Control Flow Analysis
and Loop Optimization

Program Loops

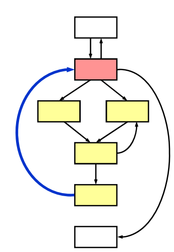
- Loop = a computation repeatedly executed until a terminating condition is reached
- High-level loop constructs:
  - While loop: while(E) S
  - Do-while loop: do S while(E)
  - For loop: for(i=1; i<=u; i+=c) S
- Why are loops important:
  - Most of the execution time is spent in loops
  - Typically: 90/10 rule, 10% code is a loop
- Therefore, loops are important targets of optimizations

Detecting Loops

- Need to identify loops in the program
  - Easy to detect loops in high-level constructs
  - Harder to detect loops in low-level code or in general control-flow graphs
- Examples where loop detection is difficult:
  - Languages with unstructured “goto” constructs: structure of high-level loop constructs may be destroyed
  - Optimizing Java bytecodes (without high-level source program): only low-level code is available

Control-Flow Analysis

- Goal: identify loops in the control flow graph
- A loop in the CFG:
  - Is a set of CFG nodes (basic blocks)
  - Has a loop header such that control to all nodes in the loop always goes through the header
  - Has a back edge from one of its nodes to the header



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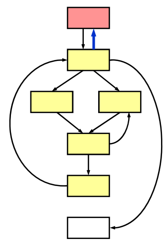
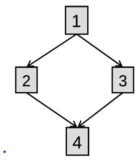


Dominators

- Use concept of dominators in CFG to identify loops
- Node d dominates node n if all paths from the entry node to n go through d



Every node dominates itself
1 dominates 1, 2, 3, 4
2 doesn't dominate 4
3 doesn't dominate 4

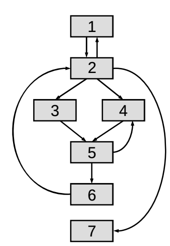
- Intuition:
  - Header of a loop dominates all nodes in loop body
  - Back edges = edges whose heads dominate their tails
  - Loop identification = back edge identification

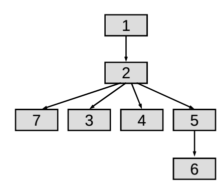
Immediate Dominators

- Properties:
  1. CFG entry node n_0 dominates all CFG nodes
  2. If d1 and d2 dominate n, then either
  - d1 dominates d2, or
  - d2 dominates d1
- d strictly dominates n if d dominates n and d≠n
- The immediate dominator idom(n) of a node n is the unique last strict dominator on any path from n_0 to n

Dominator Tree

- Build a dominator tree as follows:
  - Root is CFG entry node n_0
  - m is child of node n iff n=idom(m)
- Example:



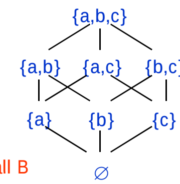


Computing Dominators

- Formulate problem as a system of constraints:
  - Define dom(n) = set of nodes that dominate n
  - $dom(n_0) = \{n_0\}$
  - $dom(n) = \cap\{ dom(m) \mid m \in pred(n) \} \cup \{n\}$

    i.e, the dominators of n are the dominators of all of n's predecessors and n itself

Dominators as a Dataflow Problem



- Let N = set of all basic blocks
- Lattice: $(2^N, \subseteq)$; has finite height
- Meet is set intersection, top element is N
- Is a forward dataflow analysis
- Dataflow equations:

  $out[B] = F_B(in[B])$, for all B

  $in[B] = \cap\{out[B'] \mid B' \in pred(B)\}$, for all B

  $in[B_s] = \{\}$
- Transfer functions: $F_B(X) = X \cup \{B\}$
  - are monotonic and distributive
- Iterative solving of dataflow equation:
  - terminates
  - computes MOP solution

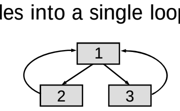
Natural Loops

- Back edge: edge $n \rightarrow h$ such that h dominates n
- Natural loop of a back edge $n \rightarrow h$:
  - h is loop header
  - Set of loop nodes is set of all nodes that can reach n without going through h
- Algorithm to identify natural loops in CFG:
  - Compute dominator relation
  - Identify back edges
  - Compute the loop for each back edge

```
for each node h in dominator tree
    for each node n for which there exists a back edge n→h
        define the loop with
            header h
            back edge n→h
            body consisting of all nodes reachable from n by a
            depth first search backwards from n that stops at h
```


Disjoint and Nested Loops

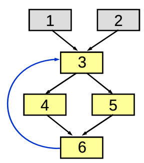
- Property: for any two natural loops in the flow graph, one of the following is true:
  1. They are disjoint
  2. They are nested
  3. They have the same header
- Eliminate alternative 3: if two loops have the same header and none is nested in the other, combine all nodes into a single loop

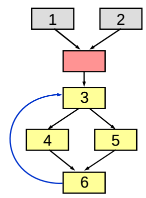


Two loops: {1,2} and {1,3}
Combine into one loop: {1,2,3}

Loop Preheader

- Several optimizations add code before header
- Insert a new basic block (called preheader) in the CFG to hold this code





Loop optimizations

- Now we know the loops
- Next: optimize these loops
  - Loop invariant code motion
  - Strength reduction of induction variables
  - Induction variable elimination

Loop Invariant Code Motion

- Idea: if a computation produces same result in all loop iterations, move it out of the loop
- Example:

```
for (i=0; i<10; i++)
    buf[i] = 10*i + x*x;
```

- Expression x*x produces the same result in each iteration; move it out of the loop:

```
t = x*x;
for (i=0; i<10; i++)
    buf[i] = 10*i + t;
```


Loop Invariant Computation

- An instruction a = b OP c is loop-invariant if each operand is:
  - Constant, or
  - Has all definitions outside the loop, or
  - Has exactly one definition, and that is a loop-invariant computation
- Reaching definitions analysis computes all the definitions of x and y that may reach t = x OP y

Algorithm

```
INV = ∅
repeat
    for each instruction I in loop such that I ∉ INV
      if operands  are constants, or operands
         have definitions outside the loop, or
         operands have exactly one definition d ∈ INV
      then INV = INV U {I}
until no changes in INV
```


Code Motion

- Next: move loop-invariant code out of the loop
- Suppose a = b OP c is loop-invariant
- We want to hoist it out of the loop

Valid Code Motion

- Code motion of a definition d: a = b OP c to pre-header is valid if:

1. Definition d dominates all loop exits where a is live
   - Use dominator tree to check whether each loop exit is dominated by d
2. There is no other definition of a in loop
   - Scan all body for any other definitions of a
3. All uses of a in loop can only be reached from definition d
   - Consult reaching definitions at each use of a for any definitions of a other than d

Valid Code Motion

- Invalid example 1: a = x*x; does not dominate break to use of a

```
a = 0;
for (i=0; i<10; i++)
  if ( f(i) ) a = x*x; else break;
b = a;
```

- Invalid example 2: there is another definition of a in loop

```
for (i=0; i<10; i++)
  if ( f(i) ) a = x*x;
  else a = 0;
```

- Invalid example 3: use of a in loop can be reached from a=0;

```
a = 0;
for (i=0; i<10; i++)
  if ( f(i) ) a = x*x;
  else buf[i] = a;
```

Other Issues

- Preserve dependencies between loop-invariant instructions when hoisting code out of the loop

```
for (i=0; i<N; i++) {          x = y+z;
   x = y+z;                    t = x*x;
   a[i] = 10*i + x*x;          for(i=0; i<N; i++)
}                                 a[i] = 10*i + t;
```

- Nested loops: apply loop-invariant code motion algorithm multiple times

```
                                   t1 = x*x;
for (i=0; i<N; i++)                for (i=0; i<N; i++) {
  for (j=0; j<M; j++)                 t2 = t1+ 10*i;
    a[i][j] = x*x + 10*i + 100*j;     for (j=0; j<M; j++)
                                         a[i][j] = t2 + 100*j; }
```


Induction Variables

- An induction variable is a variable in a loop, whose value is a function of the loop iteration number $v = f(i)$
- In compilers, this a linear function:

  $f(i) = c*i + d$

- Observation: linear combinations of linear functions are linear functions
  - Consequence: linear combinations of induction variables are induction variables

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Families of Induction Variables

- Basic induction variable: a variable whose only definition in the loop body is of the form

  i = i + c

  where c is a loop-invariant value
- Derived induction variables: Each basic induction variable i defines a family of induction variables Family(i)
  - $i \in$ Family(i)
  - $k \in$ Family(i) if there is only one definition of k in the loop body , and it has the form k = c*j or k=j+c, where

    (a) $j \in$ Family(i)

    (b) c is loop invariant

    (c) The only definition of j that reaches the definition of k is in the loop

    (d) There is no definition of i between the definitions of j and k

Representation

- Representation of induction variables in family i by triples:
  - Denote basic induction variable i by <i, 1, 0>
  - Denote induction variable k=i*a+b by triple <i, a, b>

Finding Induction Variables

Scan loop body to find all basic induction variables

do

Scan loop to find all variables k with one assignment of form k = j*b, where j is an induction variable <i,c,d>, and make k an induction variable with triple <i,c*b,d>

Scan loop to find all variables k with one assignment of form k = j±b where j is an induction variable with triple <i,c,d>, and make k an induction variable with triple <i,c,b±d>

until no more induction variables found

Strength Reduction

- Basic idea: replace expensive operations (multiplications) with cheaper ones (additions) in definitions of induction variables

```
while (i<10) {
  j = ...;   // <i,3,1>
  a[j] = a[j] –2;
  i = i+2;
}
```

⇒

```
s = 3*i+1;
while (i<10) {
  j = s;
  a[j] = a[j] –2;
  i = i+2;
  s= s+6;
}
```

- Benefit: cheaper to compute s = s+6 than j = 3*i
  - s = s+6 requires an addition
  - j = 3*i requires a multiplication

General Algorithm

- Algorithm:

For each induction variable j with triple <i,a,b> whose definition involves multiplication:

1. create a new variable s
2. replace definition of j with j=s
3. immediately after i=i+c, insert s = s+a*c
   (here a*c is constant)
4. insert s = a*i+b into preheader

- Correctness: transformation maintains invariant s = a*i+b

Strength Reduction

- Gives opportunities for copy propagation, dead code elimination

```
s = 3*i+1;
while (i<10) {
  j = s;
  a[j] = a[j] –2;
  i = i+2;
  s= s+6;
}
```

⇒

```
s = 3*i+1;
while (i<10) {

  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```


Induction Variable Elimination

- Idea: eliminate each basic induction variable whose only uses are in loop test conditions and in their own definitions i = i+c
  - rewrite loop test to eliminate induction variable

```
s = 3*i+1;
while (i<10) {
  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```

- When are induction variables used only in loop tests?
  - Usually, after strength reduction
  - Use algorithm from strength reduction even if definitions of induction variables don't involve multiplications

Induction Variable Elimination

- Rewrite test condition using derived induction variables
- Remove definition of basic induction variables (if not used after the loop)

```
s = 3*i+1;
while (i<10) {
  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```

⇒

```
s = 3*i+1;
while (s<31) {
  a[s] = a[s] –2;
  s= s+6;
}
```


Induction Variable Elimination

For each basic induction variable i whose only uses are
- The test condition i < u
- The definition of i: i = i + c

  - Take a derived induction variable k in family i, with triple <i,c,d>
  - Replace test condition i < u with k < c*u+d
  - Remove definition i = i+c if i is not live on loop exit