CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Good Morning, Colleagues

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Are there any questions?

Logistics

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- Next week's assignments up

Some important concepts

- Contrapositive vs. contradiction
- How to choose a proof technique
- WLOG
- Constructive vs. non-constructive

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- For integer n, If n^3 is even, then n is even.

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- The length of the hypotenuse of a right triangle is less than the sum of the lengths of the two legs.

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7. Combining the inequalities in lines 5 and 6 yields $c^2 > a^2 + b^2$.
8. However, according to the Pythagorean Theorem: $c^2 = a^2 + b^2$. Contradicts line 7.

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