Overview of Resolution

Resolution is a method of proof by contradiction:

- Let $F$ be in Conjunctive Normal Form (CNF), $F = F_1 \land F_2 \land \ldots \land F_n$ where each $F_i$ is a disjunction (or) of literals (predicate or its negation).
- If we want to prove $G$, $F \rightarrow G$, then $F \rightarrow G$ is True, so $\neg(F \rightarrow G) = \neg(\neg F \lor G) = (F \land \neg G)$ is False.
- We add $\neg G$ to our list of clauses and try to prove that the result is False or box, $\Box$.
- If there is a new clause $N$ such that $F \rightarrow N$, then $F \land N = F$. We can add $N$ to our set of clauses without changing the result.
- The resolution step on clauses $F_i$ and $F_j$ produces a new clause $N$ such that $F_i \land F_j \rightarrow N$.
- We add new clauses $N$ to our set; if $N = \text{False}$ or box, $\Box$, then we have proved that $(F \land \neg G)$ is False.
Resolution Step

Suppose that we have two clauses, $F_i$ and $F_j$ where $F_i = A \lor L$ and $F_j = B \lor \neg L$.

The resolution step produces a new clause $N$ by removing the complementary literals $L$ and $\neg L$ and combining everything else from the two source clauses: $N = A \lor B$. ($A$ and $B$ could be composed of multiple literals.)

The result is a logical consequence of $F_i$ and $F_j$, i.e. $F_i \land F_j \rightarrow N$.

Proof:

We want to show that $(A \lor L) \land (B \lor \neg L) \rightarrow (A \lor B)$.

- Case 1: $L = \text{True}$. If $(B \lor \neg L)$ is true, then $B$ must be true, so $(A \lor B)$ is true.

- Case 2: $L = \text{False}$. If $(A \lor L)$ is true, then $A$ must be true, so $(A \lor B)$ is true.