

Derivative Securities ¹

- **Forward:** A customized, over-the-counter contract between two parties to buy or sell an asset at a specified price on a future date.
- **Future:** Similar to forwards, but they are standardized contracts traded on an exchange. Both parties are obligated to fulfill the contract at a predetermined price and date.
- **Option:** A contract that gives the buyer the right, but not the obligation, to buy or sell an underlying asset at a specific price on or before a certain date.
 - **Put:** a contract that allows one to sell a security at a certain price
 - **Call:** a contract that allows one to buy a security at a certain price
- **Swap:** An agreement between two parties to exchange cash flows for a set period of time. A common example is an interest rate swap, where one party exchanges a fixed interest rate for a floating one.

¹Source: Google AI Overview

Example: Hedge

I own some stock in my retirement account. I am happy that the stock market has gone way up, but I am afraid that it is overvalued and might crash.

I can *hedge* against this risk by buying a **put** that allows me to sell the stock average at a price a bit below today's price. (The *strike price* of the put will determine the market price of the put.)

- If the market stays high, I will not exercise the put. I lose the cost of the put.
- If the market crashes, I will exercise the put. I lose the cost of the put plus the amount that the put was below the market when I bought it. However, I avoid a huge loss.

Example: Covered Call

I own some shares of XYZ corporation, currently at \$100. I sell a **call** of this amount of XYZ at \$105 for \$3 per share. This contract is *covered* because I can fulfill the contract by selling my shares for the agreed \$105. (If it is not covered, it is *naked*.)

- If XYZ rises over \$105, the buyer of the call will exercise it, and I have effectively sold my XYZ for \$108, a nice profit. (If XYZ is over \$108, I lose the extra.)
- If XYZ stays below \$105, the buyer of the call will not exercise it. However, I get to keep my stock and the \$3 per share from selling the option.

Conservation Laws

- Common sense:

There is no such thing as a free lunch.

You can't have your cake and eat it too.

- Physics:

Mass-energy is neither created nor destroyed.

For every action, there is an equal and opposite reaction.

- Finance:

Money is neither created nor destroyed.

Every transaction requires an equal and opposite transaction.

Double-Entry Bookkeeping

Double-entry bookkeeping dates back some 1000 years, introduced in Korea and by Jewish bankers in Egypt, then in Italian banks during the Renaissance.

The basic idea is simple: money moves from one account to another; an addition to one account must correspond to a subtraction from another account.

For example, suppose you withdraw \$100 in cash from your bank account at an ATM:

	Cash in Pocket	Bank Account
Get cash	+\$100	-\$100

Double-entry bookkeeping provides an *audit trail* that allows the flow of money to be followed.

Representing Financial Contracts

Modern financial contracts are more complex than an immediate subtraction from one account and addition to another account.

Wimpy:

I'd gladly pay you Tuesday for a hamburger today.

Can we represent this formally?

```
(and (one hamburger)
      (give (zcb tuesday 5 USD)) )
```

zcb is a zero-coupon bond, i.e. a promise to pay \$5 at a future time, Tuesday.

This kind of representation is recursive, allowing complex contracts to be represented with a small set of *combinators*.

Finance Combinators²

<code>(zero)</code>	a contract that has no rights and no obligations
<code>(one k)</code>	one unit of currency k
<code>c</code>	you immediately acquire contract c
<code>(give c)</code>	to give a contract c to another party; like negation
<code>(at t c)</code>	if you acquire contract c before time t , it becomes effective at time t
<code>(truncate t c)</code>	contract c ceases to exist after time t
<code>(and c_1 c_2)</code>	both contracts c_1 and c_2
<code>(or c_1 c_2)</code>	your choice of contracts c_1 and c_2
<code>(cond b c_1 c_2)</code>	you acquire contract c_1 if the observable b is true, else you acquire c_2
<code>(scale o c)</code>	multiply contract c by observable o
<code>(when b c)</code>	you must acquire contract c when b becomes true (but worthless if b can never be true)
<code>(anytime b c)</code>	you may acquire contract c any time b becomes true
<code>(until b c)</code>	is like contract c but must be abandoned when b becomes true

²S. Peyton Jones and J.M. Eber, “How to write a financial contract”, in The Fun of Programming, ed Gibbons and de Moor, Palgrave Macmillan 2003

Example: Covered Call

Example: Sell for \$3 per share a Covered Call of XYZ Corporation at time t at a strike price of \$105 per share.

```
(and (scale (* 3 n) USD)
      (at t
        (cond (> (value XYZ) (scale 105 USD))
              (and (scale (* 105 n) USD)
                    (give (scale n XYZ)) )
              (zero) ) ) )
```