1. Dependences (20 points)

(a) (6 points) What are flow-, anti- and output- dependences in programs? Explain each one using a couple of sentences and a small example.

(b) (4 points) What is aliasing and how does it complicate dependence analysis?

(c) (10 points) Draw the static dependence graph for the following program. Each edge in the dependence graph should be marked with its type (flow/anti/output) and the variable that gives rise to that dependence. All variables such as I and J that are not assigned to in these statements can be assumed to be assigned somewhere else in the program before these statements are executed. You may assume that the variables in the program (A,B,C,D,E,I,J,K,T) are not aliased.

\[
\begin{align*}
S1 & : A = B+C; \\
S2 & : D = A*E; \\
S3 & : T = T+5; \\
S4 & : D = T; \\
S5 & : A = D-7;
\end{align*}
\]

Answers:

(a) 2 points for each kind of dependence.

(b) (2 points) Aliasing means two or more variable names may refer to the same memory location.

(2 points) Aliasing complicates dependence analysis because there may be dependences between variables with different names if they happen to refer to the same location. Array references are the classical example.

(c) Not shown
2. Finite-differences (25 points)

Consider the ordinary differential equation
\[ \frac{d^2 y}{dx^2} + y = 1 \]
in the interval \(0 \leq x \leq \frac{\pi}{2}\), with boundary conditions \(y(0) = 1\) and \(y\left(\frac{\pi}{2}\right) = 0\). Divide the interval \(0 \leq x \leq \frac{\pi}{2}\) into four equally spaced intervals as shown in Figure 1. Write down the linear system for computing approximations to the values of \(y(0), y(h), \ldots, y\left(\frac{\pi}{2}\right)\) using centered-differences to discretize the differential equation. You do not have to solve this system.

Figure 1: Discretization of \(x\) for Problem 2
Answer:
h = \pi/8 (1 points)
y(0) = 1 (3 points)
y(4h) = 0 (3 points)
(y(2h)-2y(h)+y(0))/h^2 + y(h) = 1 (6 points)
(y(3h)-2y(2h)+y(h))/h^2 + y(2h) = 1 (6 points)
(y(4h)-2y(3h)+y(2h))/h^2 + y(3h) = 1 (6 points)

You have 3 equations in 3 unknowns \((y(h), y(2h), y(3h))\). Solve to find these values.
3. Short questions (35 points) Answer the following questions using 3-4 sentences for each one.

(a) (4 points) Name one iterative method for solving linear systems and describe briefly how it works.
   Ans: Jacobi, Gauss-Seidel, conjugate gradient, GMRES are all iterative methods for solving linear systems of equations. You start with some initial approximation to the solution. In each iteration, you compute another, hopefully better, approximation by using the current approximation and the residual. For example, consider using Jacobi for this system:
   \[ a_1x + b_1y = c_1 \]
   \[ a_2x + b_2y = c_2 \]
   You use the first equation to compute \( x(n+1) \) and the second to compute \( y(n+1) \) as follows:
   \[ x(n+1) = \frac{(c_1 - b_1y(n))}{a_1} \]
   \[ y(n+1) = \frac{(c_2 - a_2x(n))}{b_2} \]

(b) (6 points) What is meant by BLAS1, BLAS2 and BLAS3 kernels? Give examples of each kind.
   Ans: BLAS stands for Basic Linear Algebra Subroutines. 2 points for each one below. BLAS1 stands for routines that have a single loop, such as dot product and saxpy. BLAS2 stands for routines that have two loops, such as matrix-vector product and triangular solve. BLAS3 stands for routines that have three loops, such as matrix-matrix product.

(c) (6 points) What are the "3 C’s" in the context of the cache behavior of sequential programs? Explain each of them in a few sentences.
   Ans: Different kinds of cache misses: capacity, conflict, and cold.

(d) (6 points) What is meant by spatial and temporal locality in programs? Explain briefly why programs may exhibit spatial and temporal locality.
   Ans: Spatial locality: if a program accesses memory address M, it is likely to access addresses close to M such as M-1 or M+1 in the near future. Temporal locality: if a program accesses memory address M, it is likely to access address M (or the cache line containing M) in the near future.

(e) (4 points) Name a program transformation that can be used to improve spatial locality in programs, using a small example to illustrate your answer.
   Ans: Loop interchange or loop blocking. In class, we showed how this could enhance spatial locality for matrix-vector product.

(f) (3 points) What is register renaming? Explain briefly why it is used in modern processors.
The ISA deals with logical registers. In OOO processors, these are backed up by a larger number of physical registers. While executing a program, an OOO processor renames logical registers to physical registers to eliminate anti- and output dependences, while ensuring true dependences are respected. This may improve the amount of ILP.
4. *Cache organization (20 points)*

Consider a processor that has 32-bit addresses, where each address corresponds to one byte in memory. There is a 2-way set associative cache with a total of 4K entries and a line size of 16-bytes. Draw a picture of an address showing bit positions, and specify the bits that will be used for the tag, index, and byte offset to access data in this cache. Explain your answer briefly.

**Answer:**

Tag: leading 17 bits (5 points)  Index: next 11 bits (10 points)  Offset: least significant 4 bits (5 points)