Lecture 5

Partial Redundancy Elimination

I. Forms of redundancy
   - global common subexpression elimination
   - loop invariant code motion
   - partial redundancy

II. Lazy Code Motion Algorithm
   - Mathematical concept: a cut set
   - Basic technique (anticipation)
   - 3 more passes to refine algorithm

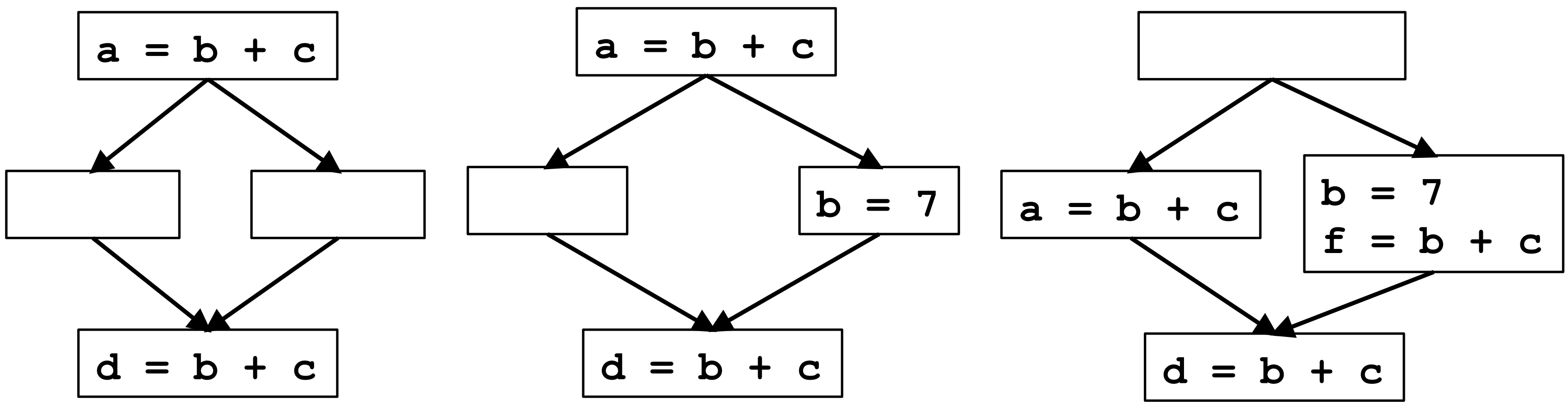
Reading: Chapter 9.5

Overview

- **Eliminates many forms of redundancy in one fell swoop**
- **Originally formulated as 1 bi-directional analysis**
- **Lazy code motion algorithm**
  - formulated as 4 separate uni-directional passes
    - backward, forward, forward, backward

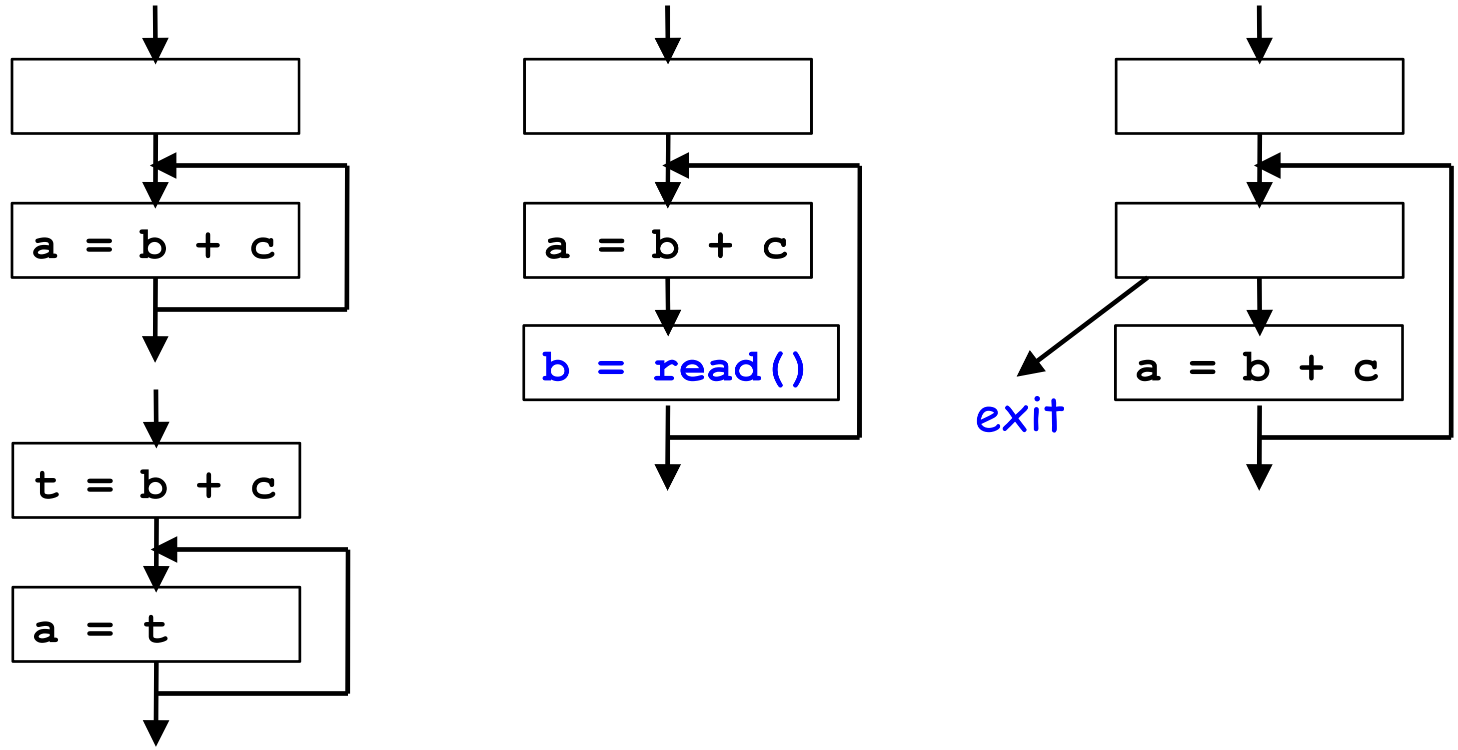
I. Common Subexpression Elimination

Build up intuition about redundancy elimination with examples of familiar concepts



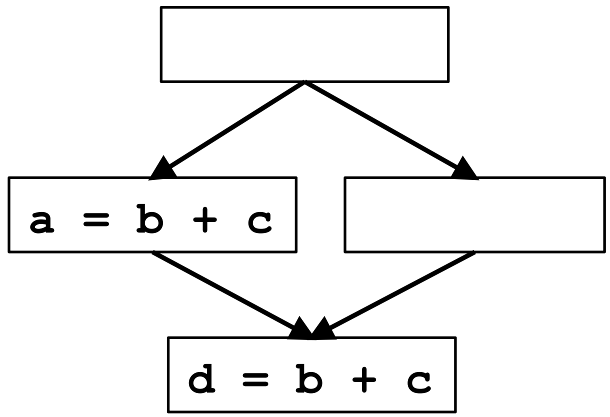
- **A common expression may have different values on different paths!**
- **On every path reaching p,**
  - expression b+c has been computed
  - b, c not overwritten after the expression

Loop Invariant Code Motion



- Given an expression (b+c) inside a loop,
  - does the value of b+c change inside the loop?
  - is the code executed at least once?

Partial Redundancy

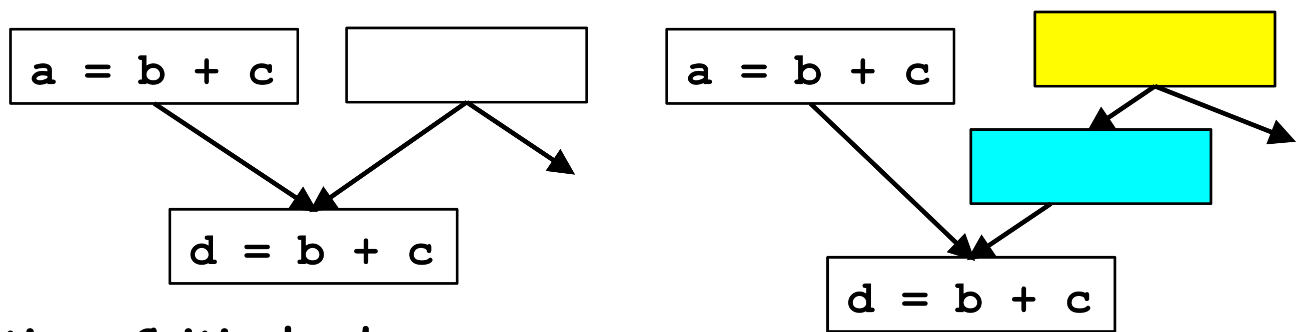


- Can we place calculations of b+c
  such that no path re-executes the same expression
- Partial Redundancy Elimination (PRE)
  - subsumes:
    - global common subexpression (full redundancy)
    - loop invariant code motion (partial redundancy for loops)

Unifying theory: More powerful, elegant → but less direct.

II. Preparing the Flow Graph

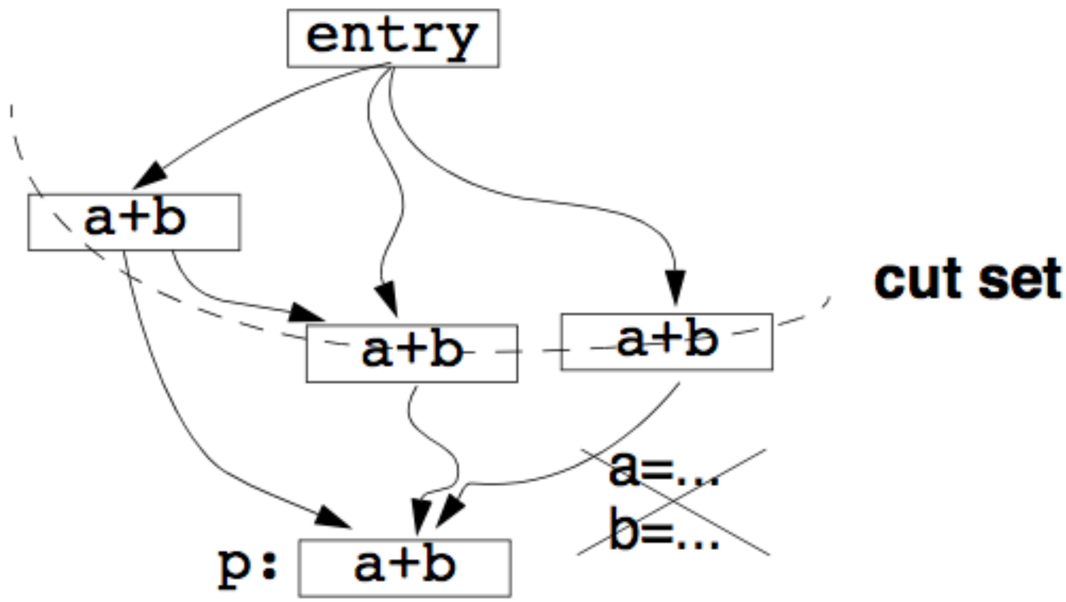
- **Key observation**
  - Can replace a bi-directional (!) data flow with several unidirectional data flows → much easier
  - Better result as well!



- **Definition: Critical edges**
  - source basic block has multiple successors
  - destination basic block has multiple predecessors
- **Modify the flow graph: (treat every statement as a basic block)**
  - To keep algorithm simple: restrict placement of instructions to the beginning of a basic block
  - Add a basic block for every edge that leads to a basic block with multiple predecessors (not just on critical edges)

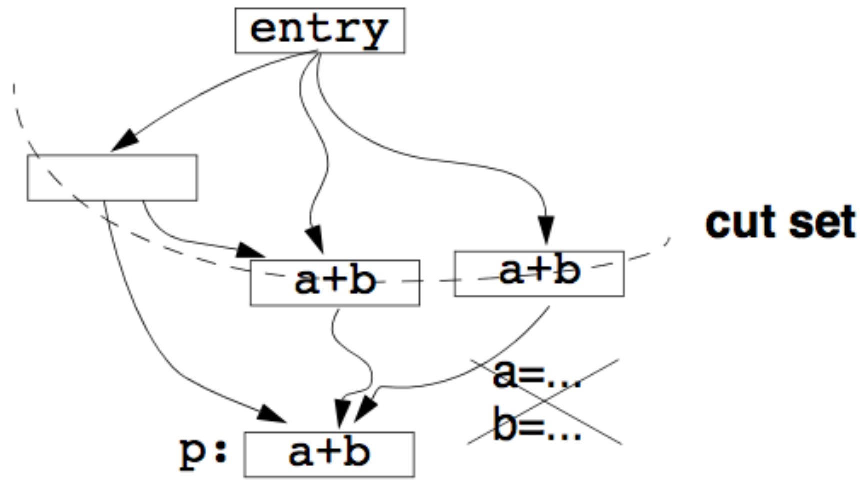
Full Redundancy: A Cut Set in a Graph

Key mathematical concept



- **Full redundancy at p: expression a+b redundant on all paths**
  - a cut set: nodes that separate entry from p
  - a cut set contains calculation of a+b
  - a, b, not redefined

Partial Redundancy: Completing a Cut Set



- **Partial redundancy at p: redundant on some but not all paths**
  - Add operations to create a cut set containing a+b
  - Note: Moving operations up can eliminate redundancy
- **Constraint on placement: no wasted operation**
  - a+b is "anticipated" at B if its value computed at B will be used along ALL subsequent paths
  - a, b not redefined, no branches that lead to exit with out use
- **Range where a+b is anticipated → Choice**

Pass 1: Anticipated Expressions

This pass does most of the heavy lifting in eliminating redundancy

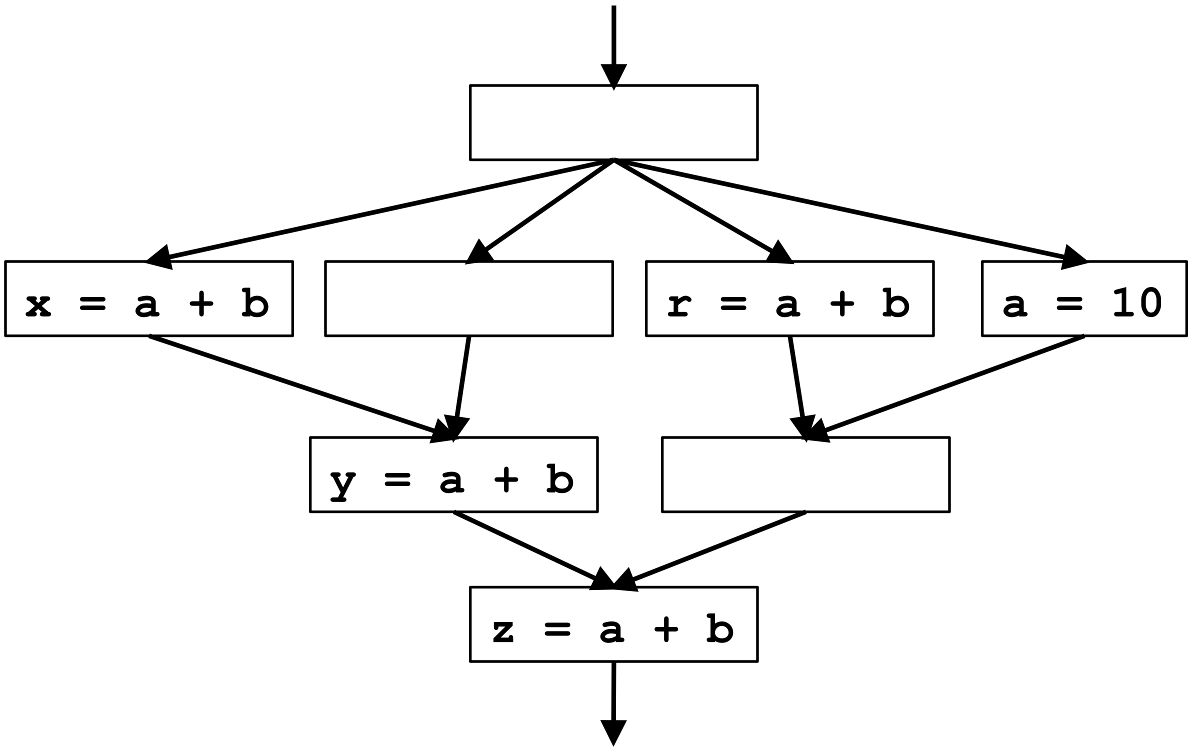
- **Backward pass: Anticipated expressions**
  Anticipated[b].in: Set of expressions anticipated at the entry of b
  - An expression is anticipated if its value computed at point p will be used along ALL subsequent paths

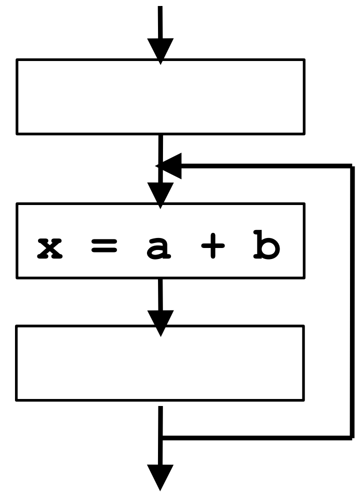
	Anticipated Expressions
Domain	Sets of expressions
Direction	backward
Transfer Function	$f_b(x) = EUse_b \cup (x - EKill_b)$ EUse: used exp, EKill: exp killed
$\wedge$	$\cap$
Boundary	in[exit] = $\emptyset$
Initialization	in[b] = {all expressions}

- **First approximation:**
  - place operations at the frontier of anticipation (boundary between not anticipated and anticipated)

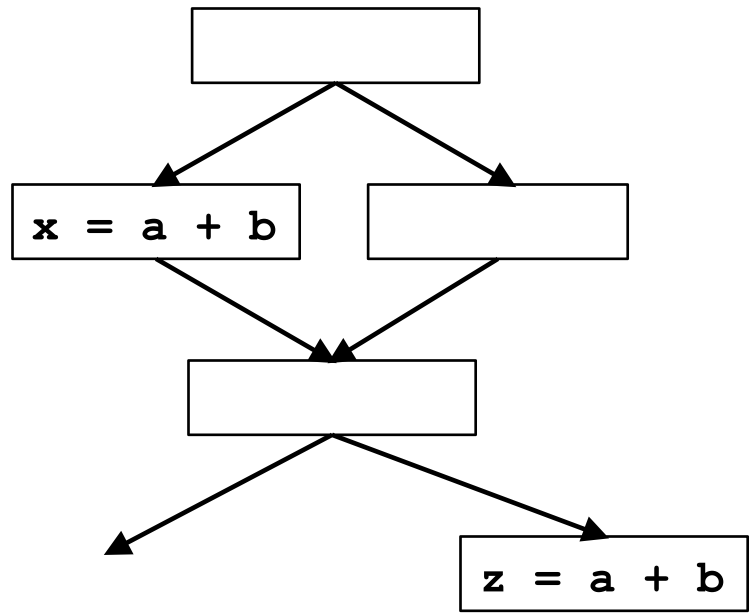
Examples (1)

See the algorithm in action





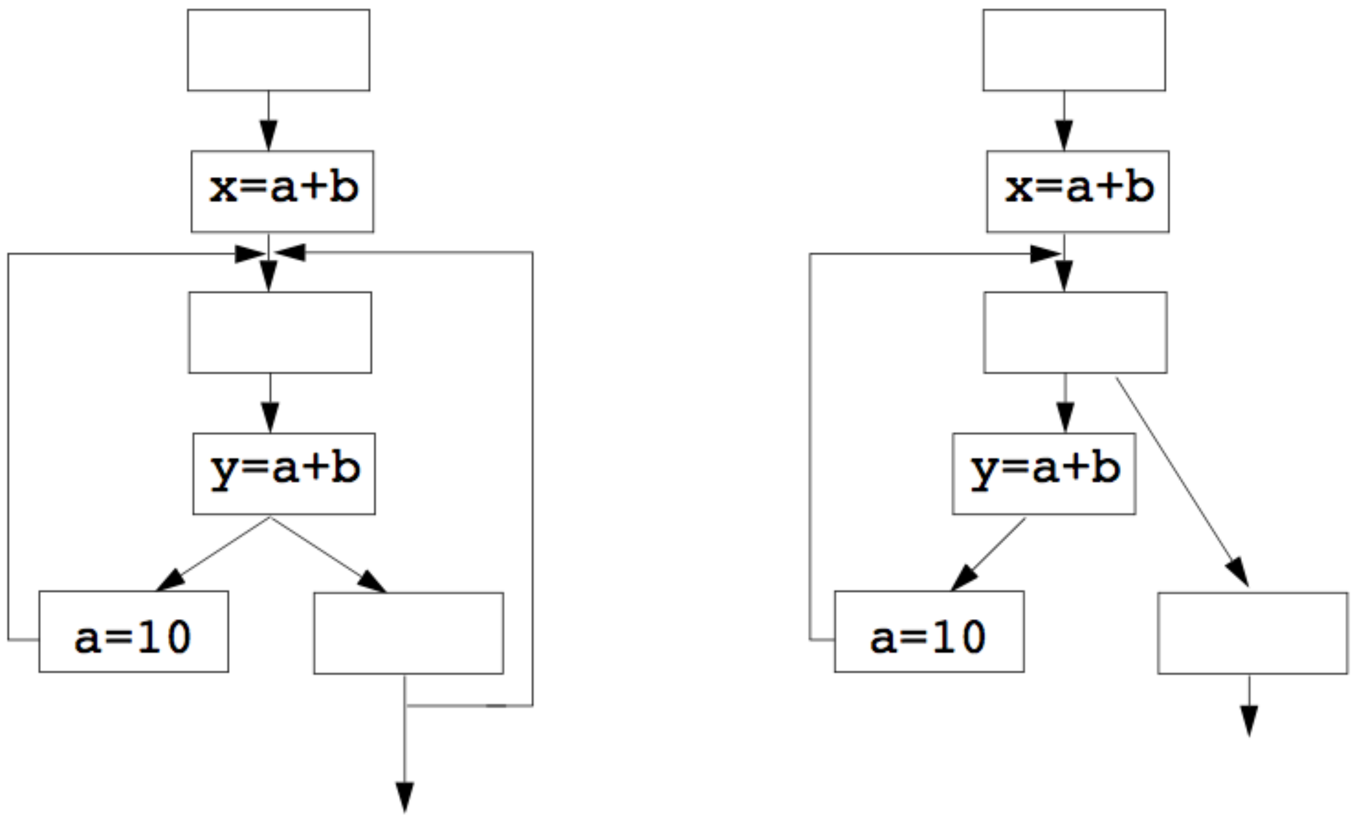
Examples (2)



- Cannot eliminate all redundancy

Examples (3)

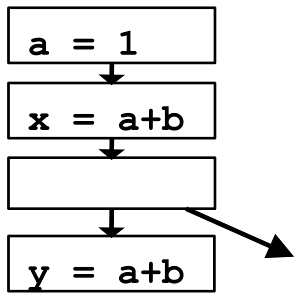
Do you know how the algorithm works without simulating it?



Pass 2: Place As Early As Possible

There is still some redundancy left!

- **First approximation: frontier between "not anticipated" & "anticipated"**
- **Complication: Anticipation may oscillate**



- **An anticipation frontier may cover a subsequent frontier.**
- **Once an expression has been anticipated,**
  it is "available" to subsequent frontiers
  → no need to re-evaluate.
- e will be **available at p** if
  e has been "anticipated but not subsequently killed" on all paths reaching p

Available Expressions

- e will be **available at p** if
  e has been "anticipated but not subsequently killed" on all paths reaching p

	Available Expressions
Domain	Sets of expressions
Direction	forward
Transfer Function	$f_b(x) = (\text{Anticipated}[b].\text{in} \cup x) - \text{EKill}_b$
$\wedge$	$\cap$
Boundary	out[entry] = $\varnothing$
Initialization	out[b] = {all expressions}

Early Placement

- **earliest(b)**
  - set of expressions added to block b under early placement
- **Place expression at the** **earliest point anticipated and not already available**
  - earliest(b) = anticipated[b].in - available[b].in
- ***Algorithm***
  - For all basic block b,

    if x+y ∈ earliest[b]

    at beginning of b:

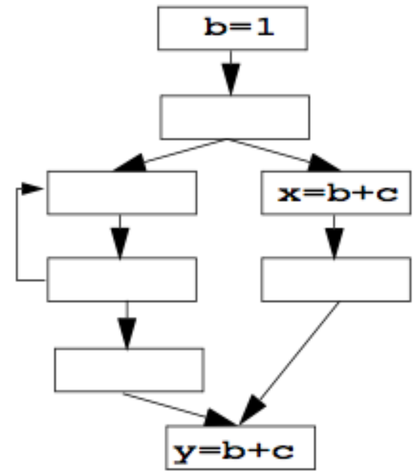
    create a new variable t

    t = x+y,

    replace every original x+y by t

Pass 3: Lazy Code Motion

Let's be lazy without introducing redundancy.



Delay without creating redundancy to reduce register pressure

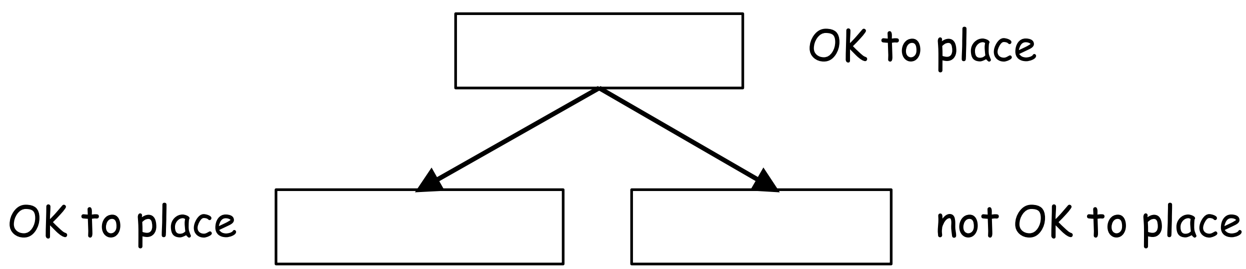
An expression e is postponable at a program point p if

- all paths leading to p
  have seen the earliest placement of e but not a subsequent use

	Postponable Expressions
Domain	Sets of expressions
Direction	forward
Transfer Function	$f_b(x) = (earliest[b] \cup x) - EUse_b$
$\wedge$	$\cap$
Boundary	$out[entry] = \emptyset$
Initialization	out[b] = {all expressions}

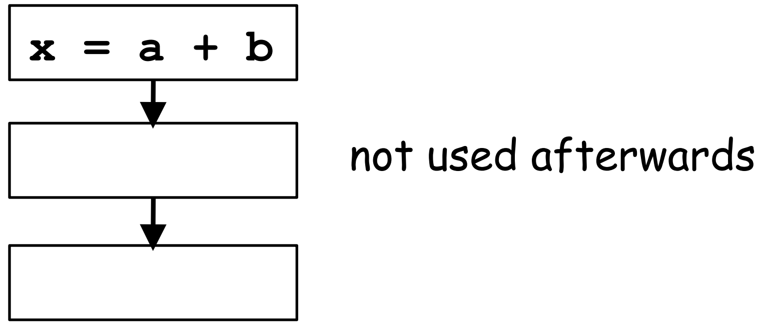
Latest: frontier at the end of "postponable" cut set

- latest[b] = (earliest[b] ∪ postponable.in[b]) ∩
  $(EUse_b \cup \neg(\bigcap_{s \in succ[b]}(earliest[s] \cup postponable.in[s])))$
  - OK to place expression: earliest or postponable
  - Need to place at b if either
    - used in b, or
    - not OK to place in one of its successors
- Works because of pre-processing step (an empty block was introduced to an edge if the destination has multiple predecessors)
  - if b has a successor that cannot accept postponement, b has only one successor
  - The following does not exist:



Pass 4: Cleaning Up

Finally… this is easy, it is like liveness



- **Eliminate temporary variable assignments unused beyond current block**
- **Compute: Used.out[b]: sets of used (live) expressions at exit of b.**

	Used Expressions
Domain	Sets of expressions
Direction	backward
Transfer Function	$f_b(x) = (EUse[b] \cup x) - latest[b]$
$\wedge$	$\cup$
Boundary	in[exit] = ∅
Initialization	in[b] = ∅

Code Transformation

Original version: For each basic block b,

```
if x+y ∈ earliest[b]
    at beginning of b:
        create a new variable t
        t = x+y,
replace every original x+y by t
```

New version: For each basic block b,

```
if (x+y) ∈ (latest[b] ∩ ¬ used.out[b]) { }
else
    if x+y ∈ latest[b]
        at beginning of b:
            create a new variable t
            t = x+y,
    replace every original x+y by t
```

4 Passes for Partial Redundancy Elimination

- **Heavy lifting**: **Cannot introduce operations not executed originally**
  - Pass 1 (backward): Anticipation: range of code motion
  - Placing operations at the frontier of anticipation gets most of the redundancy
- **Squeezing the last drop of redundancy**:
  An anticipation frontier may cover a subsequent frontier
  - Pass 2 (forward): Availability
  - Earliest: anticipated, but not yet available
- **Push the cut set out -- as late as possible**
  To minimize register lifetimes
  - Pass 3 (forward): Postponability: move it down provided it does not create redundancy
  - Latest: where it is used or the frontier of postponability
- **Cleaning up**
  - Pass 4: **Remove temporary assignment**

Remarks

- **Powerful algorithm**
  - Finds many forms of redundancy in one unified framework
- **Illustrates the power of data flow**
  - Multiple data flow problems