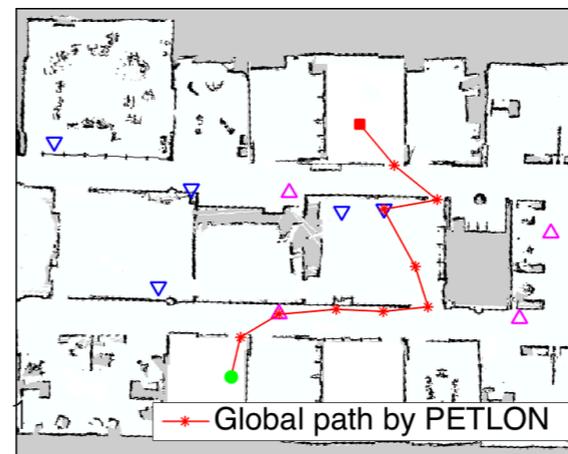
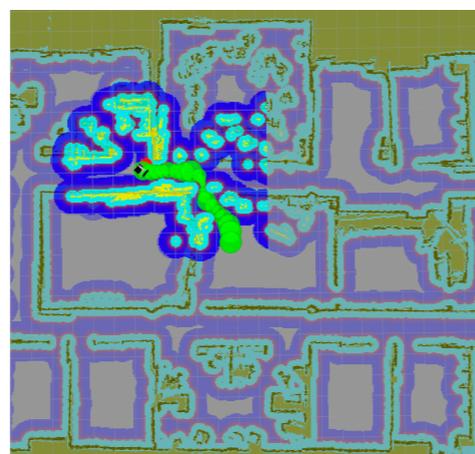


# PETLON: Planning Efficiently for Task-Level-Optimal Navigation

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# Autonomous agents are becoming more capable of:

- Long-duration deployment
- Large-scale navigation
- Knowledge-intensive reasoning

## Service robots will be desired to:

- Understand human requests
- Complete multiple tasks with high efficiency



# Task Planning

- **Task requests?**

Plan to satisfy certain specifications

- **Satisfiability problem**

logic programs

Answer set programming (ASP)

Prolog

AI planning

PDDL

STRIPS

e.g. **camping preparation (in ASP):**

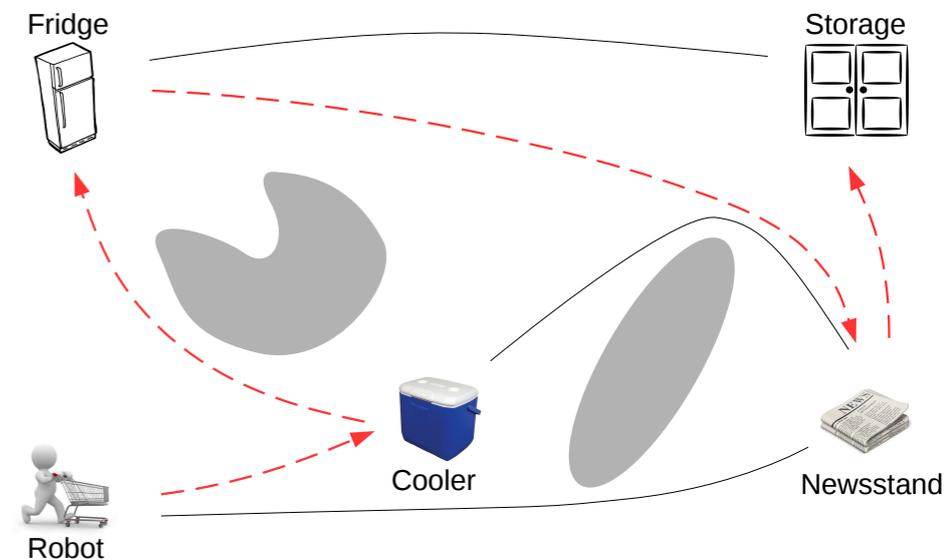
need\_ice(milk)

cooling(cooler)

-spoiled(NI) :- inside(NI,C), cooling(C), need\_ice(NI)

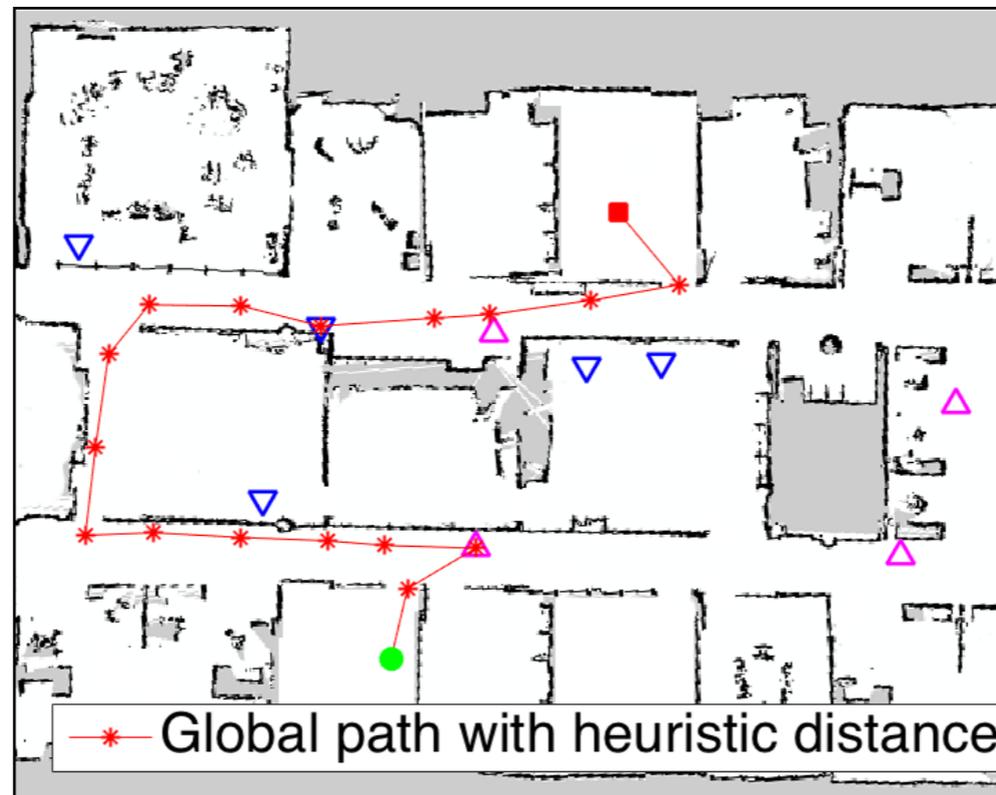
**:- delivered(S,milk,maxstep), -spoiled(milk), storage(S)**

**#minimize{X,Y:cost(X,Y)}.**



# Motion Planning

- **Navigation paths for travel cost evaluation**  
Plan to satisfy kinematics/dynamics constraints
  - **Collision-free, dynamically feasible**
- Trajectory optimization
  - CHOMP
- Sampling-based algorithms
  - RRT
  - PRM



# The Computational Challenges in Large Domain Planning

- **Large task domains** include:

- **Large number of object instances**  $\mathcal{N}$

cooling(cooler\_east1)  
cooling(fridge\_west1)  
cooling(fridge\_west2)  
hascoffee(cafe\_east1)  
hascoffee(cafe\_west1)  
⋮

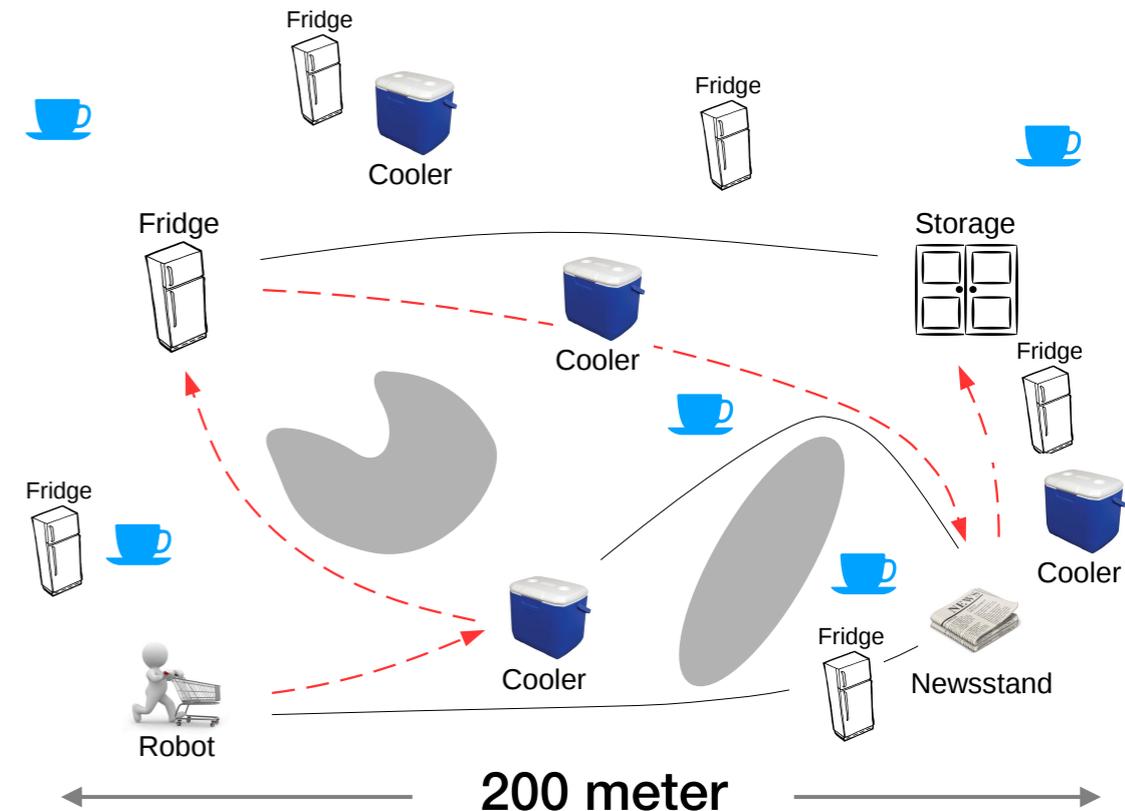
- **Large search space:**

exponential growth in task length  $L$  and  $\mathcal{N}$  in node traversals

$$O(\mathcal{N}^L)$$

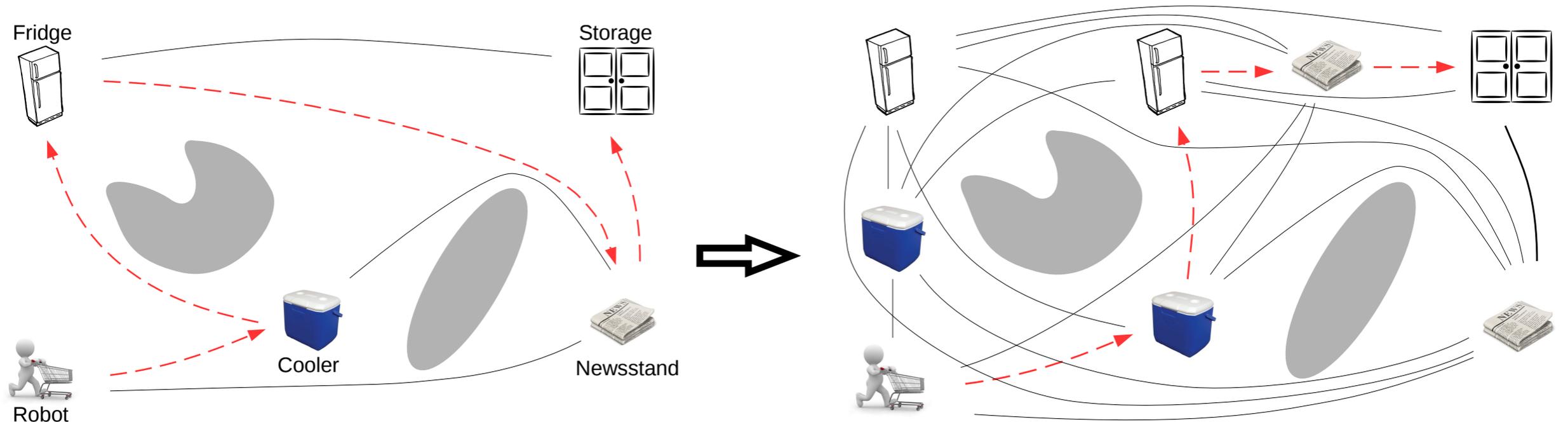
- **Large domain navigation cost:**

- evaluated by motion planner for task-level travel cost estimate



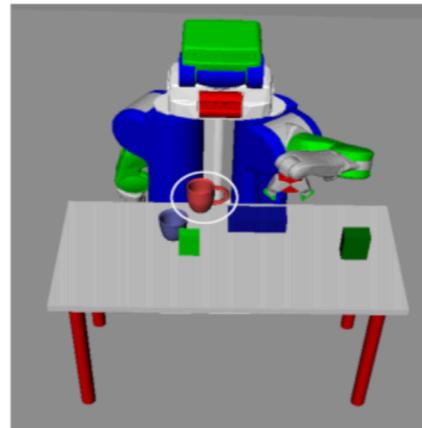
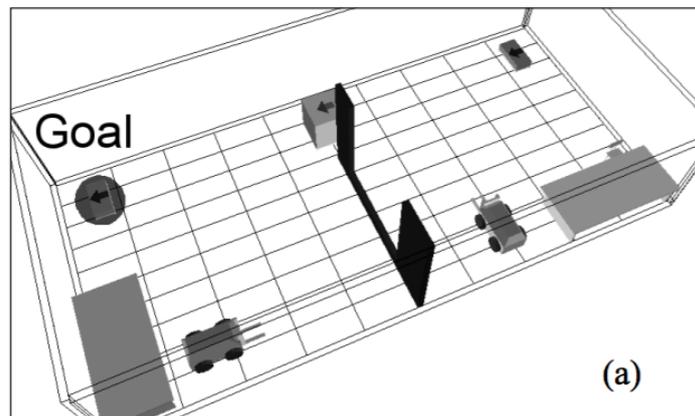
# The Challenges

- **Desire for optimality guarantee**  
suboptimal plans may have much longer travel length in large maps
- **Minimize the calls for motion evaluation**  
the intensive search **with motion evaluation** grows exponentially in  $L$  and  $N$ :  $O(N^L)$



# Related Work

- **Semantic attachments**  
symbolic planner with function calls  
Dornhege, et. al, 2009
- **Integrated task and motion planning in robotics**  
plan to solve for geometrically constrained problems



Gravot, Cambon, et. al, 2005  
Wolfe, et.al, 2010  
Plaku, et.al, 2010  
Kaelbling, et. al, 2011  
Lagriffoul, et al, 2014  
Srivastava, et al, 2014  
Garrett, et. al, 2017

## logic-geometric programming

plan to optimize certain objective

Toussaint, 2015

- **Limitations:** not addressing optimality guarantee and the corresponding motion evaluation expense problem

# PETLON:

## Planning Efficiently for Task-Level-Optimal Navigation

$\mathcal{D}^t$

Symbolic state space:  $s \in S$

Symbolic action space:  $a \in A$

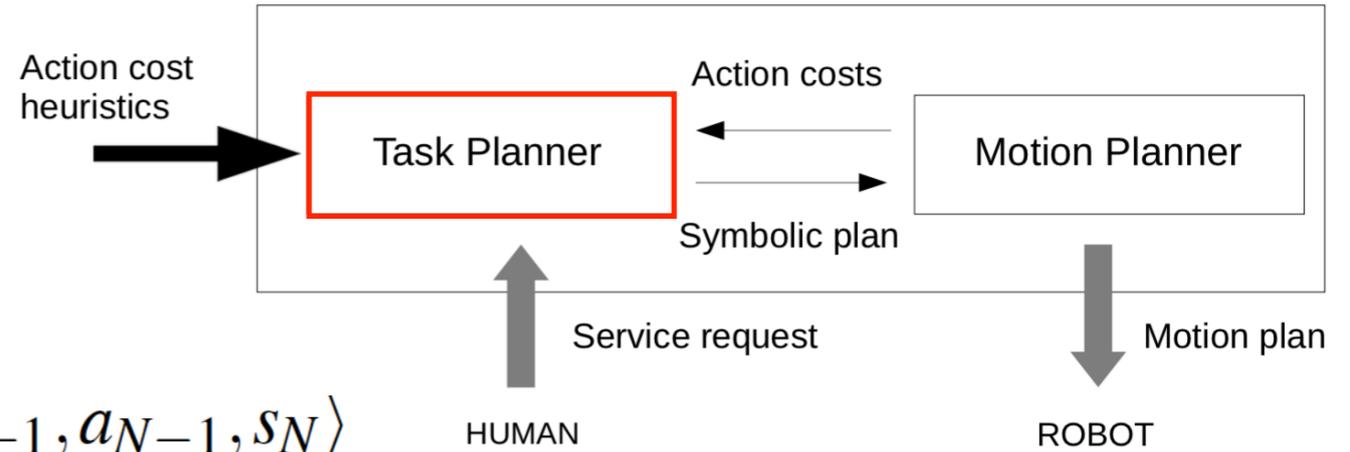
State transition function:  $\mathcal{T}^t : \langle s, a \rangle \rightarrow s'$

Task plans:  $p = \langle s_0, a_0, \dots, s_{N-1}, a_{N-1}, s_N \rangle$

Task specifications:  $S^G \subseteq S$

Feasible task plan space:  $p \in P$  s.t.  $s_0 = s^{init}, s_N \in S^G$

Cost function:  $Cost(\langle s, a, s' \rangle) \rightarrow \mathbb{R}$



- **Optimal task-level action sequence:**

$$\mathcal{P}^t : p^* = \operatorname{argmin}_{p \in P} \sum_{\langle s, a, s' \rangle \in p} Cost(\langle s, a, s' \rangle)$$

# PETLON:

## Planning Efficiently for Task-Level-Optimal Navigation

$\mathcal{D}^m$

**Numeric state space:**  $x \in X$

**Numeric action space:**  $u \in U$

**State transition function:**  $\mathcal{T}^m : \langle x, u \rangle \rightarrow x'$

**Motion plans:**  $\xi = \langle x_0, u_0, \dots, x_{L-1}, u_{L-1}, x_L \rangle$

**Robot model:**  $\mathcal{M}$

**Collision-free space:**  $Free(\mathcal{D}^m | \mathcal{M})$

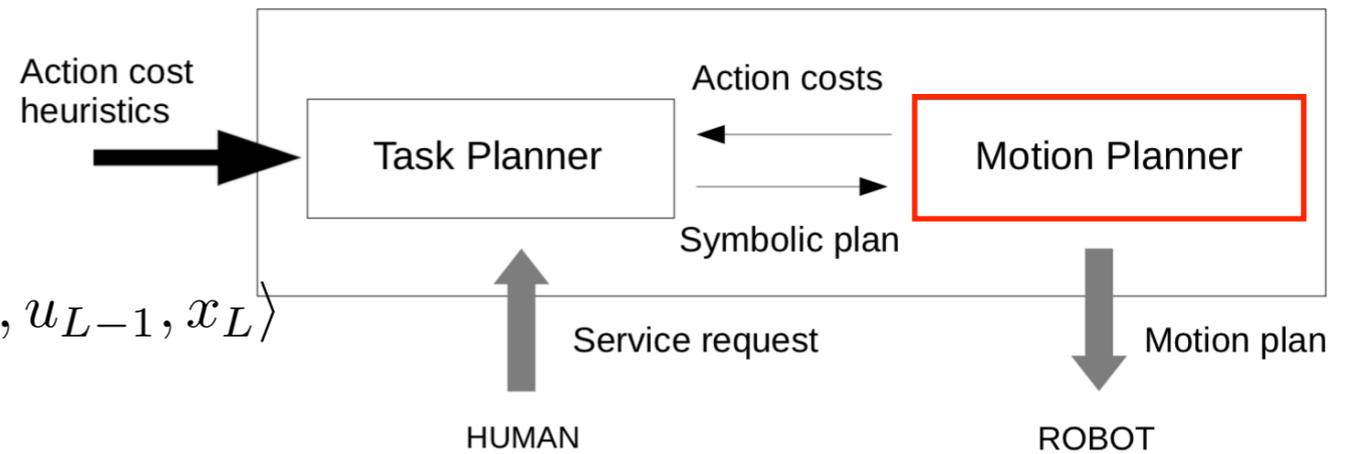
**State mapping function:**  $f : s \rightarrow X$

**Initial set:**  $X^{init} = f(s^{init}) \in X$

**Goal set:**  $X^{goal} = f(s^{goal}) \in X$

**Feasible motion plans:**  $\xi \in \Xi$  s.t.  $x_0 \in X^{init}$ ,  $x_L \in X^{Goal}$ ,  $x_0, \dots, x_L \in Free(\mathcal{D}^m | \mathcal{M})$

**Motion cost function:**  $Len : \xi \rightarrow \mathbb{R}$

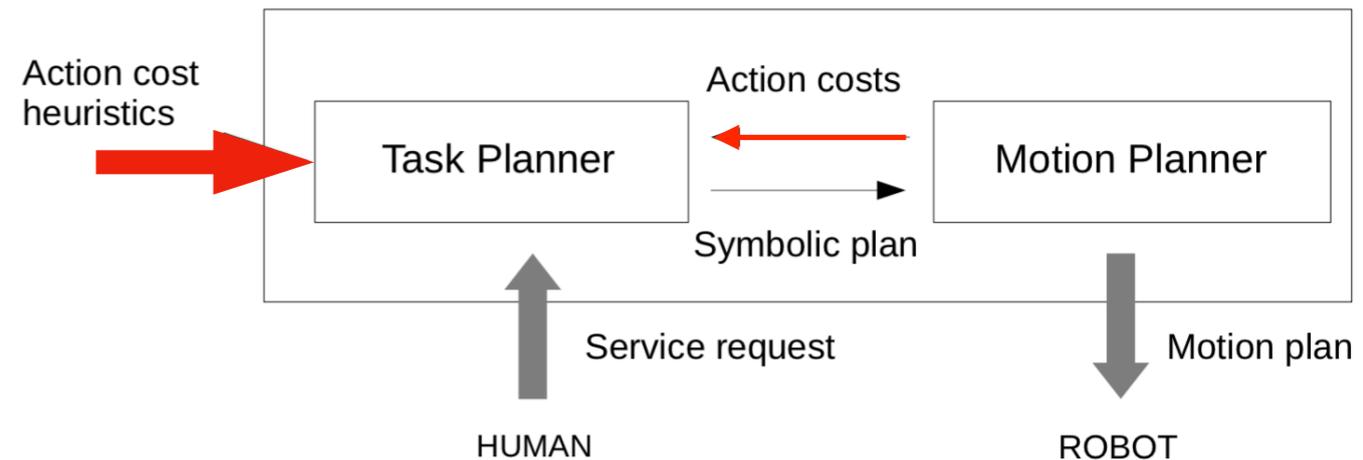


- Motion evaluation:**

$$\mathcal{P}^m : \xi^* = \operatorname{argmin}_{\xi \in \Xi} Len(\xi)$$

$$\hat{C}(x, x') = Len(\mathcal{P}^m(\langle s, a, s' \rangle, f, \mathcal{D}^m, \mathcal{M}))$$

# PETLON: Planning Efficiently for Task-Level-Optimal Navigation



## Two Types of Cost Functions:

- **Admissible heuristic** cost function  $h$  (cheap):

$$h(x, x') = \|x - x'\|_2 \quad x \in f(s), \quad x' \in f(s')$$

- **Evaluated** cost function  $\hat{C}$  (expensive):

$$\hat{C}(x, x') = \text{Len}(\mathcal{P}^m(\langle s, a, s' \rangle, f, \mathcal{D}^m, \mathcal{M}))$$

- **Maintained** cost function  $Cost$  (used in  $\mathcal{D}^t$ ):

$$h(x, x') \leq Cost(s, a, s') \leq \hat{C}(x, x')$$

# PETLON:

## Planning Efficiently for Task-Level-Optimal Navigation

- The **task and motion planning** problem:

$$\Omega : \langle \mathcal{D}^t, \mathcal{D}^m, s^{init}, S^G, x^{init}, f, \mathcal{M} \rangle \rightarrow \langle p, [\xi_0, \xi_1, \dots, \xi_{N-1}] \rangle$$

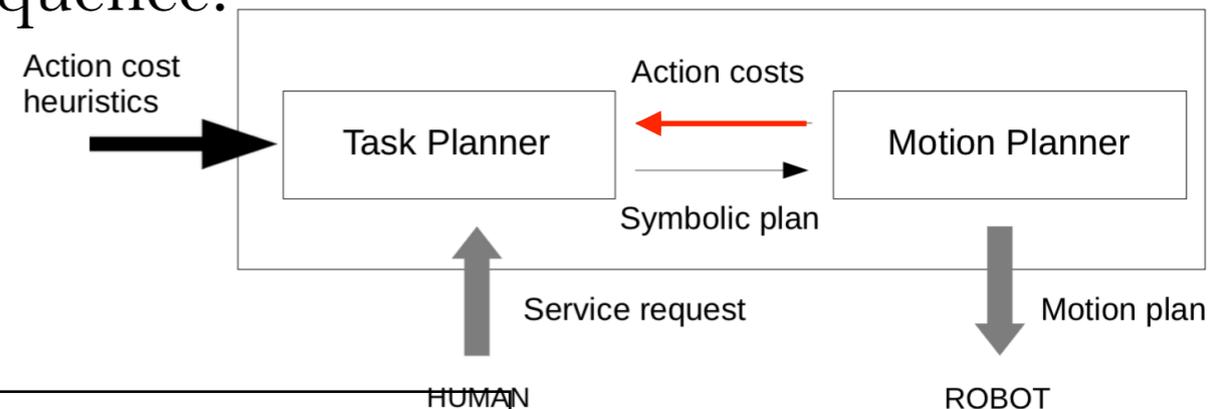
s.t.

$$\mathcal{D}^t : p(0) = s^{init}, p(N) \in S^G, |p| = N$$

$$\mathcal{D}^m : \xi_0(0) = x^{init}, \xi_i(0) \in f(s_i), \xi_i(T_i) \in f(s_{i+1}) \text{ for } i = \{0, 1, \dots, N-1\}$$

- **Optimal task-level navigation action sequence:**

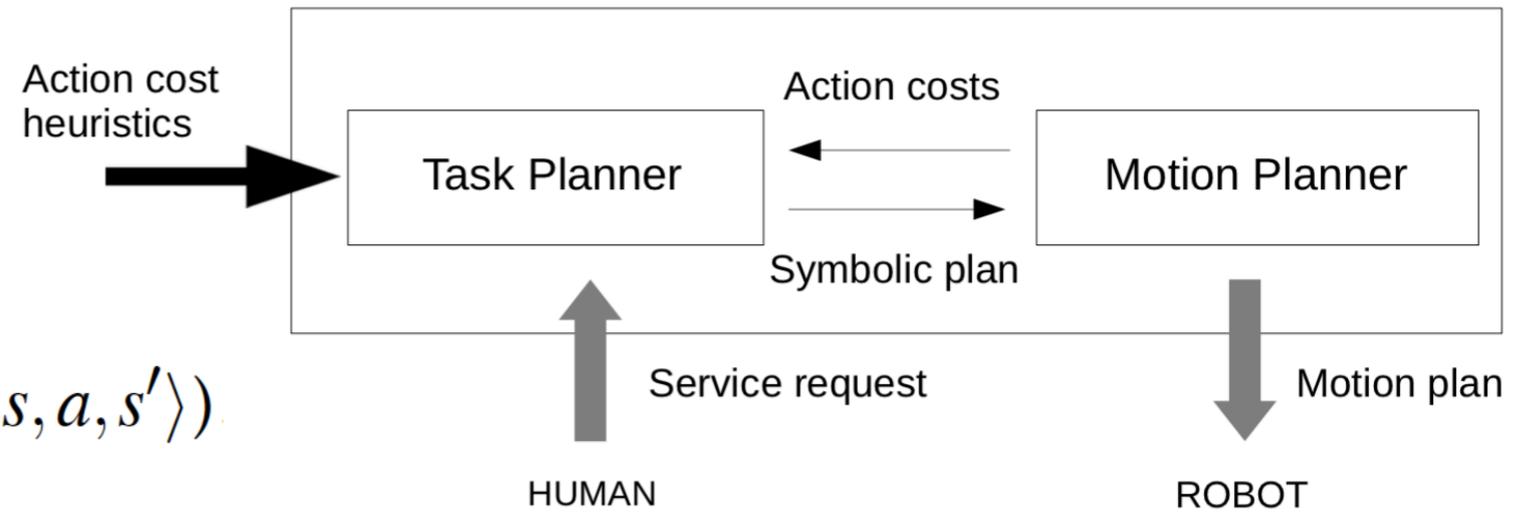
$$p^* = \operatorname{argmin}_{p \in P} \left( \sum_{0 \leq i < |p|} \operatorname{Len}(\xi_i) | \mathcal{P}^m, f \right)$$



- **PETLON:** minimizing number of calls for motion evaluation while still **guarantees task-level optimality**

**Recall:**

$$\mathcal{P}^t : p^* = \operatorname{argmin}_{p \in \mathcal{P}} \sum_{\langle s, a, s' \rangle \in p} \operatorname{Cost}(\langle s, a, s' \rangle)$$



**Proposition 1: if all task actions in  $p^*$  are evaluated<sup>(1)</sup>, it is the optimal solution.<sup>(4)</sup>**

$$\sum_{\langle s, a, s' \rangle \in p^*} \operatorname{Cost}(s, a, s') = \sum_{\langle s, a, s' \rangle \in p^*} \hat{C}(s, a, s') \quad (1)$$

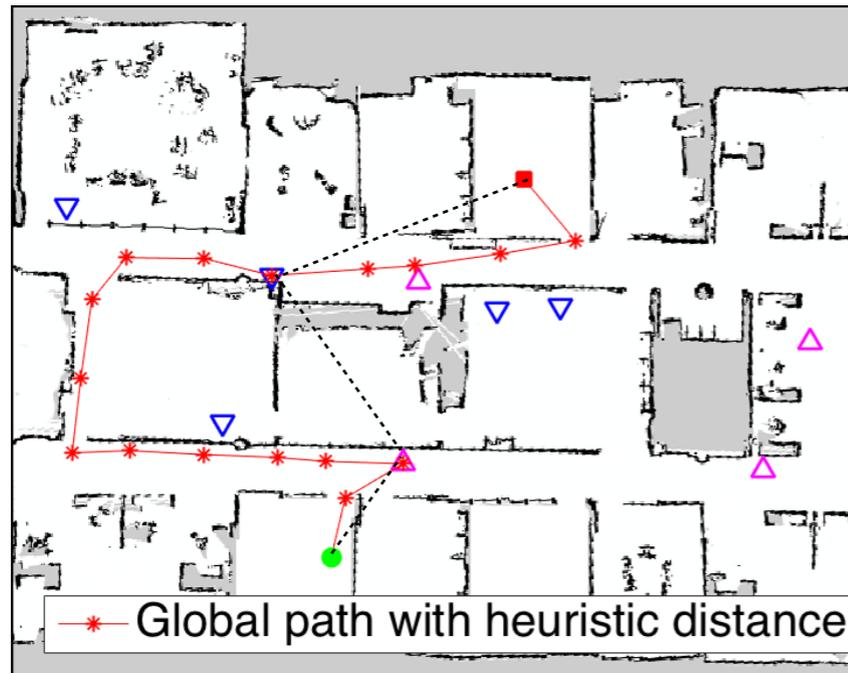
$$\sum_{\langle s, a, s' \rangle \in p^*} \operatorname{Cost}(s, a, s') \leq \sum_{\langle s, a, s' \rangle \in p} \operatorname{Cost}(s, a, s') \quad (2)$$

$$\sum_{\langle s, a, s' \rangle \in p} \operatorname{Cost}(s, a, s') \leq \sum_{\langle s, a, s' \rangle \in p} \hat{C}(s, a, s') \quad (3)$$

$$\Rightarrow \sum_{\langle s, a, s' \rangle \in p^*} \hat{C}(s, a, s') \leq \sum_{\langle s, a, s' \rangle \in p} \hat{C}(s, a, s') \quad (4) \quad \blacksquare$$

# PETLON: Planning Efficiently for Task-Level-Optimal Navigation

iter 0



----- Dash: plan value using  $Cost$ , maintained cost function  
 ——— Solid: plan true value by  $\hat{C}$ , evaluated cost function

---

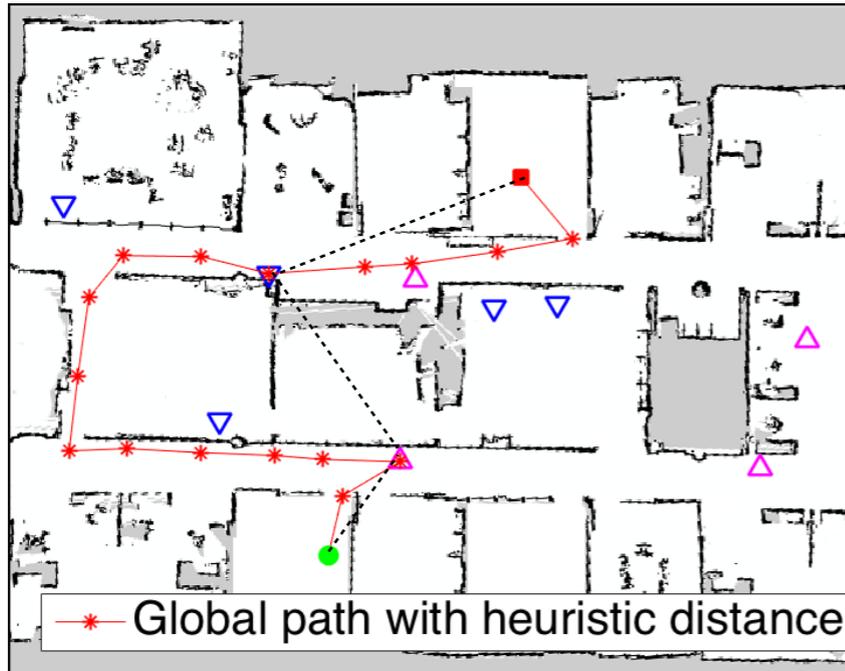
PETLON algorithm

---

1. Initialize  $Cost(s, a, s') \leftarrow h(x, x')$   
 Initialize empty state-action-state array:  $A^{evld}$
  2. while true do
    - $[\hat{p}^*, P] \leftarrow \mathcal{P}^t(s^{init}, S^G, Cost, \mathcal{D}^t)$
    - if  $\langle s, a, s' \rangle \in A^{evld}, \forall \langle s, a, s' \rangle \in \hat{p}^*$  then
    - return  $\hat{p}^*$
    - end if
  3. for each  $p \in P$  do
    - Evaluate motion cost:  $Cost(s, a, s') \leftarrow \hat{C}(x, x')$
    - Append  $\langle s, a, s' \rangle$  to  $A^{evld}$
- end for  
end while
-

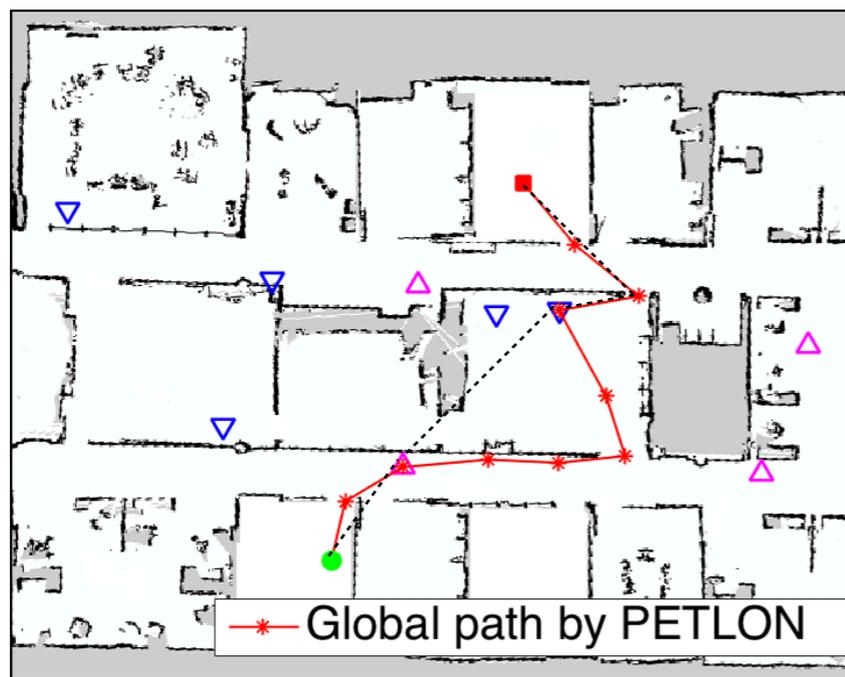
# PETLON: Planning Efficiently for Task-Level-Optimal Navigation

iter 0



----- Dash: plan value using  $Cost$ , maintained cost function  
 ——— Solid: plan true value by  $\hat{C}$ , evaluated cost function

iter 1




---

PETLON algorithm

---

1. Initialize  $Cost(s, a, s') \leftarrow h(x, x')$   
 Initialize empty state-action-state array:  $A^{evld}$

2. while true do

$[\hat{p}^*, P] \leftarrow \mathcal{P}^t(s^{init}, S^G, Cost, \mathcal{D}^t)$

⇒ if  $\langle s, a, s' \rangle \in A^{evld}, \forall \langle s, a, s' \rangle \in \hat{p}^*$  then  
 return  $\hat{p}^*$

end if

3. for each  $p \in P$

do

Evaluate motion cost:  $Cost(s, a, s') \leftarrow \hat{C}(x, x')$

Append  $\langle s, a, s' \rangle$  to  $A^{evld}$

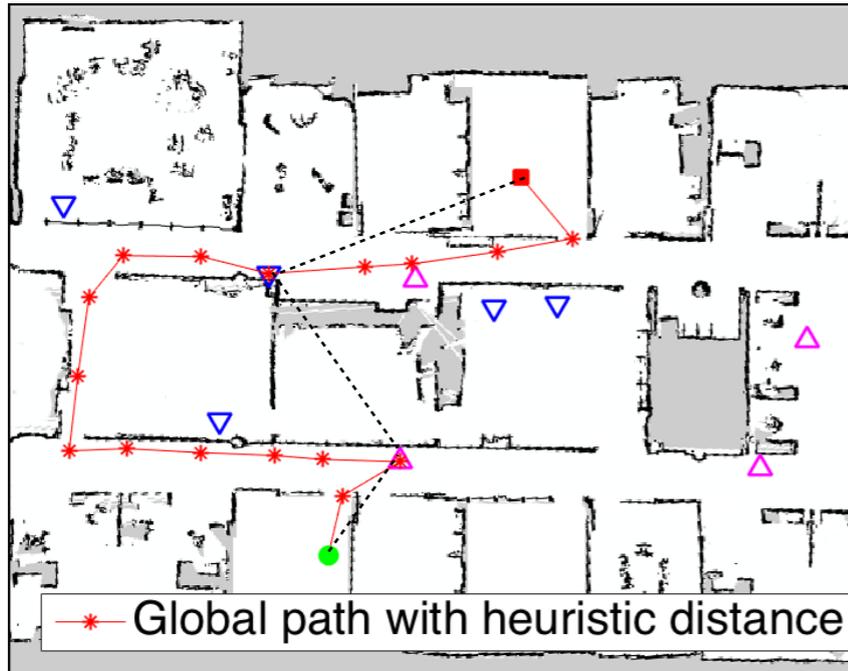
end for

end while

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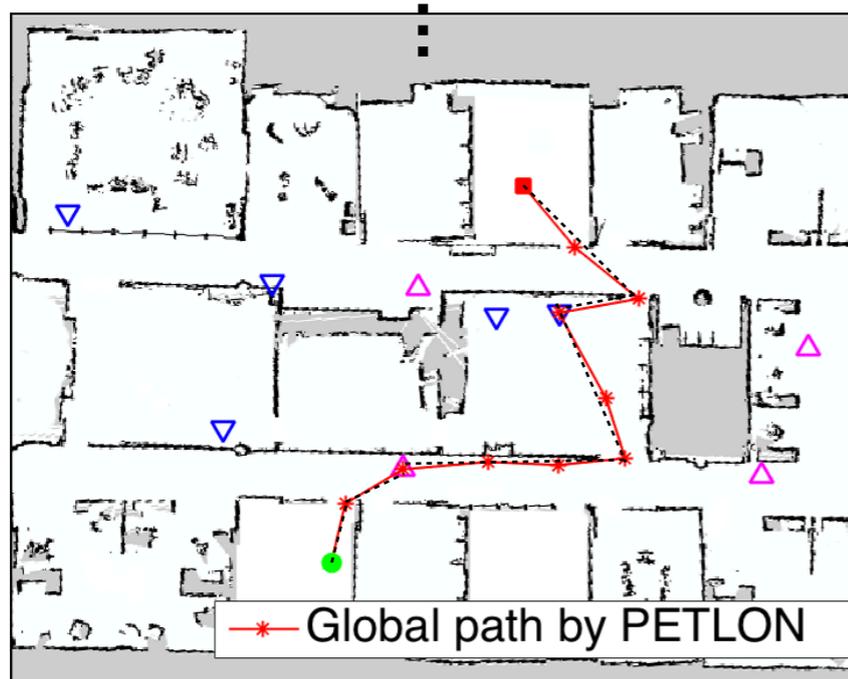
# PETLON: Planning Efficiently for Task-Level-Optimal Navigation

iter 0



----- Dash: plan value using  $Cost$ , maintained cost function  
 ——— Solid: plan true value by  $\hat{C}$ , evaluated cost function

iter N




---

PETLON algorithm

---

1. Initialize  $Cost(s, a, s') \leftarrow h(x, x')$   
 Initialize empty state-action-state array:  $A^{evld}$

2. while true do

$[\hat{p}^*, P] \leftarrow \mathcal{P}^t(s^{init}, S^G, Cost, \mathcal{D}^t)$

⇒ if  $\langle s, a, s' \rangle \in A^{evld}, \forall \langle s, a, s' \rangle \in \hat{p}^*$  then  
 return  $\hat{p}^*$

end if

3. for each  $p \in P$  do

Evaluate motion cost:  $Cost(s, a, s') \leftarrow \hat{C}(x, x')$

Append  $\langle s, a, s' \rangle$  to  $A^{evld}$

end for

end while

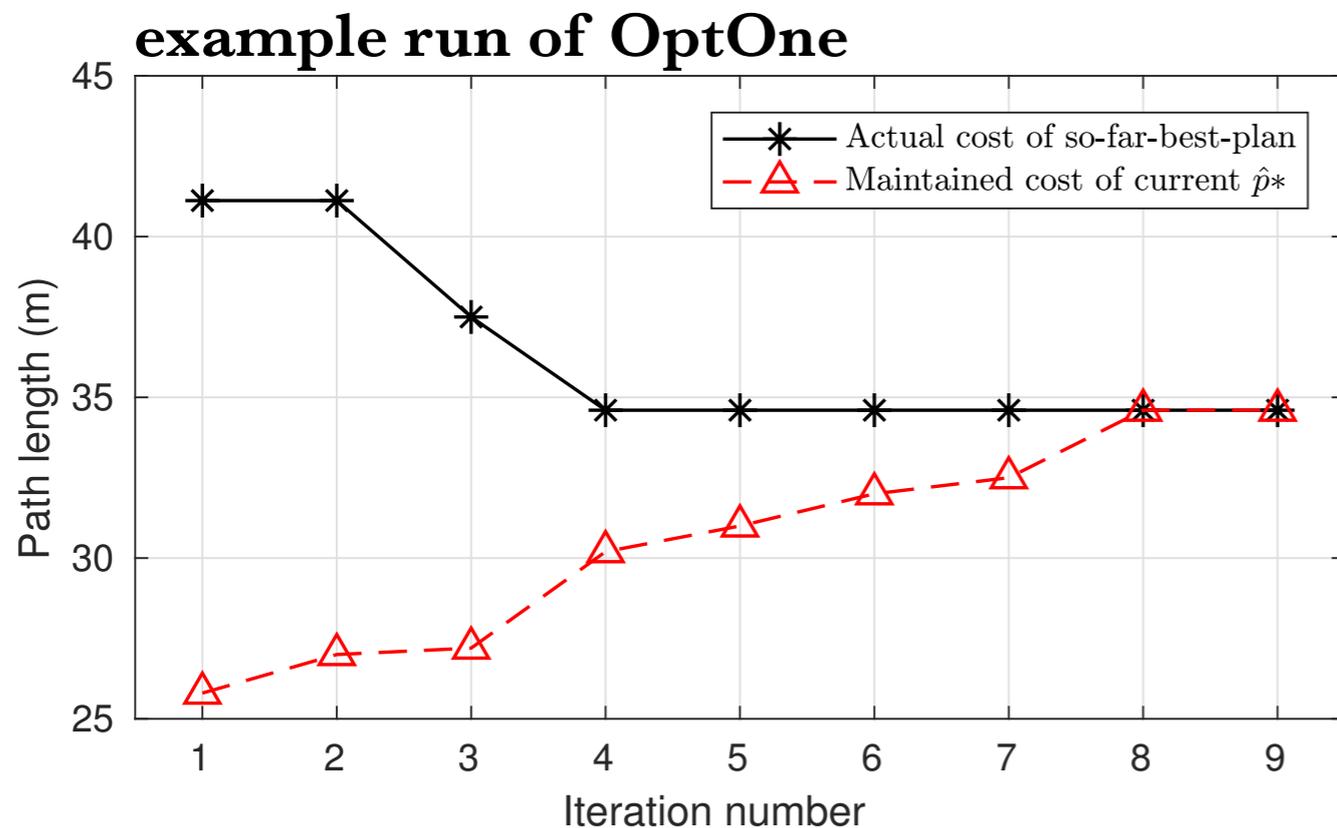
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- guarantees task-level optimality
- minimal calls for motion evaluation

- **OptOne**: evaluate the best plan  $p^*$

# PETLON: Planning Efficiently for Task-Level-Optimal Navigation

- **So-far-best cost**  $C^{sfb}$  :  
the cost value of the so-far best plan
- **Anytime** property:  
 $C^{sfb}$  monotonically decreases




---

## PETLON algorithm

---

Initialize  $Cost(s, a, s') \leftarrow h(x, x')$   
 Initialize empty state-action-state array:  $A^{evld}$

**while true do**

$[\hat{p}^*, P] \leftarrow \mathcal{P}^t(s^{init}, S^G, Cost, \mathcal{D}^t)$

**if**  $\langle s, a, s' \rangle \in A^{evld}, \forall \langle s, a, s' \rangle \in \hat{p}^*$  **then**

**return**  $\hat{p}^*$

**end if**

**for each**  $p \in P$

**do**

Evaluate motion cost:  $Cost(s, a, s') \leftarrow \hat{C}(x, x')$

Append  $\langle s, a, s' \rangle$  to  $A^{evld}$

**end for**  
**end while**

---

# PETLON:

## Planning Efficiently for Task-Level-Optimal Navigation

- **OptAll:**  
evaluate all feasible plans with costs  $< C^{sfb}$   
(also guarantees optimality)

---

### PETLON algorithm

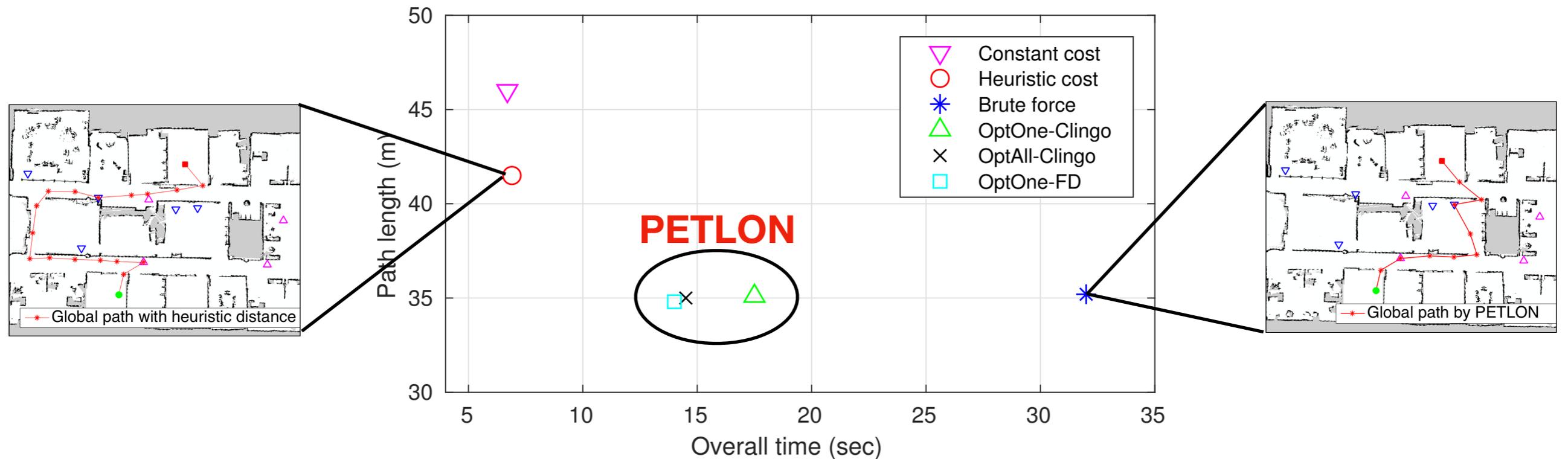
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```
Initialize  $Cost(s, a, s') \leftarrow h(x, x')$ 
Initialize empty state-action-state array:  $A^{evld}$ 
+ Initialize the so-far-best cost:  $C^{sfb} \leftarrow Inf$ 
while true do
   $[\hat{p}^*, P] \leftarrow \mathcal{P}^t(s^{init}, S^G, Cost, \mathcal{D}^t)$ 
  if  $\langle s, a, s' \rangle \in A^{evld}, \forall \langle s, a, s' \rangle \in \hat{p}^*$  then
    return  $\hat{p}^*$ 
  end if
+ for each  $p \in P$  and  $Cost(p) < C^{sfb}$  do
  Evaluate motion cost:  $Cost(s, a, s') \leftarrow \hat{C}(x, x')$ 
  Append  $\langle s, a, s' \rangle$  to  $A^{evld}$ 
+  $C_{plan} = \sum_{\langle s, a, s' \rangle \in p} Cost(s, a, s')$ 
+  $C^{sfb} = \min(C_{plan}, C^{sfb})$ 
end for
end while
```

---

- **guarantees task-level optimality**
- **more calls for motion evaluation than OptOne, fewer calls for task planner**

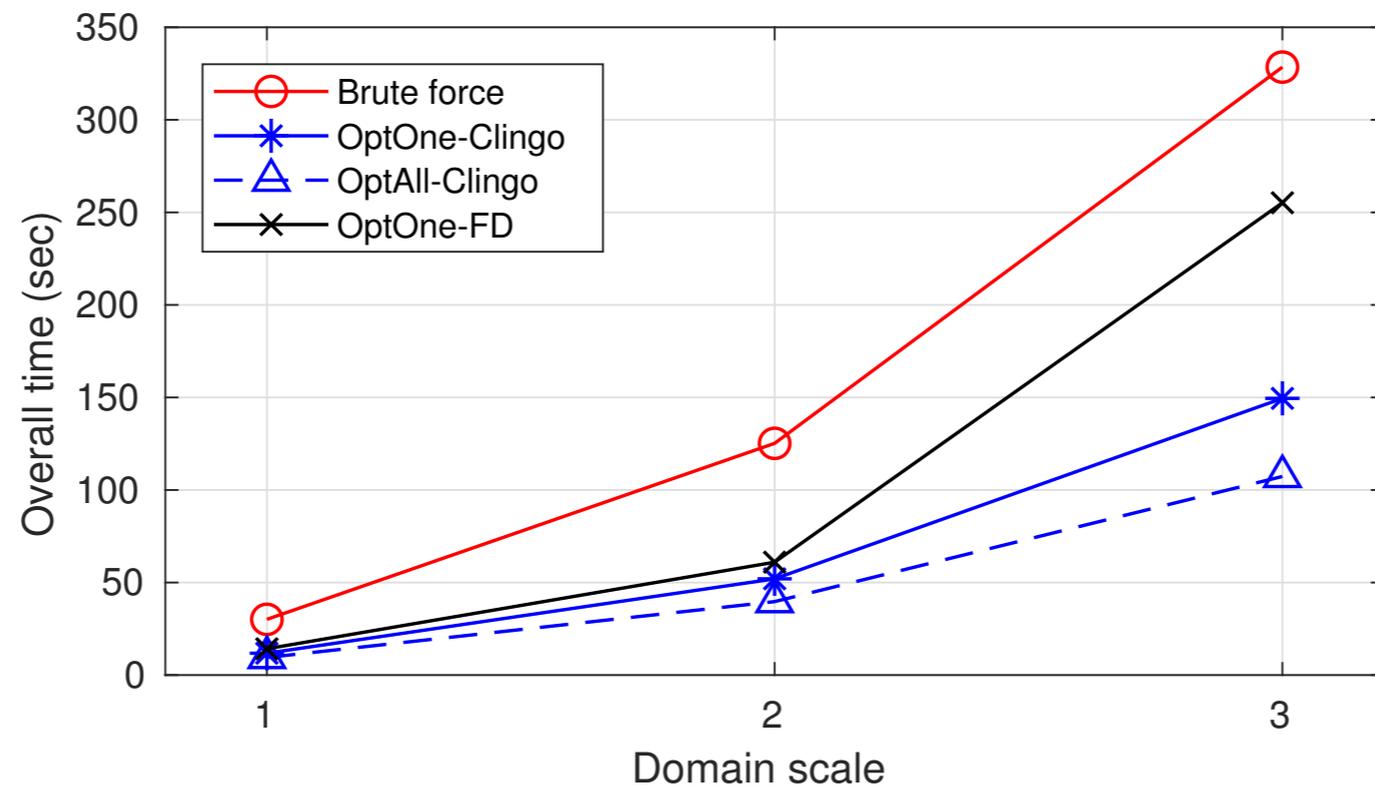
# Plan Quality v.s. Efficiency



	Domain scale			
	1	2	3	
<b>PETLON</b> (	OptOne-Clingo	10.75		
	OptOne-FD	13.60		
	OptAll-Clingo	16.00		
	brute-force baseline	325.00		

**Total number of motion evaluations**

# Experiments with Domain Scale-ups



**PETLON**

**PETLON**

	Domain scale		
	1	2	3
OptOne-Clingo	10.75	9.00	11.00
OptOne-FD	13.60	11.00	12.00
OptAll-Clingo	16.00	17.50	25.75
brute-force baseline	325.00	1295.00	2850.00

**Total number of motion evaluations**

# Conclusion

- **PETLON:**

An algorithm that *saves motion evaluations computation* yet *guarantees optimality* in large-domain navigation problem

- **Future work:**

To more efficiently re-use the task-level search

To include cost estimate in *dynamic environments*, and

to impose an *exploration mechanism* in uncertain environments

**Thank you!**