CS344M Autonomous Multiagent Systems

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Good Afternoon, Colleagues

Are there any questions?

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- How can we apply game theory to RoboCup?
- Examples of game theory that aren't modeled as a matrix?
- What about irrational agents?
- Pure vs mixed strategy?

Logistics

• Progress reports due in 2 weeks

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Game Theory Premises

Simultaneous actions: (mutual exclusivity?)

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- Simultaneous actions: (mutual exclusivity?)
- No communication
- Outcome depends on combination of actions
- Utility (payoff) encapsulates everything about preferences over outcomes

Solution Concepts

- Dominant strategy
- Nash equilibrium
- Pareto optimality
- Maximum social welfare
- Maximin strategy

Prisoner's Dilemma

| | | Column | |
|-------|------|--------|------|
| | | C(1) | D(2) |
| Row | C(1) | 3,3 | 0,5 |
| 100 W | D(2) | 5,0 | 1,1 |

Chicken

Column
C(1)
D(2)

C(1)
3,3
1,5

Row
D(2)
5,1
0,0

My wife and I agree to meet at a concert

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- Unfortunately, there are 2: Bach and Stravinsky

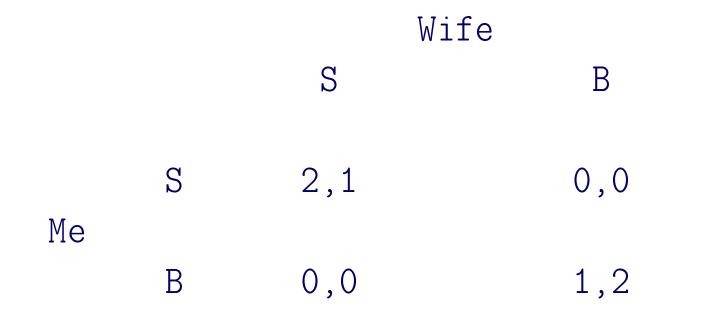
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- Propose a payoff matrix



Does every game have a pure strategy Nash equilibrium?

Matching Pennies

- We each put a penny down covered
- If they match, I win, if they don't, you win

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Nash equilibrium?

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 - Nobel prize and academy award!

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- Not known if complexity of finding one is NP-complete or in P

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- Is a Pareto optimal outcome necessarily the result of Nash equilibrium strategies?
- Is the maximum social welfare outcome necessarily Pareto optimal?
- If both players play maximin, is it necessarily a Nash equilibrium?

| | | | Player | 2 | _ |
|----------|----------|--------|--------|--------|---|
| | | Action | 1 | Action | 2 |
| Player 1 | Action 1 | 4,8 | | 2,0 | |
| | Action 2 | 6,2 | | 0,8 | |

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What if player 2 picks action 1 3/4 of the time?

Player 2
Action 1 4,8 2,0
Player 1 Action 2 6,2 0,8

• What if player 2 picks action 1 3/4 of the time? 1 = 3.5, 2 = 4.5

| | | Action | Player 1 | 2 Action | 2 |
|----------|----------|--------|-------------|-------------|---|
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| Pla | yer 1 must be indifferent between actions 1 d | and 2 |
|-----------------------|--|-------------------|
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| | | |
| | | |
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 Player 1 must be indifferent between actions 1 and 24q+2-2q = 6q

- Player 1 must be indifferent between actions 1 and 24q+2-2q = 6q
- Player 2 must be indifferent between actions 1 and 2

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- Player 2 must be indifferent between actions 1 and 28p+2-2p = 8-8p

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Rock/Paper/Scissors

• Nash equilibrium?

Rock/Paper/Scissors

- Nash equilibrium?
- Why is anything else not an equilibrium?

Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky

| | | Wif | е |
|----|---|-----|-----|
| | | S | В |
| Me | S | 2,1 | 0,0 |
| | В | 0,0 | 1,2 |

Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky

Want only S,S or B,B - 50% each