSE [APPEND [FLATTEN A2] [FLATTEN A]]]]]
            T
            [*2]]
      T
      [*1]]


GENERALIZE COMMON SUBTERMS BY REPLACING [FLATTEN A] BY GENRL1 AND [FLATTEN A2] B
Y GENRL2.


THE GENERALIZED TERM IS:

[COND [COND [COND A1
```

```
                  T
                                                           [T 7 9]
                  [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
                         [REVERSE [APPEND GENRL2 GENRL1]]]]
            T
            [*2]]
      T
      [*1]]



MUST TRY INDUCTION.



INDUCT ON GENRL2.




THE THEOREM TO BE PROVED IS NOW:

[AND
 [COND [COND [COND A1
 .                 T
 .                 [EQUAL [APPEND [REVERSE GENRL1] [REVERSE NIL]]
 .                        [REVERSE [APPEND NIL GENRL1]]]]
 .           T
 .           [*2]]
 .     T
 .     [*1]]
 [IMPLIES
  [COND [COND [COND A1
  .                 T
  .                 [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
  .                        [REVERSE [APPEND GENRL2 GENRL1]]]]
  .           T
  .           [*2]]
  .     T
  .     [*1]]
  [COND
   [COND [COND A1
   .           T
   .           [EQUAL [APPEND [REVERSE GENRL1] [REVERSE [CONS GENRL21 GENRL2]]]
   .                  [REVERSE [APPEND [CONS GENRL21 GENRL2] GENRL1]]]]
   .     T
   .     [*2]]
   T
   [*1]]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND
 [COND [COND [COND A1 T [EQUAL [APPEND [REVERSE GENRL1] NIL] [REVERSE GENRL1]]]
 .            T
 .            [*2]]
 .      T
 .      [*1]]
 [COND
 .A1
 .T
 .[COND
 . [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
 . .      [REVERSE [APPEND GENRL2 GENRL1]]]
 . [COND
 . .[COND
 . . [EQUAL
 . . .   [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
 . . .   [APPEND [REVERSE [APPEND GENRL2 GENRL1]] [CONS GENRL21 NIL]]]
 . . T
 . . [*2]]
 . .T
 . .[*1]]
 . T]]
 NIL]
```

```
FERTILIZE WITH [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
                      [REVERSE [APPEND GENRL2 GENRL1]]].
```

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 [COND [COND [COND A1 T [EQUAL [APPEND [REVERSE GENRL1] NIL] [REVERSE GENRL1]]]
 .            T
 .            [*2]]
 .      T
 .      [*1]]
 [COND
 .A1
 .T
 .[COND
 . [COND
 . .[COND
 . . [EQUAL
 . . .  [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
 . . .  [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
 . . T
 . . [*2]]
```

```
. .T
                                                             [T 7 9]

. .[*1]]
. T
. [*3]]]
NIL]
```

(WORK ON FIRST CONJUNCT ONLY)

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE GENRL1] BY GENRL3.

THE GENERALIZED TERM IS:

```
[COND [COND [COND A1 T [EQUAL [APPEND GENRL3 NIL] GENRL3]] T [*2]] T [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL3.

THE THEOREM TO BE PROVED IS NOW:

```
[COND
 [AND
 .[COND [COND [COND A1 T [EQUAL [APPEND NIL NIL] NIL]] T [*2]] T [*1]]
 .[IMPLIES
 . [COND [COND [COND A1 T [EQUAL [APPEND GENRL3 NIL] GENRL3]] T [*2]] T [*1]]
 . [COND
 .  [COND
 .  .    [COND A1
 .  .    .     T
 .  .    .     [EQUAL [APPEND [CONS GENRL31 GENRL3] NIL] [CONS GENRL31 GENRL3]]]
 .  .    T
 .  .    [*2]]
```

```
.  T
                                                                      [T 7 9]

.  [*1]]]]
[COND
.A1
.T
.LCOND
. [COND
. .[COND
. . [EQUAL
. . .  [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
. . .  [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
. . T
. . [*2]]
. .T
. .[*1]]
. T
. [*3]]]]
NIL]
```

WHICH IS EQUIVALENT TO:

```
[COND
 A1
 T
 [COND
  [COND
  .[COND
  . [EQUAL
  . .  [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
  . .  [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
  . T
  . [*2]]
  .T
  .[*1]]
  T
  [*3]]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE GENRL2] BY GENRL4 AND [REVERSE GENRL1] BY GENRL5.

THE GENERALIZED TERM IS:

```
[COND
   A1
   T
   [COND [COND [COND [EQUAL [APPEND GENRL5 [APPEND GENRL4 [CONS GENRL21 NIL]]]
```

```
                                                             [APPEND [APPEND GENRL5 GENRL4] [CONS GENRL21 NIL]]]
                                               T
                                                                                                                       [T 7 9]

                                               [*2]]
                                   T
                                   [*1]]
                       T
                       [*3]]]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL5.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND
 .       A1
 .       T
 .       [COND [COND [COND [EQUAL [APPEND NIL [APPEND GENRL4 [CONS GENRL21 NIL]]]
 .                                      [APPEND [APPEND NIL GENRL4] [CONS GENRL21 NIL]]]
 .                                  T
 .                                  [*2]]
 .                       T
 .                       [*1]]
 .             T
 .             [*3]]]]
 [IMPLIES
  [COND
  .A1
  .T
  .[COND [COND [COND [EQUAL [APPEND GENRL5 [APPEND GENRL4 [CONS GENRL21 NIL]]]
  .                                [APPEND [APPEND GENRL5 GENRL4] [CONS GENRL21 NIL]]]
  .                            T
  .                            [*2]]
  .                 T
  .                 [*1]]
  .       T
  .       [*3]]]]
  [COND
   A1
   T
   [COND
    [COND
    .[COND
```

```
    . [EQUAL [APPEND [CONS GENRL51 GENRL5] [APPEND GENRL4 [CONS GENRL21 NIL]]]
    . .      [APPEND [APPEND [CONS GENRL51 GENRL5] GENRL4] [CONS GENRL21 NIL]]]
                                                             [T 7 9]

    . T
    . [*2]]
    .T
    .[*1]]
    T
    [*3]]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[SWAPTREE
 [LAMBDA
  [X]
  [COND
   X
   [COND
   .  [CAR X]
   .  X
   .  [COND [CDR X]
   .        [CONS NIL [CONS [SWAPTREE [CDR [CDR X]]] [SWAPTREE [CAR [CDR X]]]]]
   .        X]]
   NIL]]]

[FLATTEN
 [LAMBDA
     [X]
     [COND X
           [COND [CAR X]
                 [CONS X NIL]
                 [COND [CDR X]
                       [APPEND [FLATTEN [CAR [CDR X]]] [FLATTEN [CDR [CDR X]]]]
                       [CONS X NIL]]]
           [CONS NIL NIL]]]]

[REVERSE
       [LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

FERTILIZERS:

```
*1 = [COND [EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]] NIL T]

*2 = [COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]] NIL T]

*3 = [COND [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
                  [REVERSE [APPEND GENRL2 GENRL1]]]
           NIL
```

```
[T 7 10]      [ 16.36 18 JULY 1973]
```

THEOREM TO BE PROVED:

```
[EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]
```

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

```
[AND [EQUAL [LENGTH [FLATTEN NIL]] [TIPCOUNT NIL]]
     [AND [EQUAL [LENGTH [FLATTEN [CONS A1 NIL]]] [TIPCOUNT [CONS A1 NIL]]]
          [IMPLIES [AND [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]]
                        [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]]
                   [EQUAL [LENGTH [FLATTEN [CONS A1 [CONS A2 A]]]]
                          [TIPCOUNT [CONS A1 [CONS A2 A]]]]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]]
      [COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]
            [COND A1
                  T
                  [EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]]
                         [PLUS [TIPCOUNT A2] [TIPCOUNT A]]]]
            T]
      T]
```

FERTILIZE WITH [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]
            [COND A1
                  T
                  [EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]]
                         [PLUS [LENGTH [FLATTEN A2]] [TIPCOUNT A]]]]
            T]
      T
      [*1]]
```

FERTILIZE WITH [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [COND A1
                  T
                  [EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]]
                         [PLUS [LENGTH [FLATTEN A2]] [LENGTH [FLATTEN A]]]]]]
            T
            [*2]]
      T
      [*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [FLATTEN A] BY GENRL1 AND [FLATTEN A2] B
Y GENRL2.

THE GENERALIZED TERM IS:

```
[COND [COND [COND A1
                  T
                  [EQUAL [LENGTH [APPEND GENRL2 GENRL1]]
                         [PLUS [LENGTH GENRL2] [LENGTH GENRL1]]]]
```

```
     T
                                                                      [T 7 10]

     [*2]]
  T
  [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL2.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND
 .[COND
 . [COND
 . .  A1
 . .  T
 . .  [EQUAL [LENGTH [APPEND NIL GENRL1]] [PLUS [LENGTH NIL] [LENGTH GENRL1]]]]
 . T
 . [*2]]
 .T
 .[*1]]
 [IMPLIES
  [COND [COND [COND A1
  .                 T
  .                 [EQUAL [LENGTH [APPEND GENRL2 GENRL1]]
  .                        [PLUS [LENGTH GENRL2] [LENGTH GENRL1]]]]
  .           T
  .           [*2]]
  .     T
  .     [*1]]
  [COND
     [COND [COND A1
     .           T
     .           [EQUAL [LENGTH [APPEND [CONS GENRL21 GENRL2] GENRL1]]
     .                  [PLUS [LENGTH [CONS GENRL21 GENRL2]] [LENGTH GENRL1]]]]
     .     T
     .     [*2]]
     T
     [*1]]]]
```

[T 7 10]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[FLATTEN
 [LAMBDA
     [X]
     [COND X
           [COND [CAR X]
                 [CONS X NIL]
                 [COND [CDR X]
                       [APPEND [FLATTEN [CAR [CDR X]]] [FLATTEN [CDR [CDR X]]]]
                       [CONS X NIL]]]
           [CONS NIL NIL]]]]

[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

[TIPCOUNT
 [LAMBDA
  [X]
  [COND
   X
   [COND
   . [CAR X]
   . 1
   . [COND [CDR X] [PLUS [TIPCOUNT [CAR [CDR X]]] [TIPCOUNT [CDR [CDR X]]]] 1]]
   1]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

[PLUS [LAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
```

FERTILIZERS:

*1 = [COND [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]] NIL T]

*2 = [COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]] NIL T]

GENERALIZATIONS:

```
GENRL2 = [FLATTEN A2]
                                                        [T 7 10]

GENRL1 = [FLATTEN A]


PROFILE:  [/ S2 [A] , / E N R / E N R / E N R F , / F , / G [GENRL2] , / E N R /
 E N R .]


TIME:  25.0 SECS.
```

```
[T 8 2]     [ 16,37 18 JULY 1973]


THEOREM TO BE PROVED:

[EQUAL [LINEAR [BINARYOF N]] N]


MUST TRY INDUCTION.


INDUCT ON N.


THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [LINEAR [BINARYOF NIL]] NIL]
     [IMPLIES [EQUAL [LINEAR [BINARYOF N]] N]
              [EQUAL [LINEAR [BINARYOF [CONS NIL N]]] [CONS NIL N]]]]


WHICH IS EQUIVALENT TO:

[COND [EQUAL [LINEAR [BINARYOF N]] N]
      [EQUAL [LINEAR [BINADD [CONS 1 NIL] [BINARYOF N]]] [CONS NIL N]]
      T]


FERTILIZE WITH [EQUAL [LINEAR [BINARYOF N]] N].


THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] [BINARYOF N]]]
             [CONS NIL [LINEAR [BINARYOF N]]]]
      T
```

```
      [*1]]
```

GENERALIZE COMMON SURTERMS BY REPLACING [BINARYOF N] BY GENRL1.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]
      T
      [*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
[AND
 [COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] NIL]] [CONS NIL [LINEAR NIL]]]
 .      T
 .      [*1]]
 [IMPLIES
  [COND
  .     [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]
  .     T
  .     [*1]]
  [COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] [CONS GENRL11 GENRL1]]]
               [CONS NIL [LINEAR [CONS GENRL11 GENRL1]]]]
         T
         [*1]]]]
```

WHICH IS EQUIVALENT TO:

```
[COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]
      [COND [COND GENRL11
                  [EQUAL [DOUBLE [LINEAR [BINADD [CONS 1 NIL] GENRL1]]]
                         [CONS NIL [CONS NIL [DOUBLE [LINEAR GENRL1]]]]]
                  T]
            T
            [*1]]
      T]
```

FERTILIZE WITH [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]]
[CONS NIL [LINEAR GENRL1]]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [COND [COND GENRL11
                  [EQUAL [DOUBLE [CONS NIL [LINEAR GENRL1]]]
                         [CONS NIL [CONS NIL [DOUBLE [LINEAR GENRL1]]]]]
                  T]
            T
            [*1]]
      T
      [*2]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

```
[BINARYOF [LAMBDA [X] [COND X [BINADD [CONS 1 NIL] [BINARYOF [CDR X]]] NIL]]]

[LINEAR [LAMBDA [X]
               [COND X
                     [COND [CAR X]
                           [CONS NIL [DOUBLE [LINEAR [CDR X]]]]
                           [DOUBLE [LINEAR [CDR X]]]]
                     NIL]]]

[BINADD
 [LAMBDA
  [X Y]
```

```
 [COND
     X
     [COND Y
     .     [COND [CAR X]
     .           [COND [CAR Y]
     .                 [CONS 0 [BINADD [CONS 1 NIL] [BINADD [CDR X] [CDR Y]]]]
     .                 [CONS 1 [BINADD [CDR X] [CDR Y]]]]
     .           [CONS [CAR Y] [BINADD [CDR X] [CDR Y]]]]
     .     X]
     Y]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[DOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]



FERTILIZERS:

*1 = [COND [EQUAL [LINEAR [BINARYOF N]] N] NIL T]

*2 = [COND
      [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]
      NIL
      T]



GENERALIZATIONS:

GENRL1 = [BINARYOF N]



PROFILE:  [/ [N] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R / E N R F
, / E N R .]



TIME:  19.81 SECS.
```

16.38HRS. 18 JULY 1973. ** POP **

[]