Finite State Machines: Finite State Transducers; Specifying Control Logic

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Finite State Transducers

- A *finite state transducer* is a variant of the notion of a finite state machine
- Recall that a finite state machine corresponds to a language, i.e., to the set of input strings that it accepts
- A finite state transducer corresponds to a function from strings to strings
  - The output alphabet may be different than the input alphabet
  - Example: The function that maps each binary string to its complement

Finite State Transducer: Definition

- A finite state transducer is defined in the same way as an FSM except that:
  - In addition to the input alphabet, an output alphabet is specified
  - Typically there is no notion of acceptance of a string, so no accepting states are specified
  - Each transition has an associated output string (i.e., string over the output alphabet)
- Like an FSM, a finite state transducer performs a particular sequence of transitions on a given input string
  - The output of the finite state transducer is the concatenation of the output strings associated with these transitions

Finite State Transducer: Pictorial Representation

- A finite state transducer is drawn in the same way as an FSM except that:
  - Typically there is no notion of acceptance of a string, so no state is designated as accepting
  - Each transition is marked with a symbol or set of symbols as in the case of an FSM, followed by a forward slash, followed by an output string

Example: Serial Binary Adder

- Input: Two nonnegative integers
  - Represented as equal-length binary strings
  - Presented as a sequence of pairs of bits
  - Low-order bit first
- Output: Assuming the input strings are of length k, the output string is the k low-order bits of the sum of the input values
  - Low-order bit first
- Draw a finite state transducer for solving this problem
- How could we modify this problem slightly to ensure that the output string corresponds to the sum of the two input values, and not to just the low-order bits of the sum?

Example: Parity Generator

- Alice wants to send a sequence of $3k$ bits to Bob
- Each block of 3 bits should be followed by a parity bit for that block
- Draw a finite state transducer that Alice can use to send the message
- Draw a finite state machine that Bob can use to accept or reject each 4-bit block

Example: Base Conversion

- In each of the following cases, draw a finite state transducer that implements the desired function, or argue that no such finite state transducer exists:
  - Convert a binary string to base 4, assuming that the input value is specified low-order bit first
  - Convert a binary string to base 4, assuming that the input value is specified high-order bit first
  - Convert a binary string to base 3, assuming that the input value is specified low-order bit first

Specifying Control Logic

- A surprisingly wide range of systems can be modeled by finite state machines/transducers
  - In such cases, the finite state machine/transducer provides a precise description of the system behavior
- Of course, many systems —even some relatively simple ones— cannot be modeled using only a finite number of states
  - In such cases, a finite state machine/transducer may still be useful for modeling some important part of the system, e.g., an “outer loop” of control logic

Example: Soda Machine

- We wish to model the following soda machine as a finite state transducer:
  - The machine accepts only nickels and dimes and dispenses two products, A and B, costing 15 and 20 cents, respectively
  - If the user presses the appropriate button (i.e., a for A and b for B) after depositing at least the correct amount, the machine dispenses the item and returns change, if any, in nickels
  - If the user inserts additional coins after depositing 20 cents or more, the last coin is returned
  - If the user asks for an item before depositing the appropriate amount, a warning light flashes for 2 seconds
  - The user may cancel the transaction at any time, at which point the deposit, if any, is returned in nickels