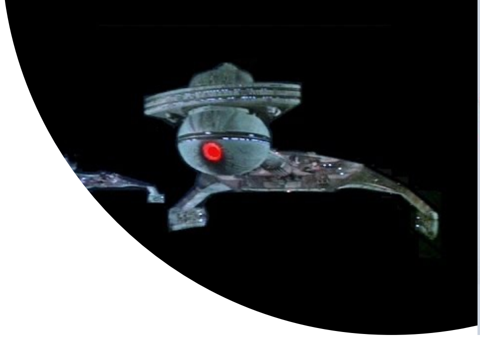
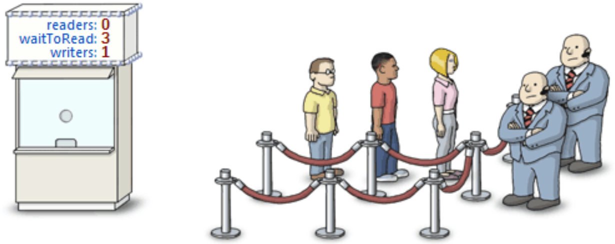
Synchronization: Monitors, Barriers

Chris Rossbach

Today



- Questions?
- Administrivia
  - Lab 1 due date moved (thx CS dept!)
  - Start looking at Lab 2 anyway, esp if you’re done with Lab 1
- Material for the day
  - Coherence redux
  - Some thoughts on work efficiency and instrumentation
  - Monitors
  - Barriers



- Acknowledgements
  - Thanks to Gadi Taubenfield: I borrowed and modified some of his slides on barriers
- Image credits
  - https://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=2ahUKEwjxi4uip8LdAhWFq1MKHbBeD4sQjRx6BAgBEAU&url=http%3A%2F%2Fpreshing.com%2F20150316%2Fsemaphores-are-surprisingly-versatile&psig=AOvVaw20Zw2eU9WAmbX8qxDSLSRd&ust=1537282884760655
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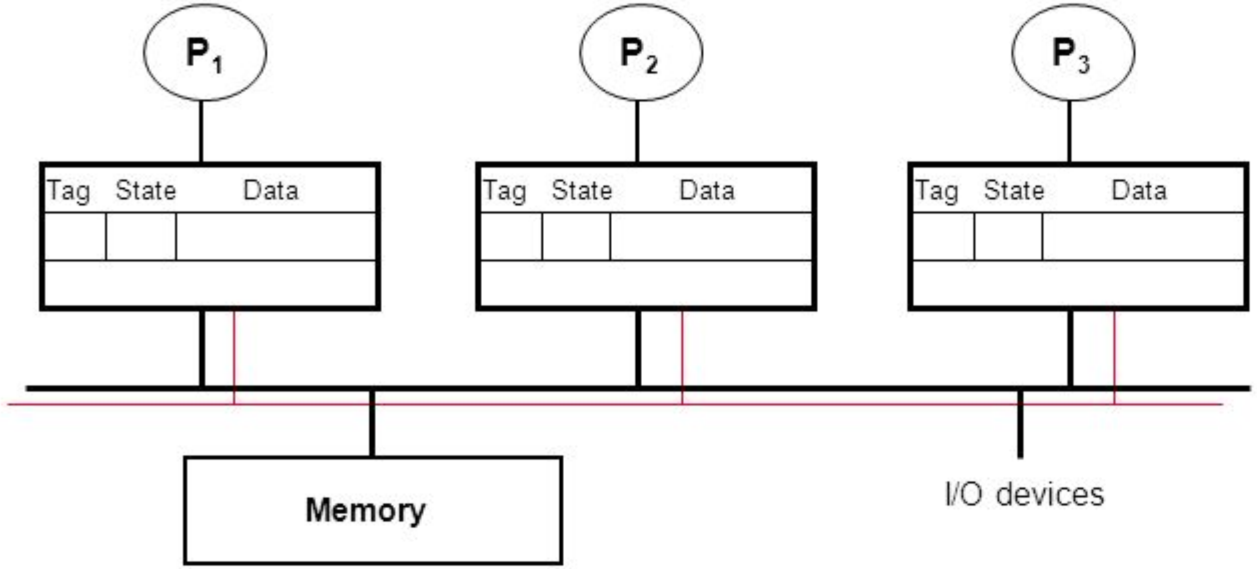
Faux Quiz (answer any 2, 5 min)

- What is the difference between Mesa and Hoare monitors?
- Why recheck the condition on wakeup from a monitor wait?
- How can you build a barrier with spinlocks?
- How can you build a barrier with monitors?
- How can you build a barrier without spinlocks or monitors?
- What is the difference between mutex and semaphores?
- How are monitors and semaphores related?
- Why does pthread_cond_init accept a pthread_mutex_t parameter? Could it use a pthread_spinlock_t? Why [not]?
- Why do modern CPUs have both coherence and HW-supported RMW instructions? Why not just one or the other?
- What is priority inheritance?

Review: Basic MESI Cache Coherence



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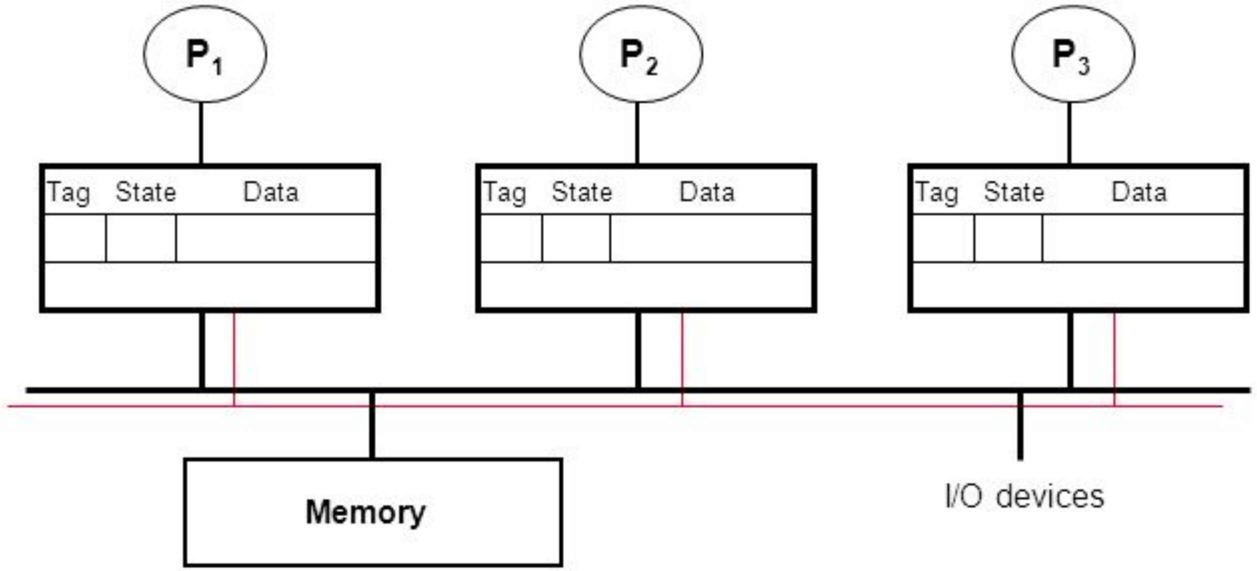

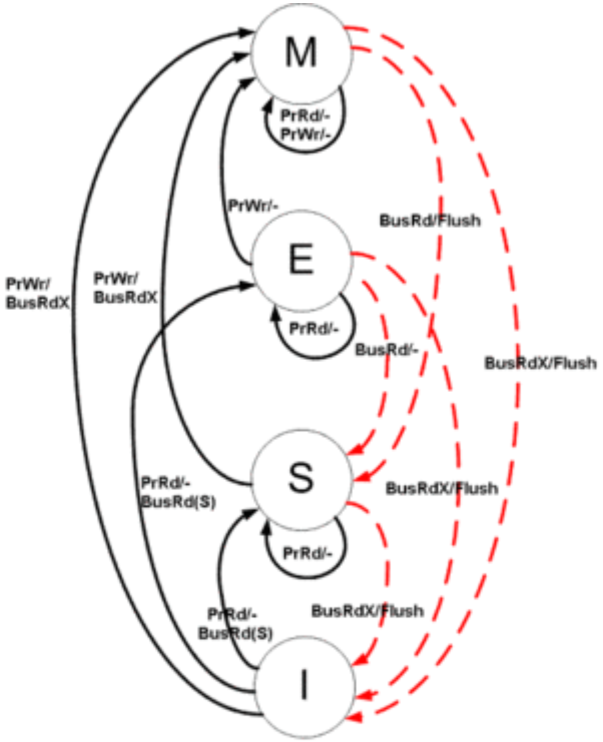

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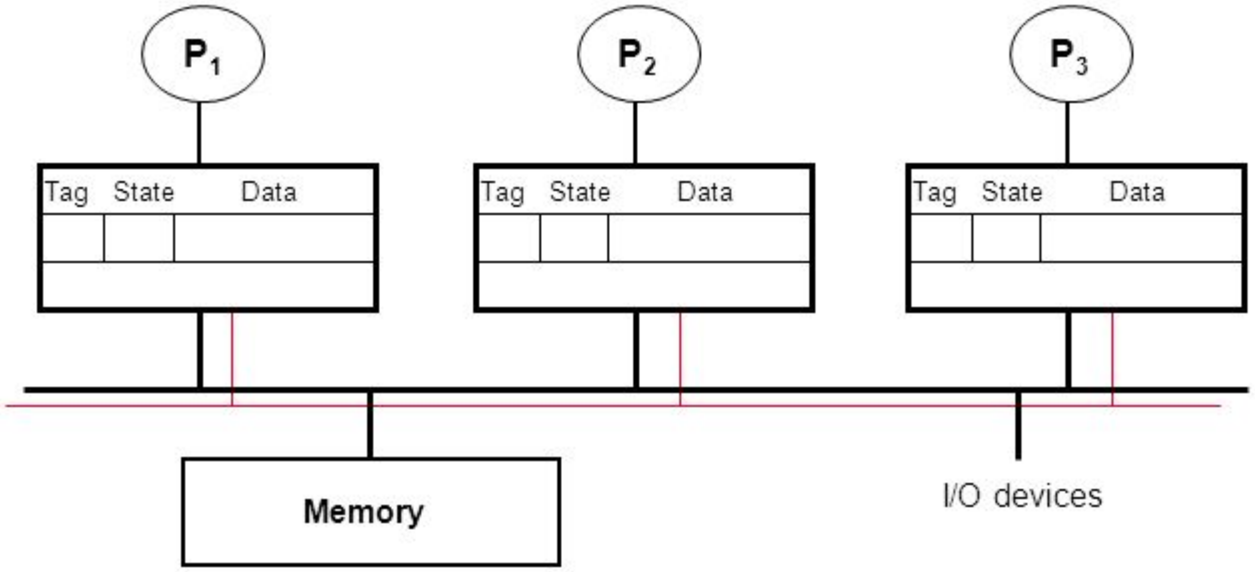
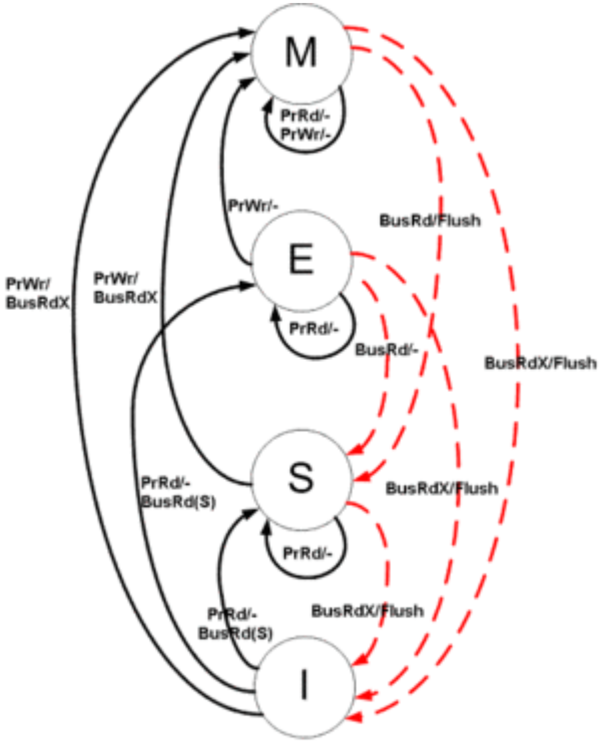
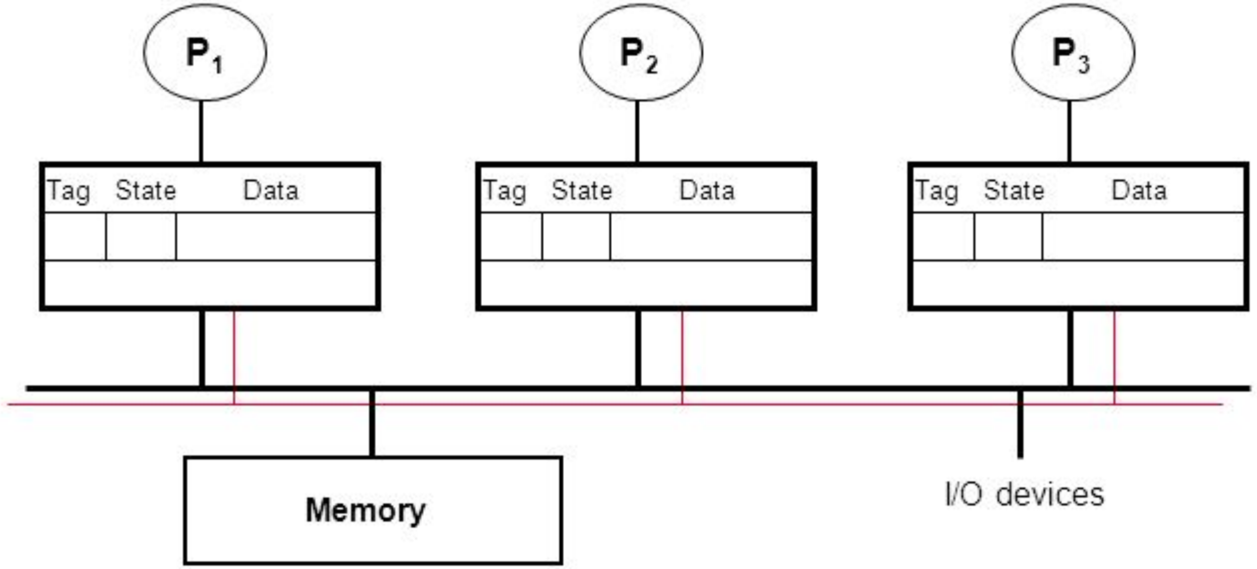
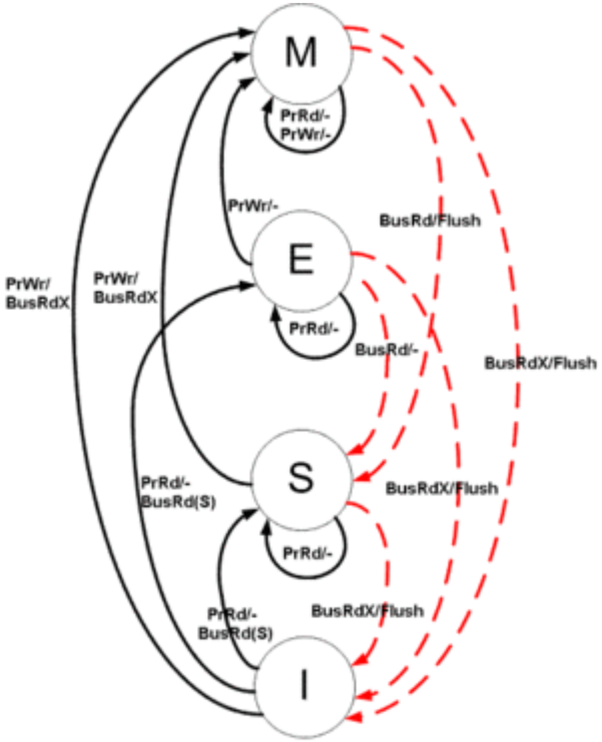




Each cache line has a state (M, E, S, I)

Review: Basic MESI Cache Coherence

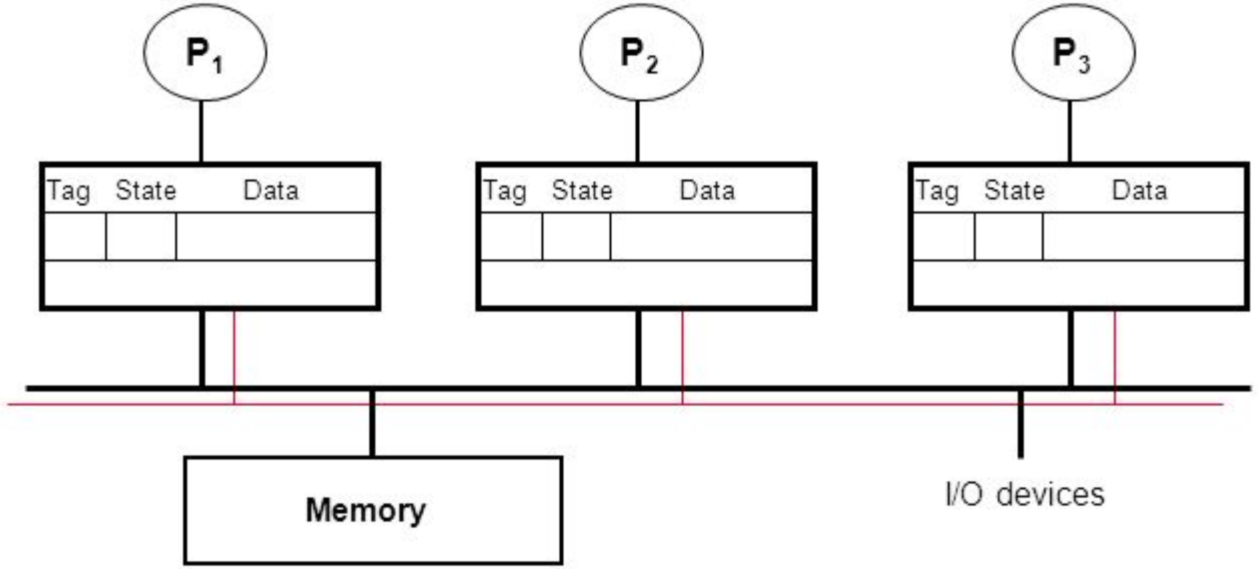


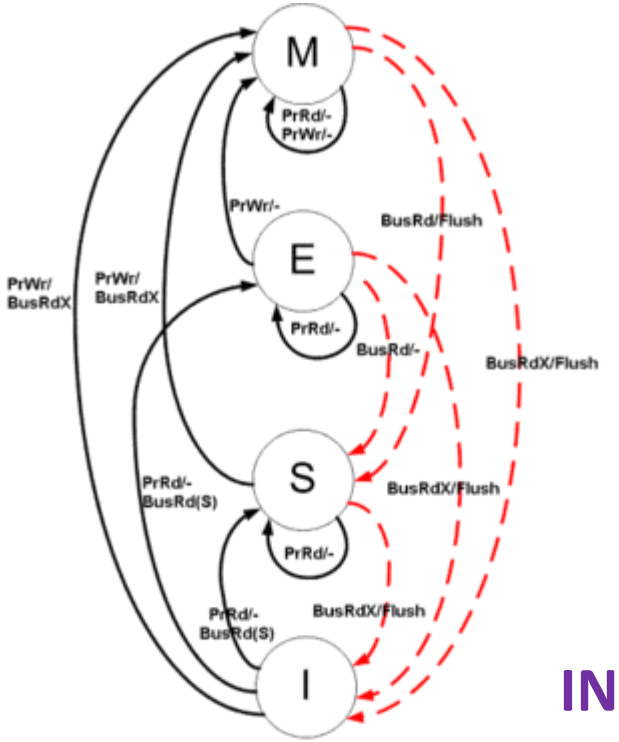


Each cache line has a state (M, E, S, I)

- Processors “snoop” bus to maintain states

Review: Basic MESI Cache Coherence



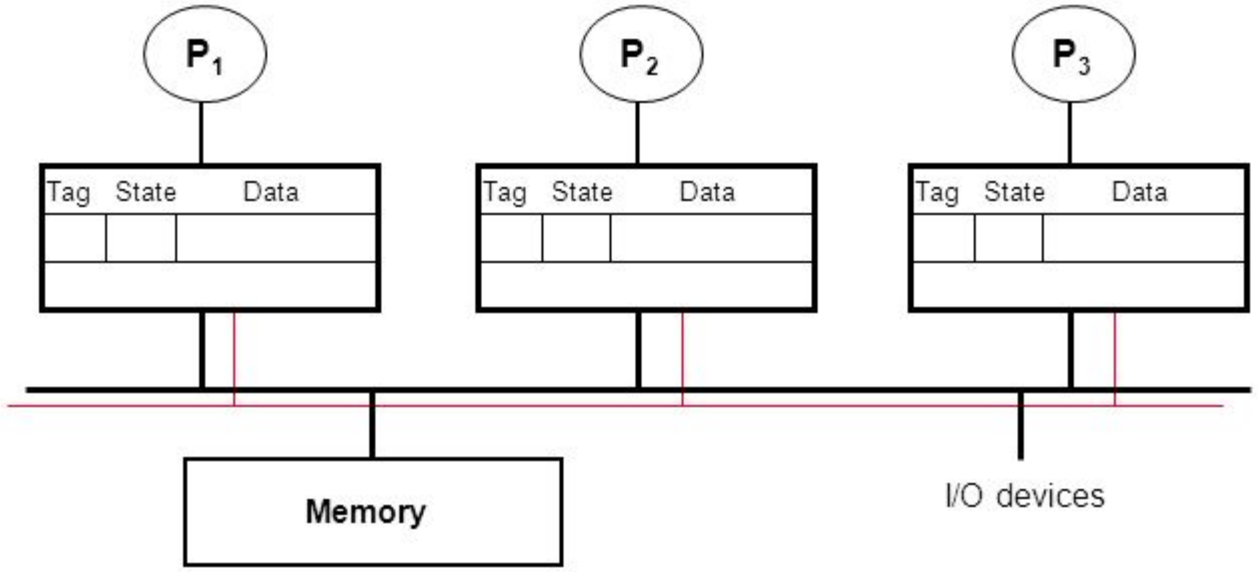


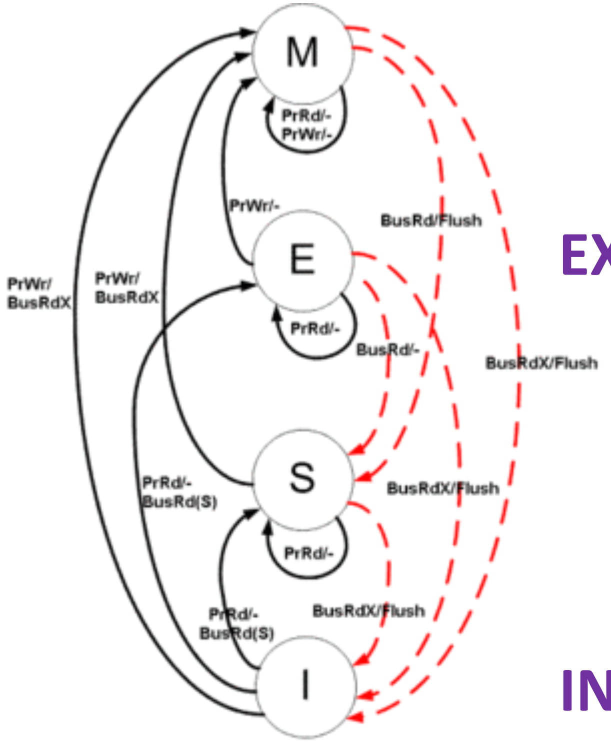
INVALID

Each cache line has a state (M, E, S, I)

- Processors "snoop" bus to maintain states
- Initially → 'I' → Invalid

Review: Basic MESI Cache Coherence





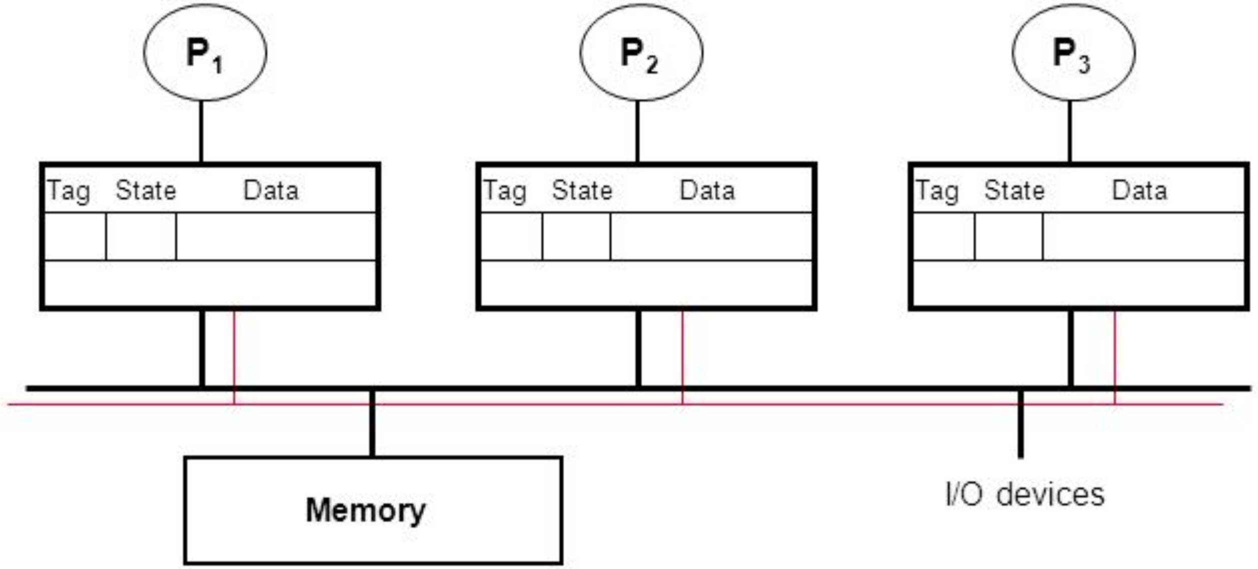
EXCLUSIVE

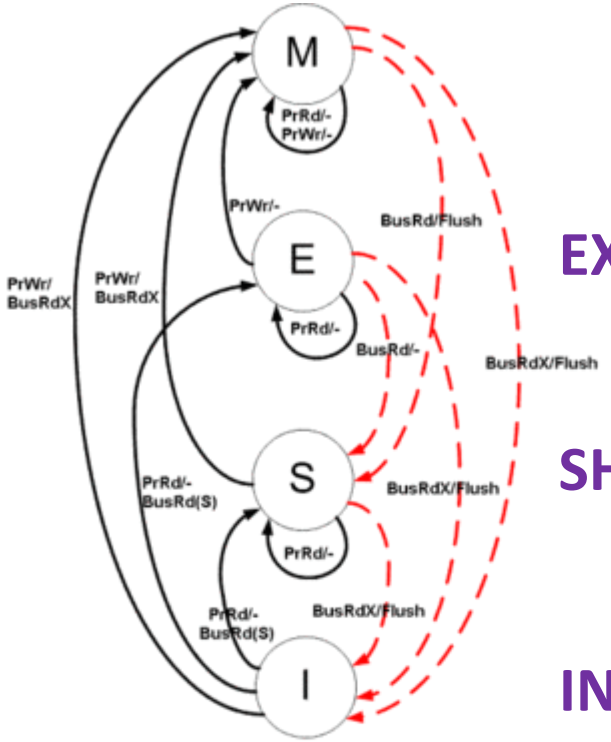
INVALID

Each cache line has a state (M, E, S, I)

- Processors "snoop" bus to maintain states
- Initially → 'I' → Invalid
- Read one → 'E' → exclusive

Review: Basic MESI Cache Coherence





EXCLUSIVE

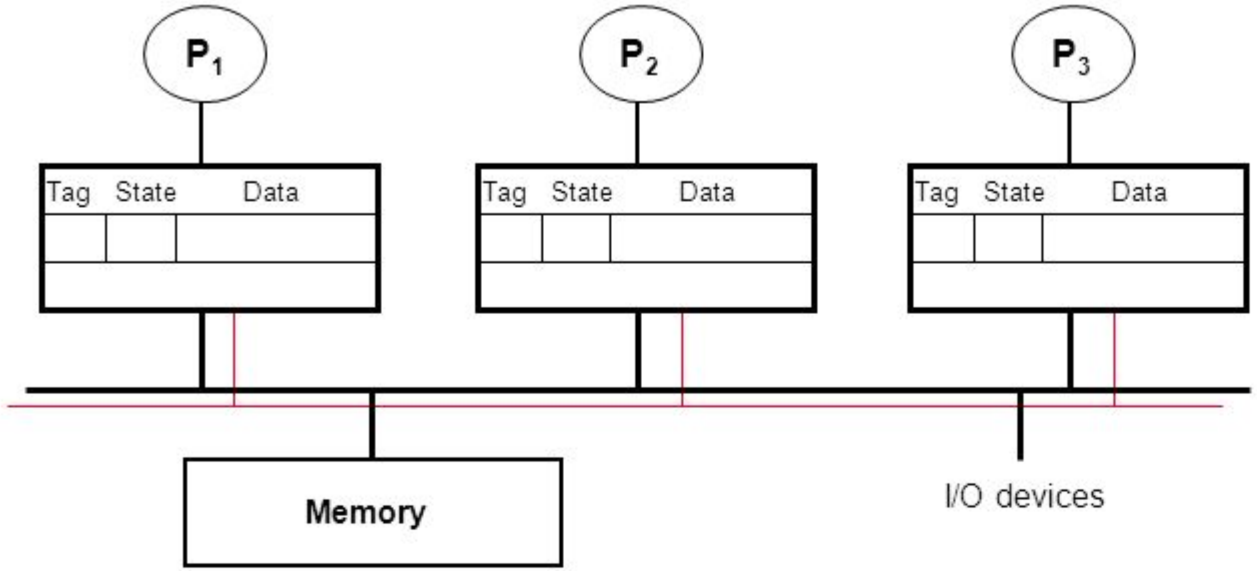
SHARED

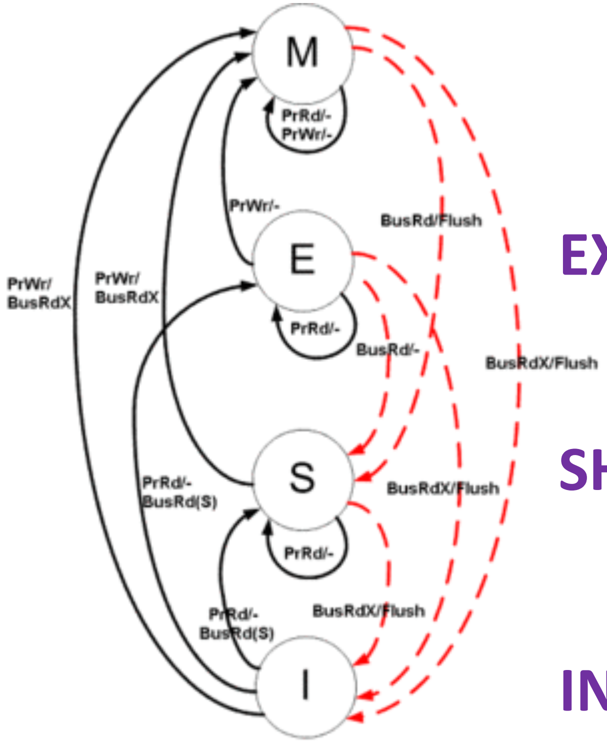
INVALID

Each cache line has a state (M, E, S, I)

- Processors “snoop” bus to maintain states
- Initially → ‘I’ → Invalid
- Read one → ‘E’ → exclusive
- Reads → ‘S’ → multiple copies possible

Review: Basic MESI Cache Coherence





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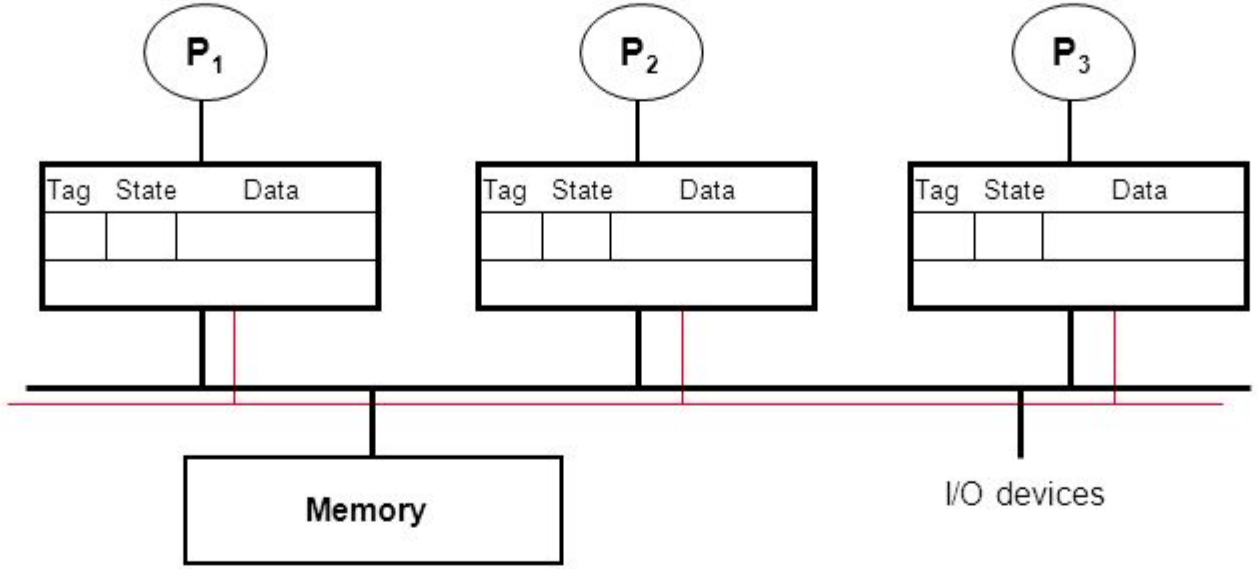
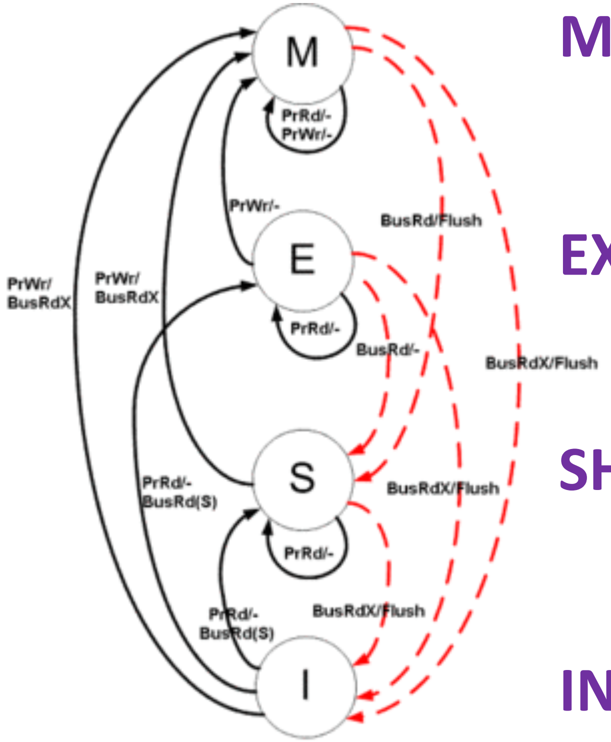
- Reads → ‘S’ → multiple copies possible
- Write → ‘M’ → single copy → lots of cache coherence traffic

Review: Basic MESI Cache Coherence





MODIFIED

EXCLUSIVE

SHARED

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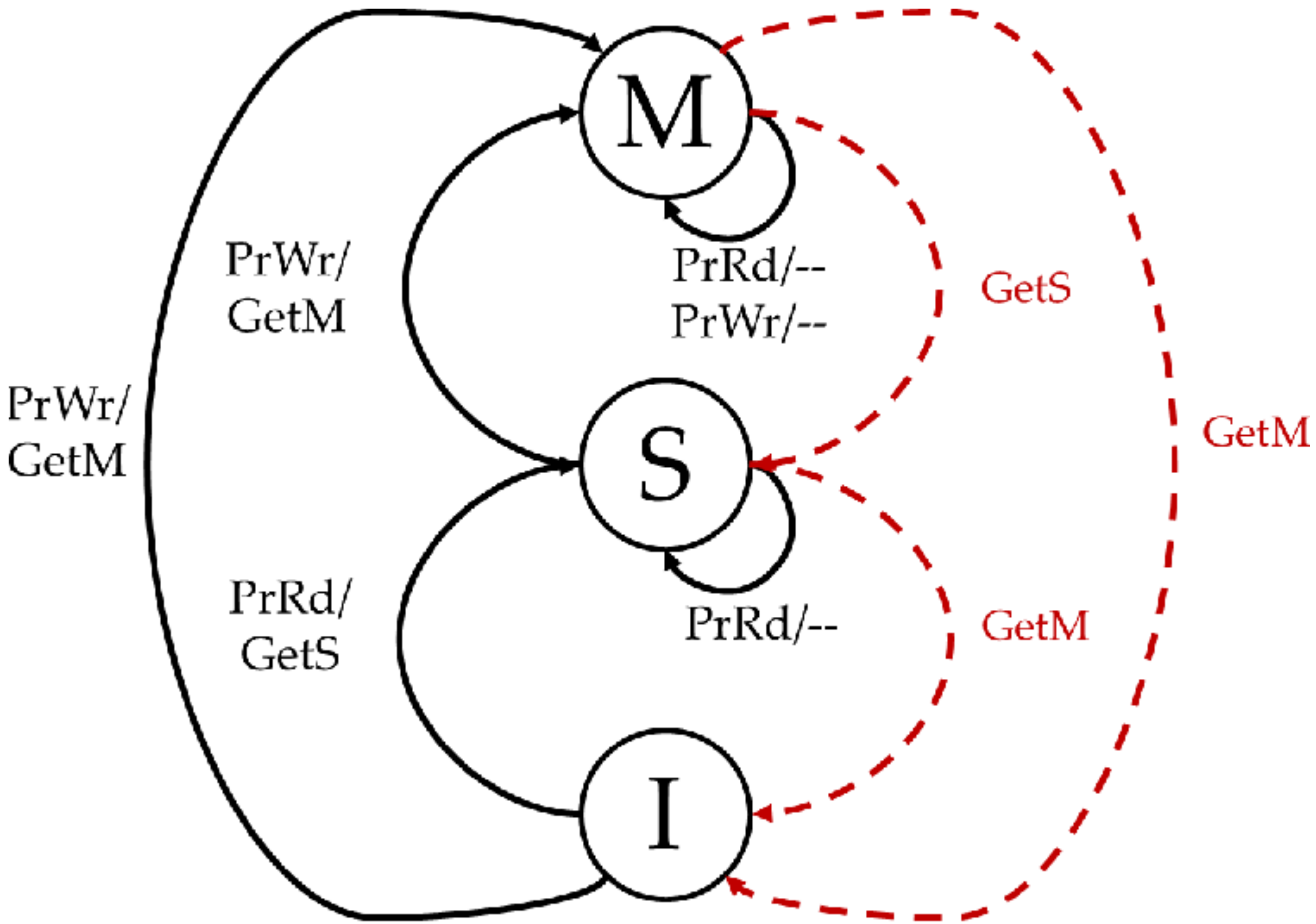
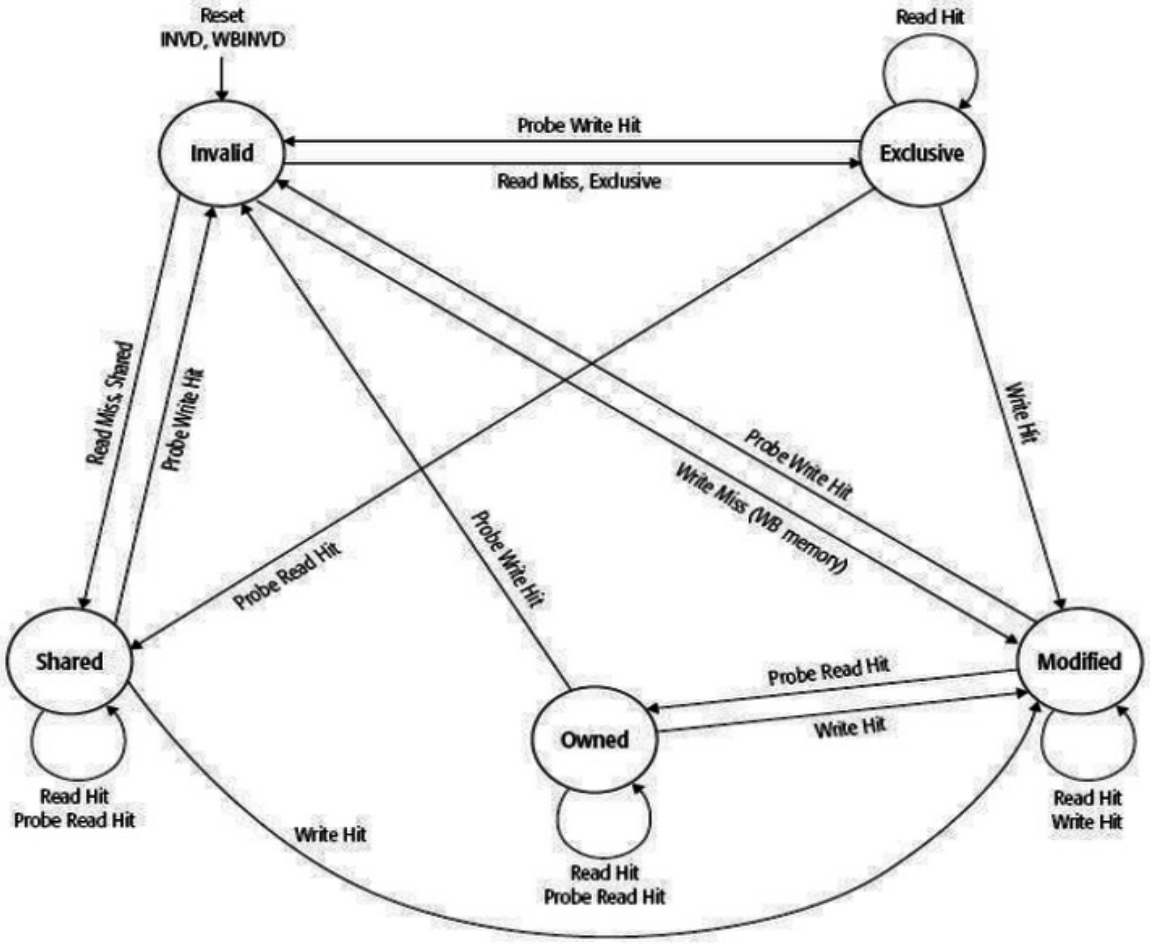
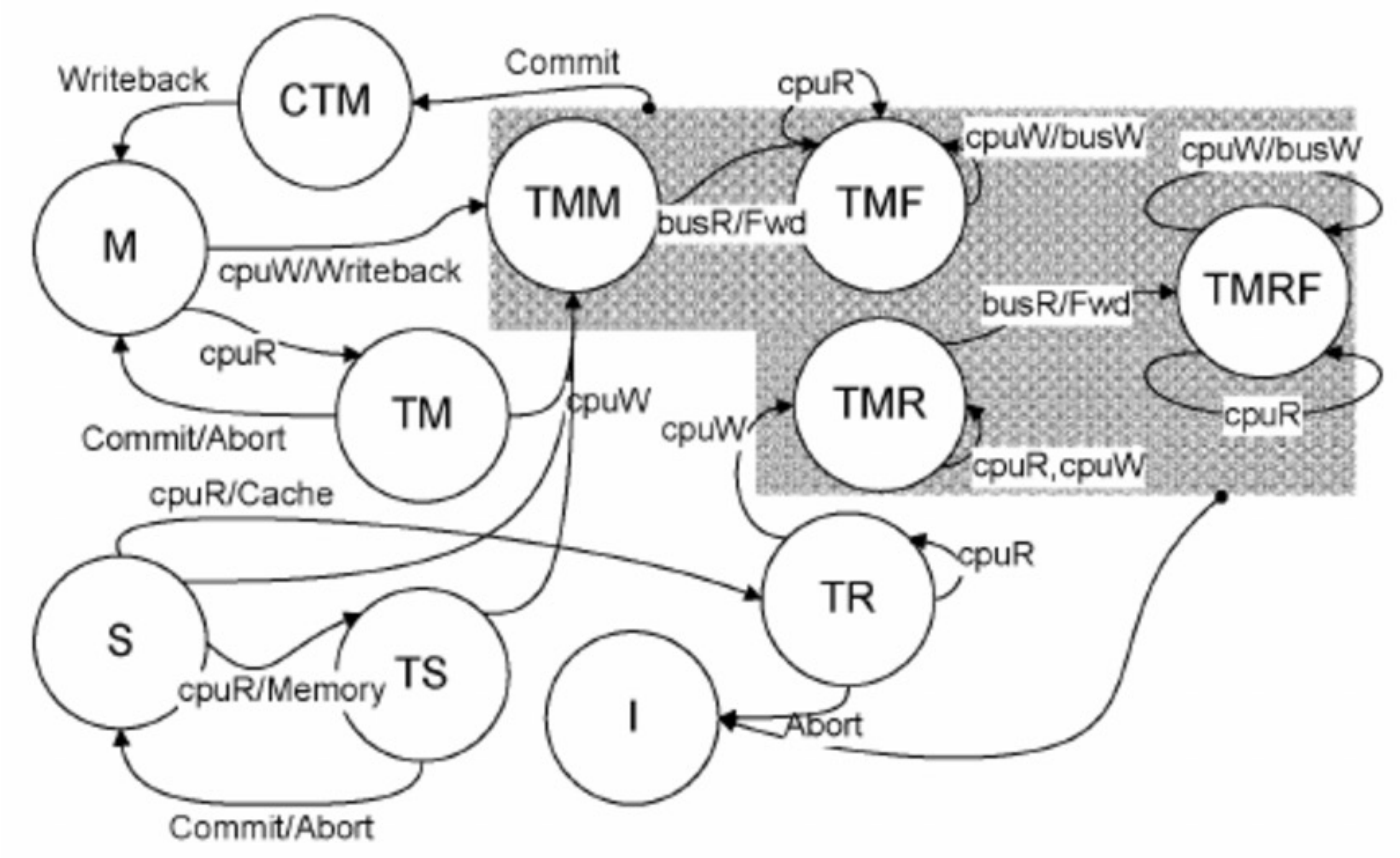
Other Coherence Protocols: MSI



Other Coherence Protocols: MOESI



Other Coherence Protocols: FRMSI

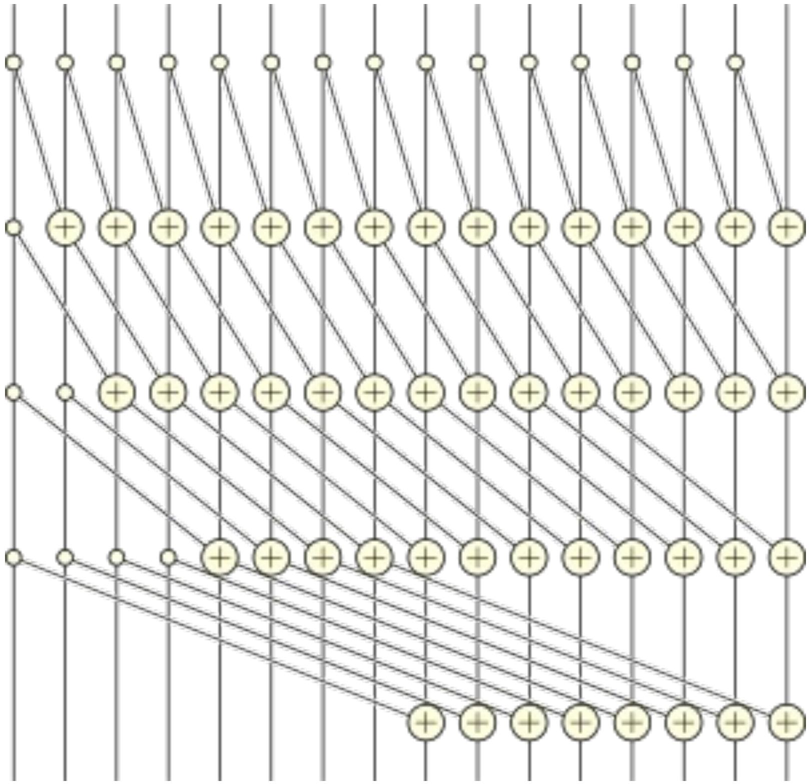


On Work-efficiency

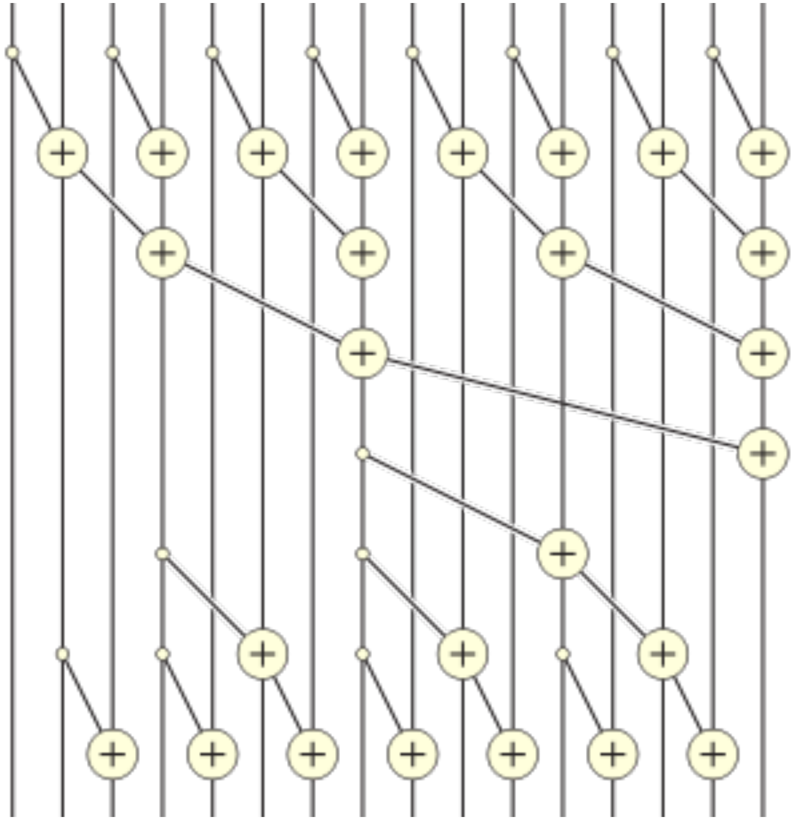
- ***Work efficient*** *(informal)**:***
  - Performs within a constant factor of the total amount of work
  - In the same complexity class as serial version
- For prefix sum, this is O(n)

On Work-efficiency

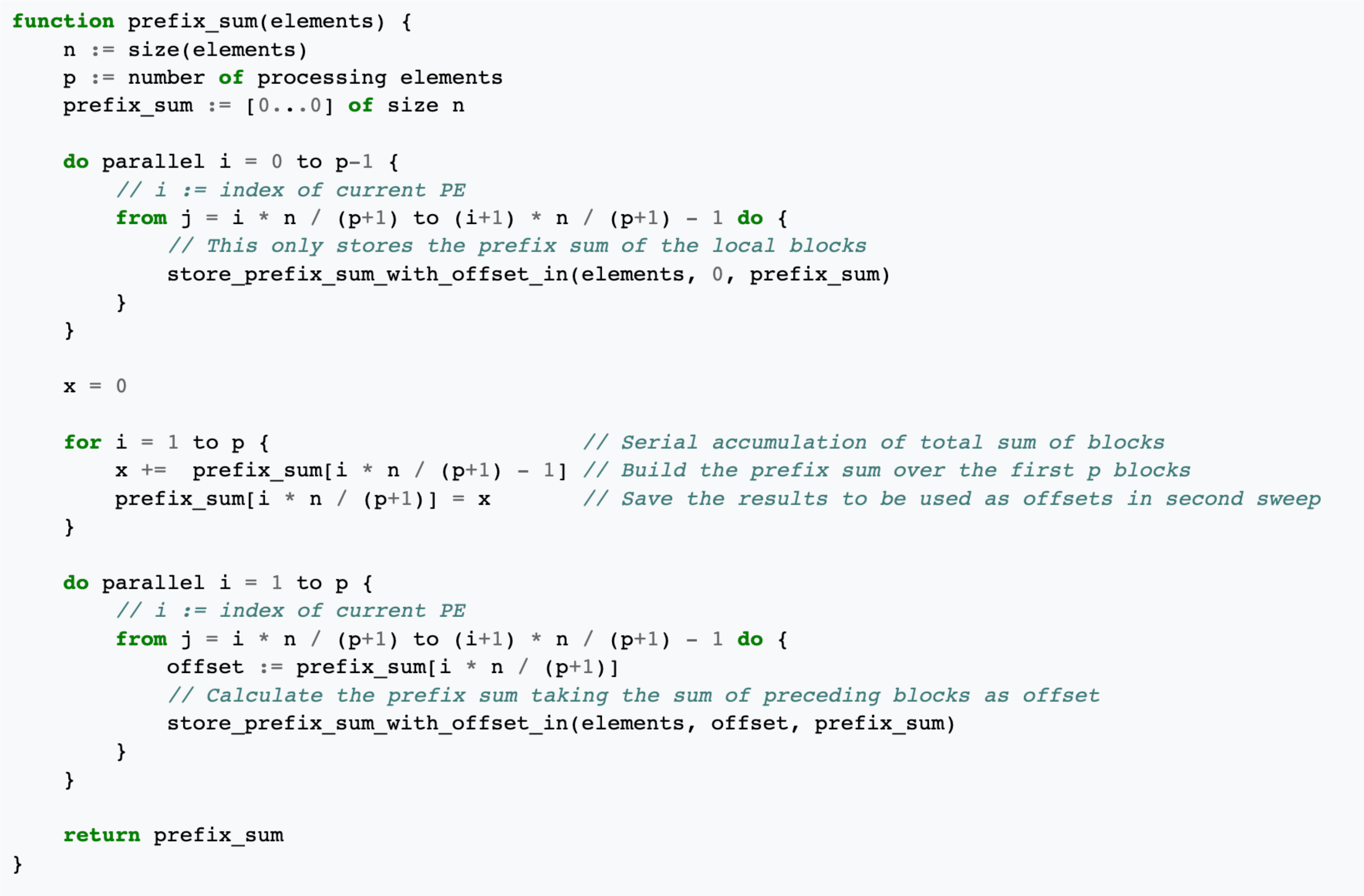
Work-inefficient (does log_n more work asymptotically)



Work-efficient (within a constant factor 2 of the seq)



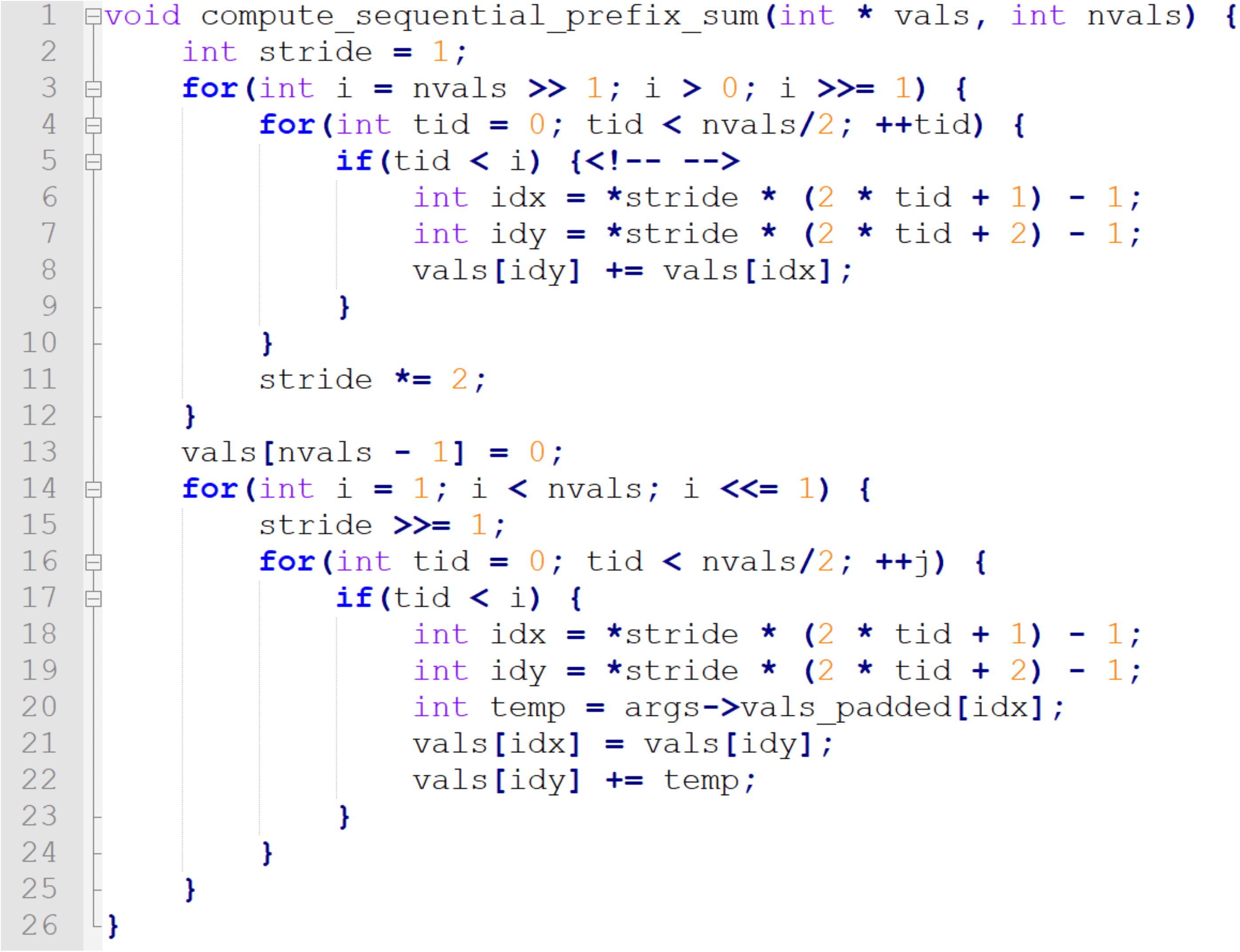
Is the “shared two-level algorithm” work efficient?



```
function prefix_sum(elements) {
    n := size(elements)
    p := number of processing elements
    prefix_sum := [0...0] of size n

    do parallel i = 0 to p-1 {
        // i := index of current PE
        from j = i * n / (p+1) to (i+1) * n / (p+1) - 1 do {
            // This only stores the prefix sum of the local blocks
            store_prefix_sum_with_offset_in(elements, 0, prefix_sum)
        }
    }

    x = 0

    for i = 1 to p {                        // Serial accumulation of total sum of blocks
        x +=  prefix_sum[i * n / (p+1) - 1] // Build the prefix sum over the first p blocks
        prefix_sum[i * n / (p+1)] = x       // Save the results to be used as offsets in second sweep
    }

    do parallel i = 1 to p {
        // i := index of current PE
        from j = i * n / (p+1) to (i+1) * n / (p+1) - 1 do {
            offset := prefix_sum[i * n / (p+1)]
            // Calculate the prefix sum taking the sum of preceding blocks as offset
            store_prefix_sum_with_offset_in(elements, offset, prefix_sum)
        }
    }

    return prefix_sum
}
```

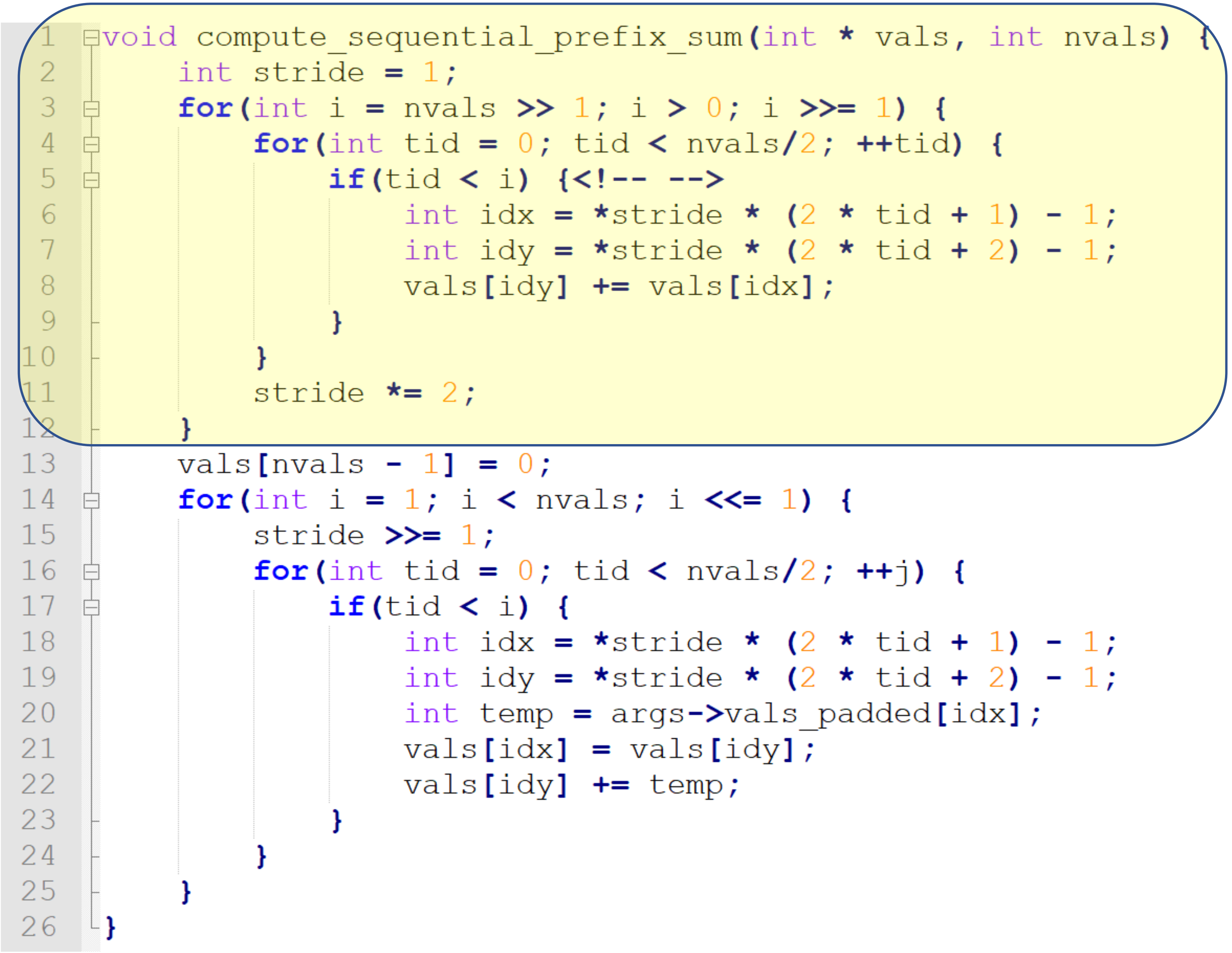
O(n+p)
Will I accept it anyway?
yes

Lab 1: Algorithm in Sequential Context



```
void compute_sequential_prefix_sum(int * vals, int nvals) {
    int stride = 1;
    for(int i = nvals >> 1; i > 0; i >>= 1) {
        for(int tid = 0; tid < nvals/2; ++tid) {
            if(tid < i) {<!-- -->
                int idx = *stride * (2 * tid + 1) - 1;
                int idy = *stride * (2 * tid + 2) - 1;
                vals[idy] += vals[idx];
            }
        }
        stride *= 2;
    }
    vals[nvals - 1] = 0;
    for(int i = 1; i < nvals; i <<= 1) {
        stride >>= 1;
        for(int tid = 0; tid < nvals/2; ++j) {
            if(tid < i) {
                int idx = *stride * (2 * tid + 1) - 1;
                int idy = *stride * (2 * tid + 2) - 1;
                int temp = args->vals_padded[idx];
                vals[idx] = vals[idy];
                vals[idy] += temp;
            }
        }
    }
}
```

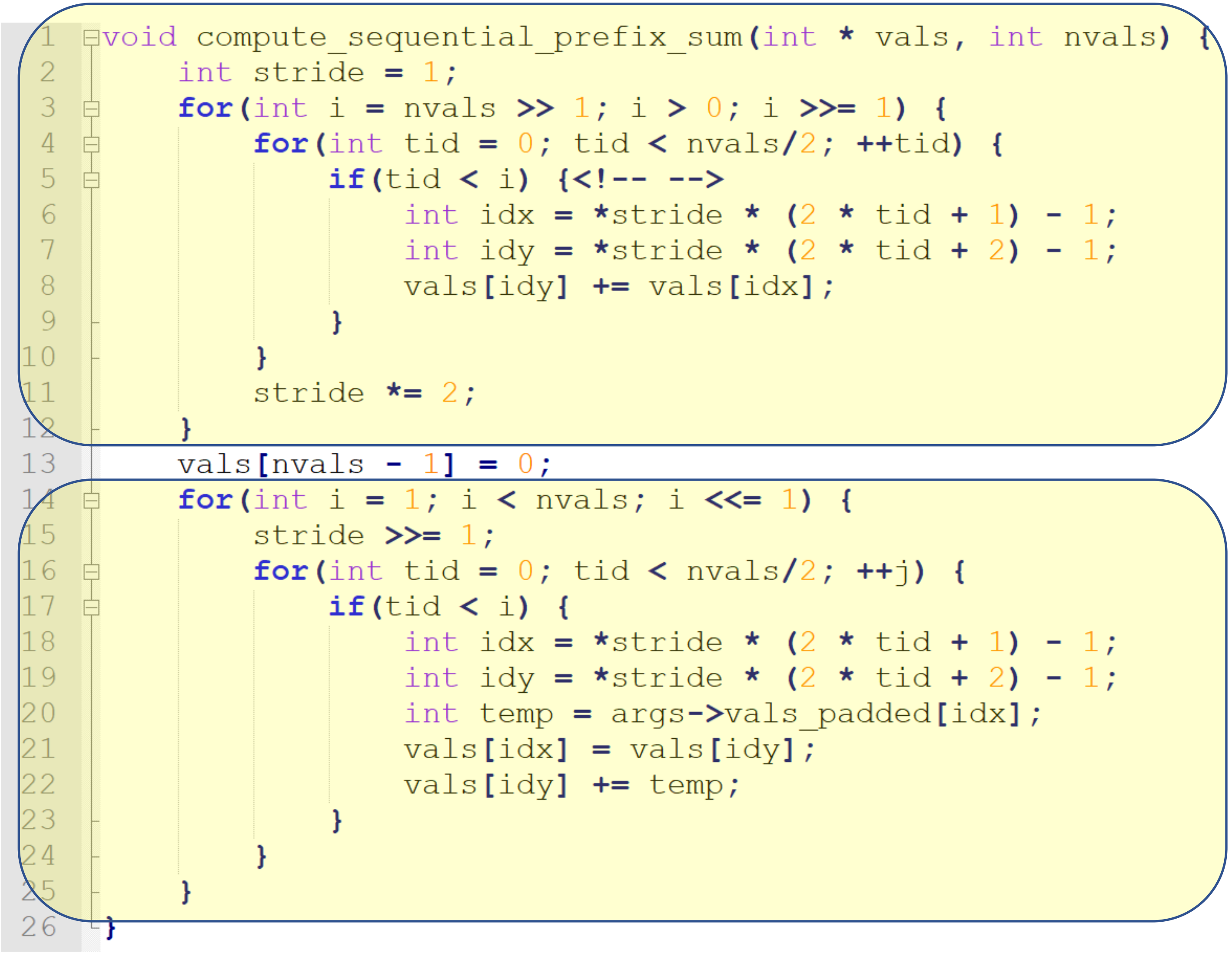
Lab 1: Algorithm in Sequential Context



```
void compute_sequential_prefix_sum(int * vals, int nvals) {
    int stride = 1;
    for(int i = nvals >> 1; i > 0; i >>= 1) {
        for(int tid = 0; tid < nvals/2; ++tid) {
            if(tid < i) {<!-- -->
                int idx = *stride * (2 * tid + 1) - 1;
                int idy = *stride * (2 * tid + 2) - 1;
                vals[idy] += vals[idx];
            }
        }
        stride *= 2;
    }
    vals[nvals - 1] = 0;
    for(int i = 1; i < nvals; i <<= 1) {
        stride >>= 1;
        for(int tid = 0; tid < nvals/2; ++j) {
            if(tid < i) {
                int idx = *stride * (2 * tid + 1) - 1;
                int idy = *stride * (2 * tid + 2) - 1;
                int temp = args->vals_padded[idx];
                vals[idx] = vals[idy];
                vals[idy] += temp;
            }
        }
    }
}
```

Upsweep

Lab 1: Algorithm in Sequential Context



```
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    int stride = 1;
    for(int i = nvals >> 1; i > 0; i >>= 1) {
        for(int tid = 0; tid < nvals/2; ++tid) {
            if(tid < i) {<!-- -->
                int idx = *stride * (2 * tid + 1) - 1;
                int idy = *stride * (2 * tid + 2) - 1;
                vals[idy] += vals[idx];
            }
        }
        stride *= 2;
    }
    vals[nvals - 1] = 0;
    for(int i = 1; i < nvals; i <<= 1) {
        stride >>= 1;
        for(int tid = 0; tid < nvals/2; ++j) {
            if(tid < i) {
                int idx = *stride * (2 * tid + 1) - 1;
                int idy = *stride * (2 * tid + 2) - 1;
                int temp = args->vals_padded[idx];
                vals[idx] = vals[idy];
                vals[idy] += temp;
            }
        }
    }
}
```
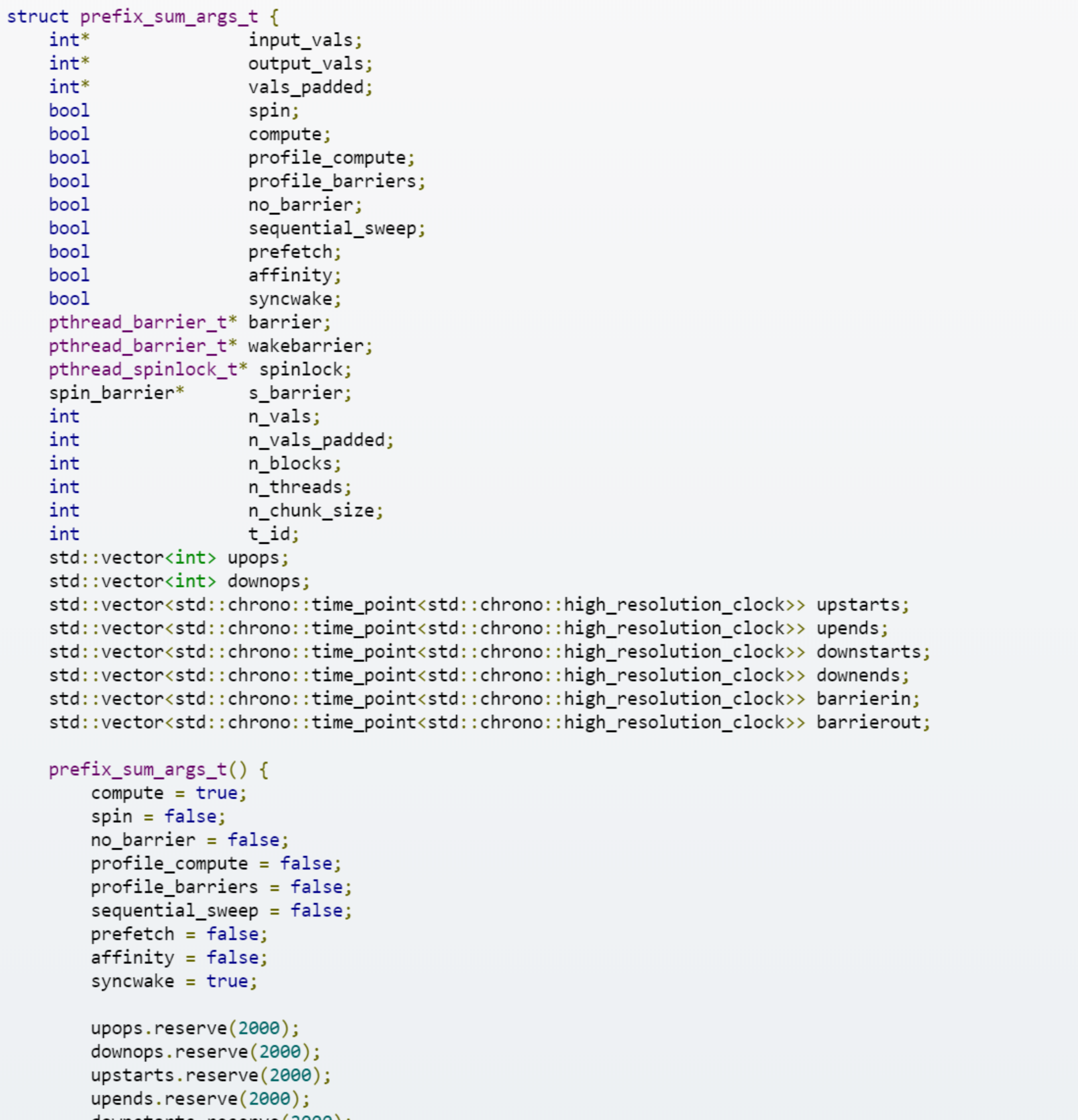

Upsweep

Downsweep

On Instrumentation

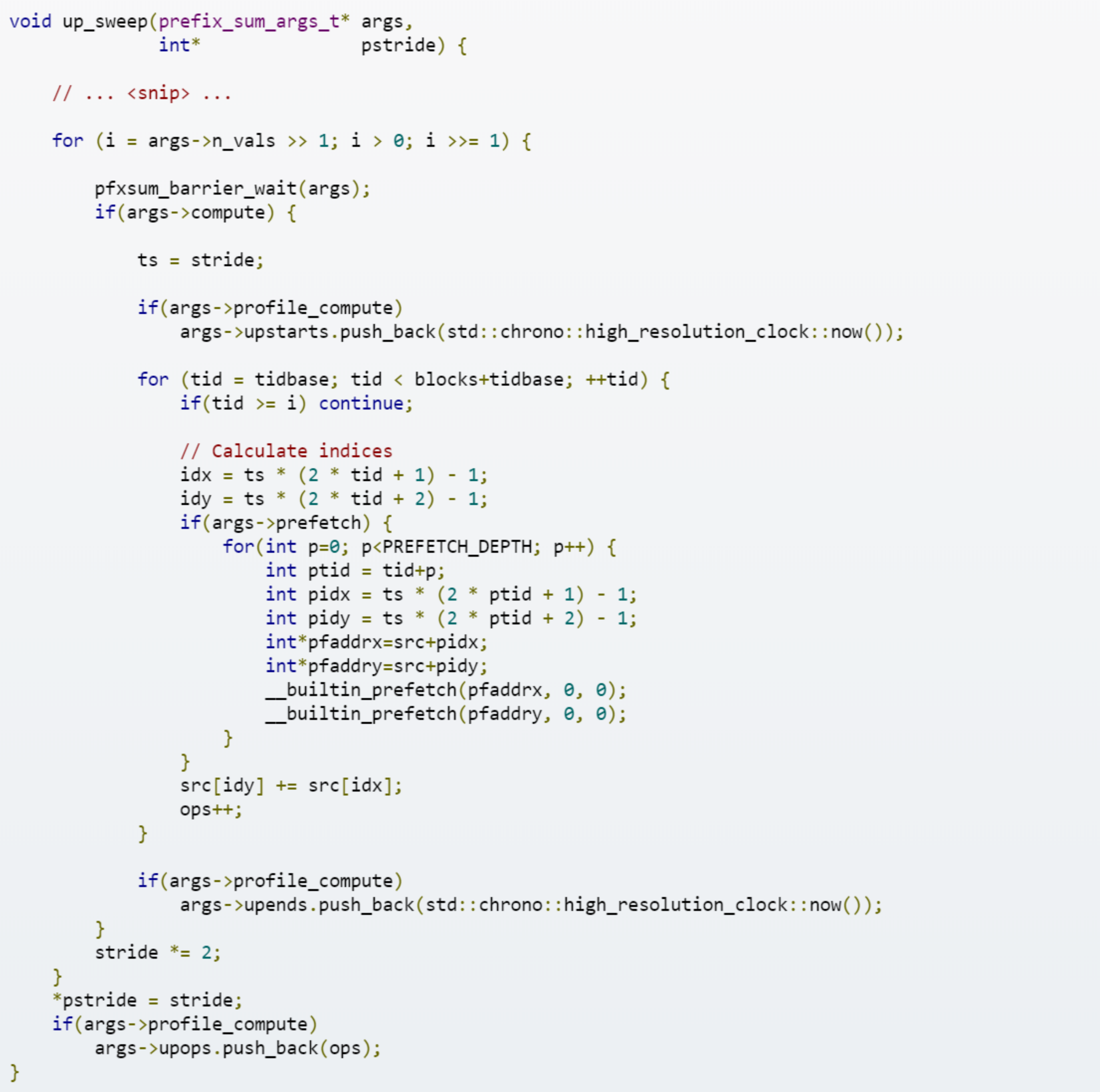
On Instrumentation



```
struct prefix_sum_args_t {
    int*               input_vals;
    int*               output_vals;
    int*               vals_padded;
    bool               spin;
    bool               compute;
    bool               profile_compute;
    bool               profile_barriers;
    bool               no_barrier;
    bool               sequential_sweep;
    bool               prefetch;
    bool               affinity;
    bool               syncwake;
    pthread_barrier_t* barrier;
    pthread_barrier_t* wakebarrier;
    pthread_spinlock_t* spinlock;
    spin_barrier*      s_barrier;
    int                n_vals;
    int                n_vals_padded;
    int                n_blocks;
    int                n_threads;
    int                n_chunk_size;
    int                t_id;
    std::vector<int> upops;
    std::vector<int> downops;
    std::vector<std::chrono::time_point<std::chrono::high_resolution_clock>> upstarts;
    std::vector<std::chrono::time_point<std::chrono::high_resolution_clock>> upends;
    std::vector<std::chrono::time_point<std::chrono::high_resolution_clock>> downstarts;
    std::vector<std::chrono::time_point<std::chrono::high_resolution_clock>> downends;
    std::vector<std::chrono::time_point<std::chrono::high_resolution_clock>> barrierin;
    std::vector<std::chrono::time_point<std::chrono::high_resolution_clock>> barrierout;

    prefix_sum_args_t() {
        compute = true;
        spin = false;
        no_barrier = false;
        profile_compute = false;
        profile_barriers = false;
        sequential_sweep = false;
        prefetch = false;
        affinity = false;
        syncwake = true;

        upops.reserve(2000);
        downops.reserve(2000);
        upstarts.reserve(2000);
        upends.reserve(2000);
        downstarts.reserve(2000);
```

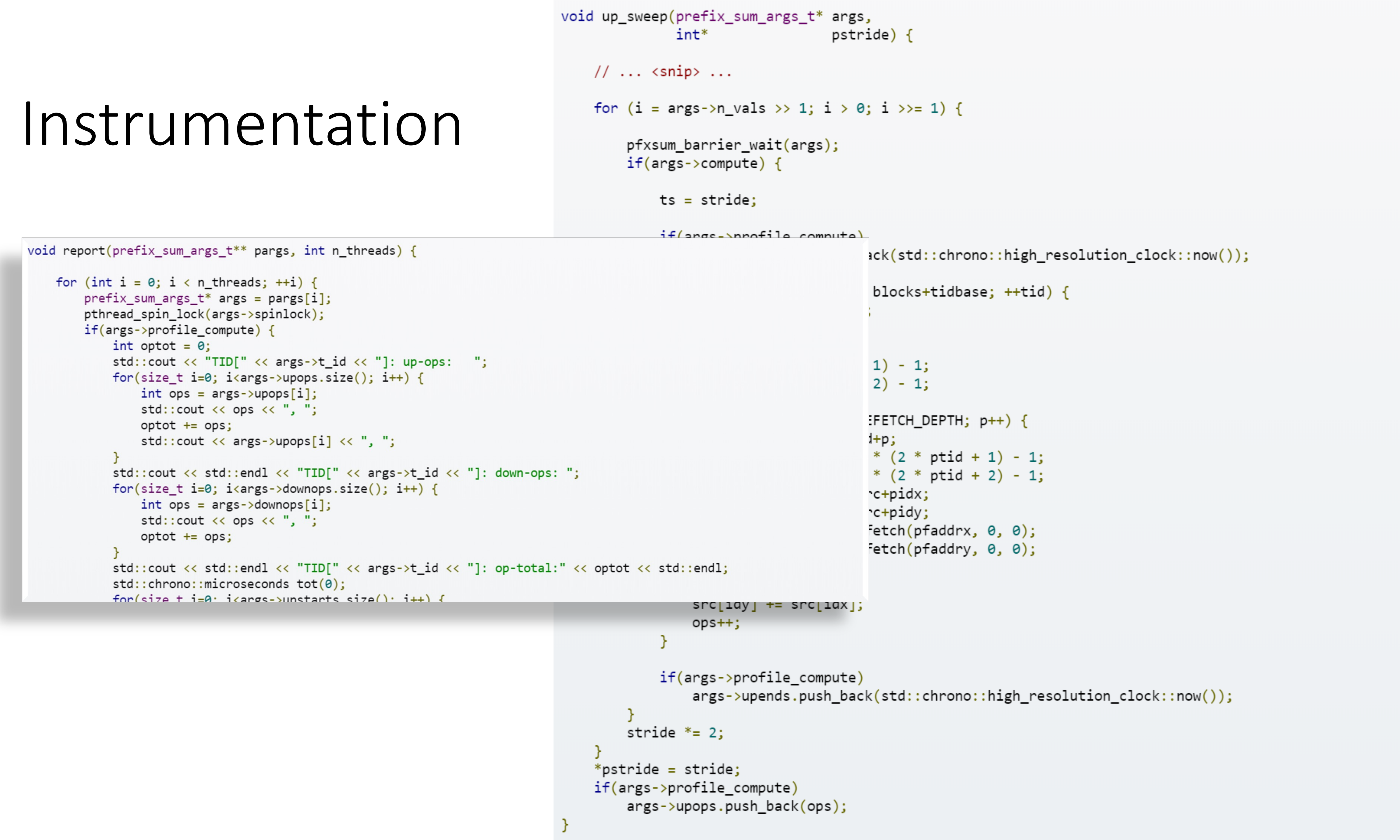
Instrumentation

Instrumentation



```
void up_sweep(prefix_sum_args_t* args,
              int*               pstride) {

  // ... <snip> ...

  for (i = args->n_vals >> 1; i > 0; i >>= 1) {

    pfxsum_barrier_wait(args);
    if(args->compute) {

      ts = stride;

      if(args->profile_compute)
        args->upstarts.push_back(std::chrono::high_resolution_clock::now());

      for (tid = tidbase; tid < blocks+tidbase; ++tid) {
        if(tid >= i) continue;

        // Calculate indices
        idx = ts * (2 * tid + 1) - 1;
        idy = ts * (2 * tid + 2) - 1;
        if(args->prefetch) {
          for(int p=0; p<PREFETCH_DEPTH; p++) {
            int ptid = tid+p;
            int pidx = ts * (2 * ptid + 1) - 1;
            int pidy = ts * (2 * ptid + 2) - 1;
            int*pfaddrx=src+pidx;
            int*pfaddry=src+pidy;
            __builtin_prefetch(pfaddrx, 0, 0);
            __builtin_prefetch(pfaddry, 0, 0);
          }
        }
        src[idy] += src[idx];
        ops++;
      }

      if(args->profile_compute)
        args->upends.push_back(std::chrono::high_resolution_clock::now());
    }
    stride *= 2;
  }
  *pstride = stride;
  if(args->profile_compute)
    args->upops.push_back(ops);
}
```

Instrumentation

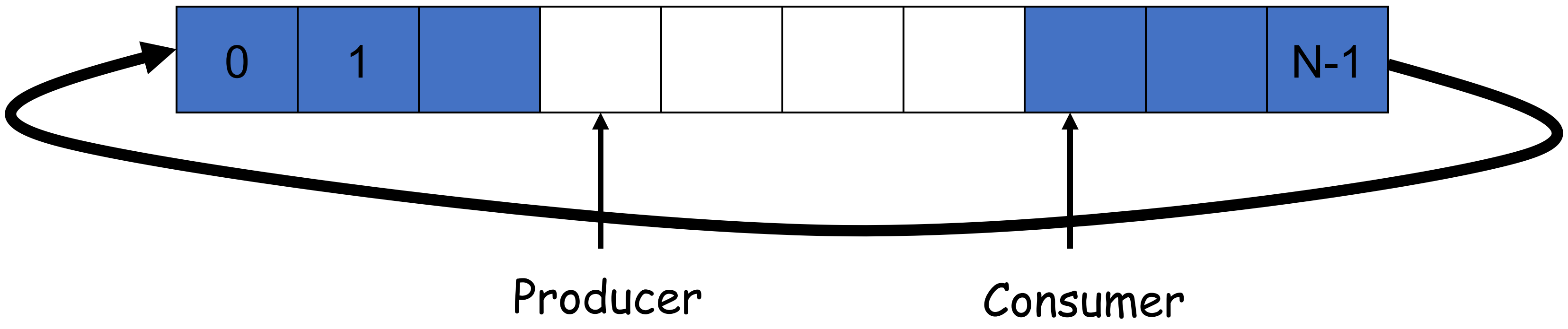


```
void report(prefix_sum_args_t** pargs, int n_threads) {

    for (int i = 0; i < n_threads; ++i) {
        prefix_sum_args_t* args = pargs[i];
        pthread_spin_lock(args->spinlock);
        if(args->profile_compute) {
            int optot = 0;
            std::cout << "TID[" << args->t_id << "]: up-ops:   ";
            for(size_t i=0; i<args->upops.size(); i++) {
                int ops = args->upops[i];
                std::cout << ops << ", ";
                optot += ops;
                std::cout << args->upops[i] << ", ";
            }
            std::cout << std::endl << "TID[" << args->t_id << "]: down-ops: ";
            for(size_t i=0; i<args->downops.size(); i++) {
                int ops = args->downops[i];
                std::cout << ops << ", ";
                optot += ops;
            }
            std::cout << std::endl << "TID[" << args->t_id << "]: op-total:" << optot << std::endl;
            std::chrono::microseconds tot(0);
            for(size_t i=0; i<args->upstarts.size(); i++) {
```

```
void up_sweep(prefix_sum_args_t* args,
              int*                pstride) {

    // ... <snip> ...

    for (i = args->n_vals >> 1; i > 0; i >>= 1) {

        pfxsum_barrier_wait(args);
        if(args->compute) {

            ts = stride;

            if(args->profile_compute)
                                         ack(std::chrono::high_resolution_clock::now());

                                          blocks+tidbase; ++tid) {


                                         1) - 1;
                                         2) - 1;

                                         EFETCH_DEPTH; p++) {
                                         d+p;
                                          * (2 * ptid + 1) - 1;
                                          * (2 * ptid + 2) - 1;
                                         c+pidx;
                                         c+pidy;
                                         etch(pfaddrx, 0, 0);
                                         etch(pfaddry, 0, 0);

                src[idy] += src[idx];
                ops++;
            }

            if(args->profile_compute)
                args->upends.push_back(std::chrono::high_resolution_clock::now());
        }
        stride *= 2;
    }
    *pstride = stride;
    if(args->profile_compute)
        args->upops.push_back(ops);
}
```

Producer-Consumer (Bounded-Buffer) Problem

- Bounded buffer: size ‘N’
  - Access entry 0… N-1, then “wrap around” to 0 again
- Producer process writes data to buffer
  - Must not write more than ‘N’ items more than consumer “consumes”
- Consumer process reads data from buffer
  - Should not try to consume if there is no data



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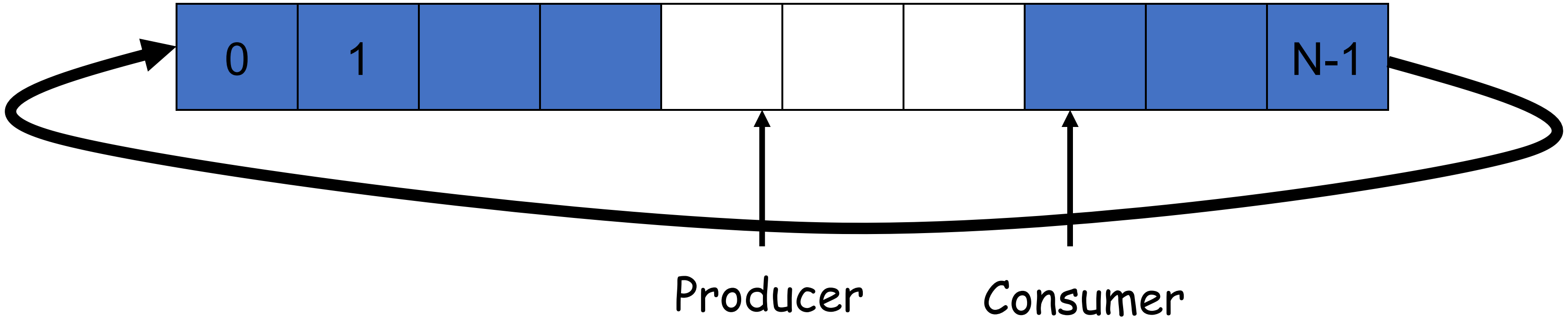
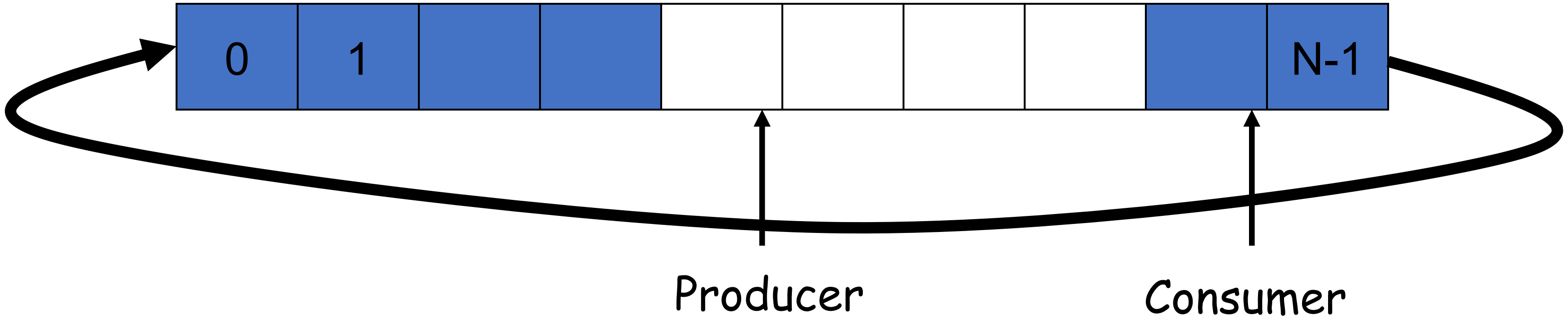


Producer-Consumer (Bounded-Buffer) Problem

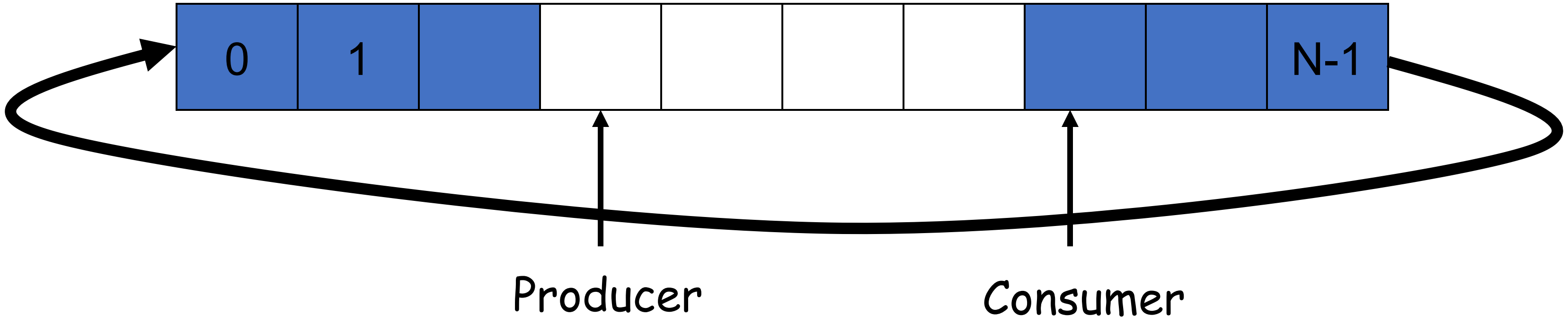
- Bounded buffer: size ‘N’
  - Access entry 0… N-1, then “wrap around” to 0 again
- Producer process writes data to buffer
  - Must not write more than ‘N’ items more than consumer “consumes”
- Consumer process reads data from buffer
  - Should not try to consume if there is no data



OK, let's write some code for this
(using locks only)

- Bounded buffer: size 'N'
  - Access entry 0… N-1, then "wrap around" to 0 again
- Producer writes data
- Consumer reads data

```
object array[N]
void enqueue(object x);
object dequeue();
```



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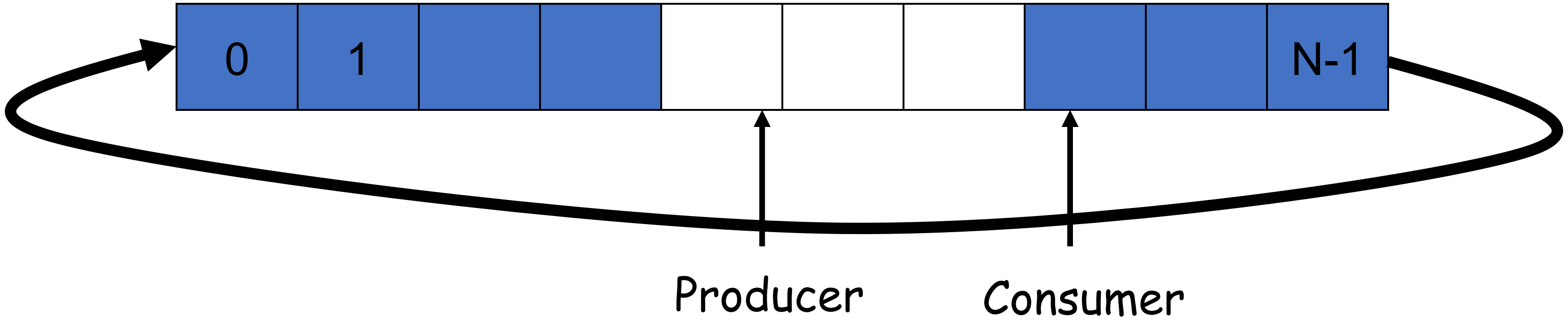

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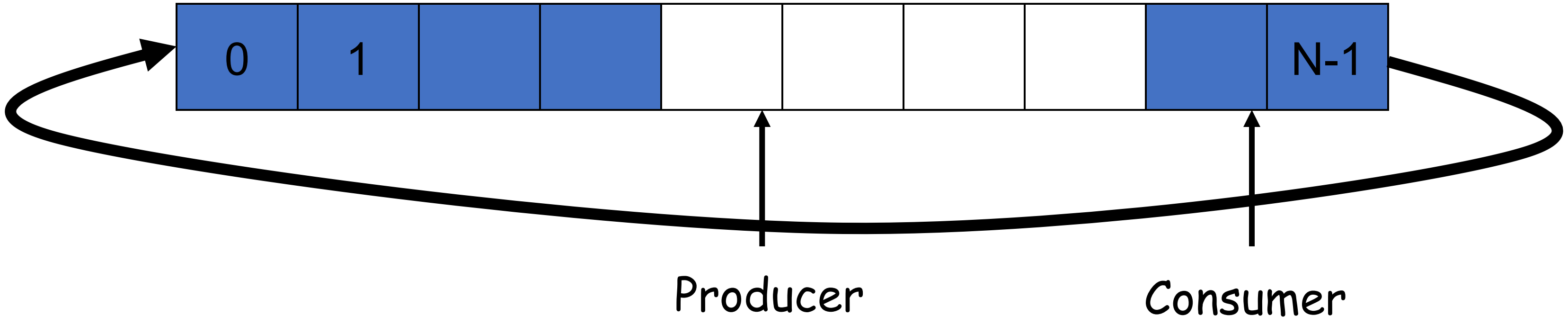

Semaphore Motivation

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- Problem with locks: mutual exclusion, but *no ordering*

Semaphore Motivation

- Problem with locks: mutual exclusion, but *no ordering*
- Inefficient for producer-consumer (and lots of other things)
  - Producer: creates a resource
  - Consumer: uses a resource
  - bounded buffer between them
  - You need synchronization for correctness, *and…*
  - Scheduling order:
    - producer waits if buffer full, consumer waits if buffer empty

Semaphores

- Synchronization variable
  - Integer value
    - Can’t access value directly
    - Must initialize to some value
      - sem_init(sem_t *s, int pshared, unsigned int value)
  - Two operations
    - sem_wait, or down(), P()
    - sem_post, or up(), V()

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    - sem_wait, or down(), P()
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```
int sem_wait(sem_t *s) {
    wait until value of semaphore s
          is greater than 0
    decrement the value of
          semaphore s by 1
}
```

```
int sem_post(sem_t *s) {
    increment the value of
          semaphore s by 1
    if there are 1 or more
            threads waiting, wake 1
}
```

Semaphores

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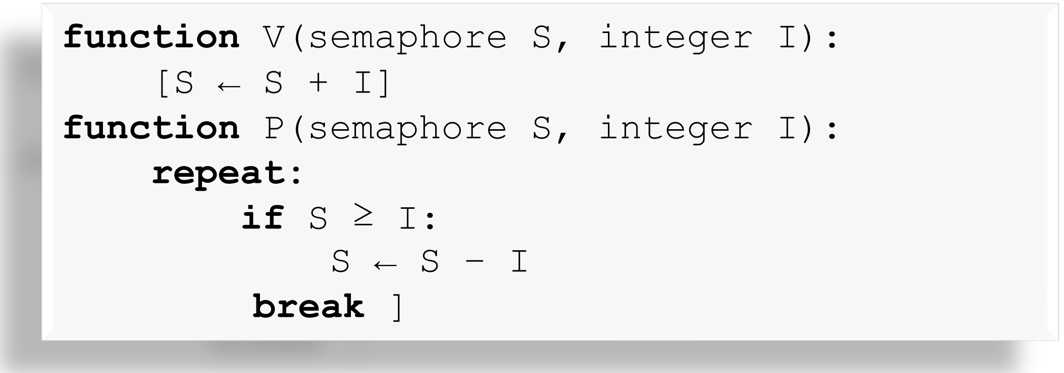

```
function V(semaphore S, integer I):
    [S ← S + I]
function P(semaphore S, integer I):
    repeat:
        if S ≥ I:
            S ← S - I
        break ]
```

```
int sem_wait(sem_t *s) {
    wait until value of semaphore s
        is greater than 0
    decrement the value of
        semaphore s by 1
}
```

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}
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Semaphore Uses

- Mutual exclusion
  - Semaphore as mutex
  - What should initial value be?

```
// initialize to X
sem_init(s, 0, X)

sem_wait(s);
// critical section
sem_post(s);
```

Semaphore Uses

- Mutual exclusion
  - Semaphore as mutex
  - What should initial value be?
    - Binary semaphore: X=1

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- Scheduling order
  - One thread waits for another

```
//thread 0
… // 1st half of computation
sem_post(s);
```

```
// thread 1

sem_wait(s);
… //2nd half of computation
```

Semaphore Uses

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// initialize to X
sem_init(s, 0, X)

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  - One thread waits for another
  - What should initial value be?

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Producer-Consumer with semaphores

- Two semaphores
    - sem_t full; // # of filled slots
    - sem_t empty; // # of empty slots

Producer-Consumer with semaphores

- Two semaphores
  - sem_t full; // # of filled slots
  - sem_t empty; // # of empty slots

```
sem_init(&full, 0, 0);
sem_init(&empty, 0, N);
```

```
producer() {
    sem_wait(empty);
    … // fill a slot
    sem_post(full);
}
```

```
consumer() {
    sem_wait(full);
    … // empty a slot
    sem_post(empty);
}
```

Producer-Consumer with semaphores

- Two semaphores
  - sem_t full; // # of filled slots
  - sem_t empty; // # of empty slots

Is this correct?

```
sem_init(&full, 0, 0);
sem_init(&empty, 0, N);
```

```
producer() {
    sem_wait(empty);
    … // fill a slot
    sem_post(full);
}
```

```
consumer() {
    sem_wait(full);
    … // empty a slot
    sem_post(empty);
}
```

Producer-Consumer with semaphores

- Two semaphores
  - sem_t full; // # of filled slots
  - sem_t empty; // # of empty slots

```
sem_init(&full, 0, 0);
sem_init(&empty, 0, N);
```

```
producer() {
    sem_wait(empty);
    … // fill a slot
    sem_post(full);
}
```

```
consumer() {
    sem_wait(full);
    … // empty a slot
    sem_post(empty);
}
```

Producer-Consumer with semaphores

- Two semaphores
  - sem_t full; // # of filled slots
  - sem_t empty; // # of empty slots

- Problem: mutual exclusion?

```
sem_init(&full, 0, 0);
sem_init(&empty, 0, N);
```

```
producer() {
    sem_wait(empty);
    … // fill a slot
    sem_post(full);
}
```

```
consumer() {
    sem_wait(full);
    … // empty a slot
    sem_post(empty);
}
```

Producer-Consumer with semaphores

- Three semaphores
  - sem_t full; // # of filled slots
  - sem_t empty; // # of empty slots
  - sem_t mutex; // mutual exclusion

```
sem_init(&full, 0, 0);
sem_init(&empty, 0, N);
sem_init(&mutex, 0, 1);
```

```
producer() {
    sem_wait(empty);
    sem_wait(&mutex);
    … // fill a slot
    sem_post(&mutex);
    sem_post(full);
}
```

```
consumer() {
    sem_wait(full);
    sem_wait(&mutex);
    … // empty a slot
    sem_post(&mutex);
    sem_post(empty);
}
```

Pthreads and Semaphores

- Type: `pthread_semaphore_t`

```
int pthread_semaphore_init(pthread_spinlock_t *lock);
int pthread_semaphore_destroy(pthread_spinlock_t *lock);
…
```

- ?????

Pthreads and Semaphores

Pthreads and Semaphores

- No pthread_semaphore_t!

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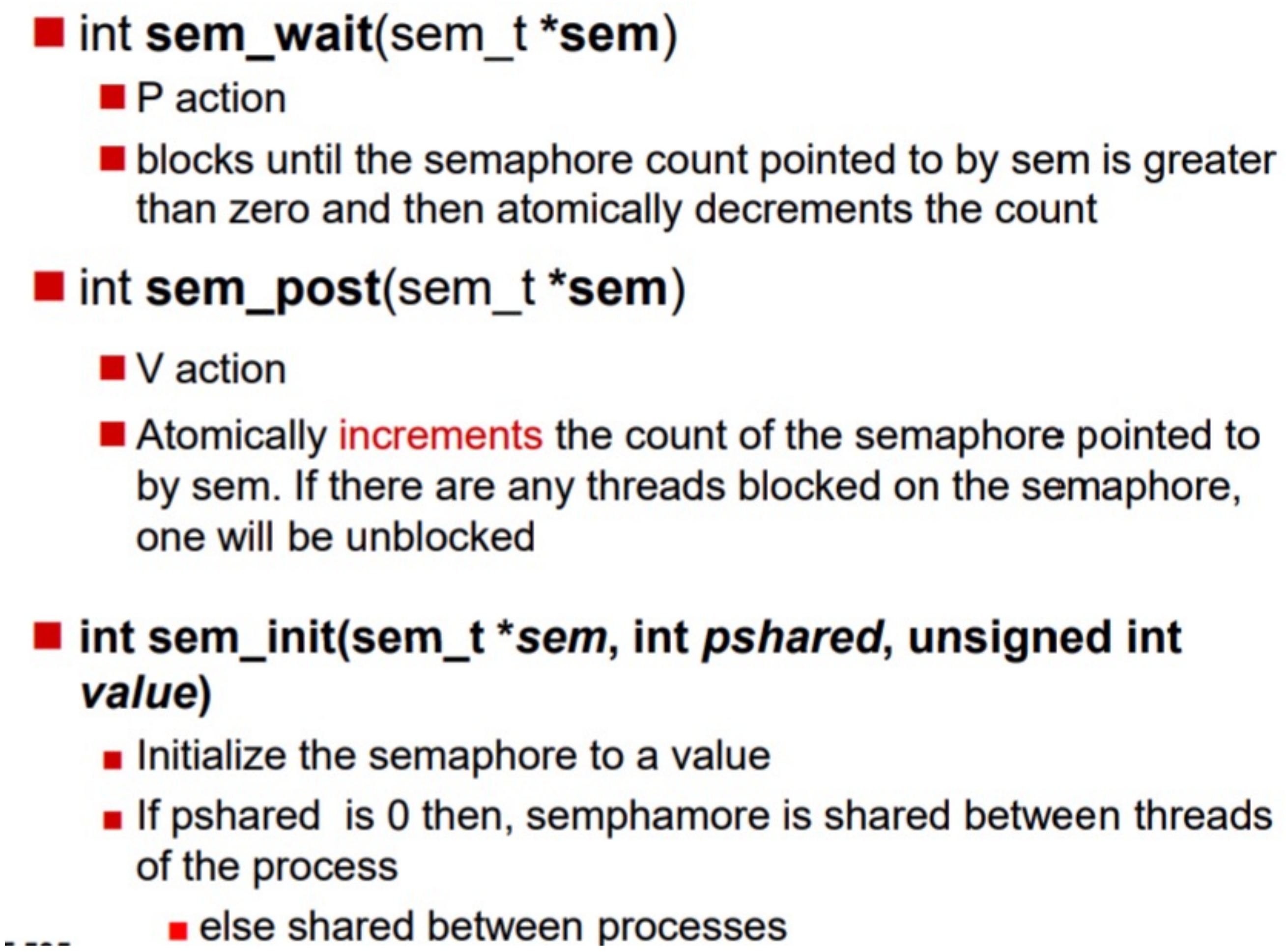
Pthreads and Semaphores

- No pthread_semaphore_t!
- POSIX does define standard
- #include <semaphore.h>



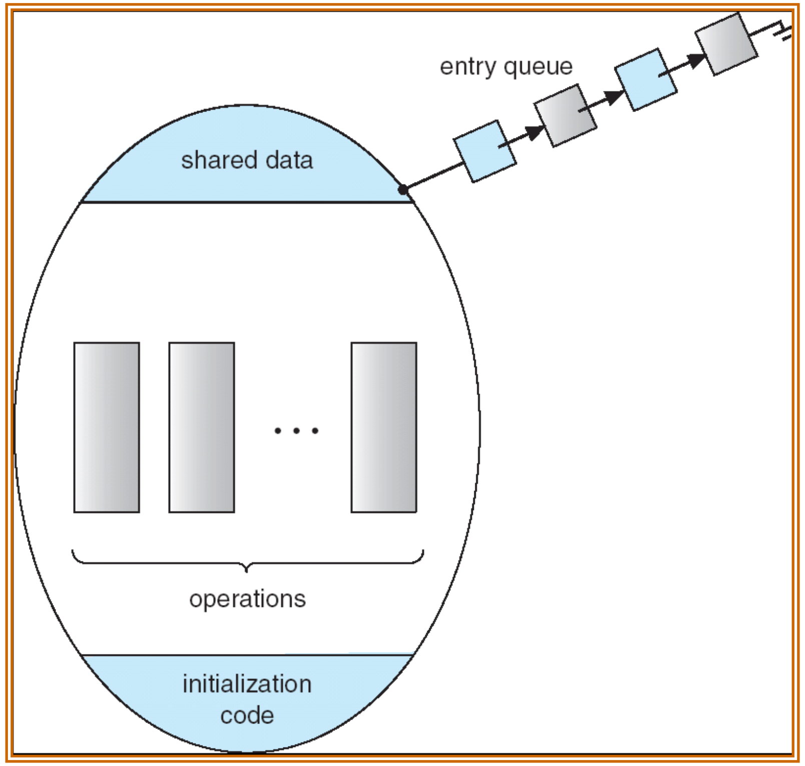
- int **sem_wait**(sem_t ***sem**)
  - P action
  - blocks until the semaphore count pointed to by sem is greater than zero and then atomically decrements the count
- int **sem_post**(sem_t ***sem**)
  - V action
  - Atomically increments the count of the semaphore pointed to by sem. If there are any threads blocked on the semaphore, one will be unblocked
- **int sem_init(sem_t ****sem***, int** ***pshared***, unsigned int** ***value*****)**
  - Initialize the semaphore to a value
  - If pshared is 0 then, semphamore is shared between threads of the process
    - else shared between processes

What is a monitor?

What is a monitor?

- ❑ Monitor: one big lock for set of operations/ methods
- ❑ Language-level implementation of mutex

- Entry procedure: called from outside
- Internal procedure: called within monitor
- Wait within monitor releases lock



Many variants…

Pthreads and conditions/monitors

- Type `pthread_cond_t`

```
int pthread_cond_init(pthread_cond_t *cond,
                      const pthread_condattr_t *attr);
int pthread_cond_destroy(pthread_cond_t *cond);
int pthread_cond_wait(pthread_cond_t *cond,
                      pthread_mutex_t *mutex);
int pthread_cond_signal(pthread_cond_t *cond);
int pthread_cond_broadcast(pthread_cond_t *cond);
```

Pthreads and conditions/monitors

Why the pthread_mutex_t parameter for pthread_cond_wait?

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Java:

`synchronized keyword`

`wait()/notify()/notifyAll()`

C#: Monitor class

`Enter()/Exit()/`

`Pulse()/PulseAll()`

Does this code work?

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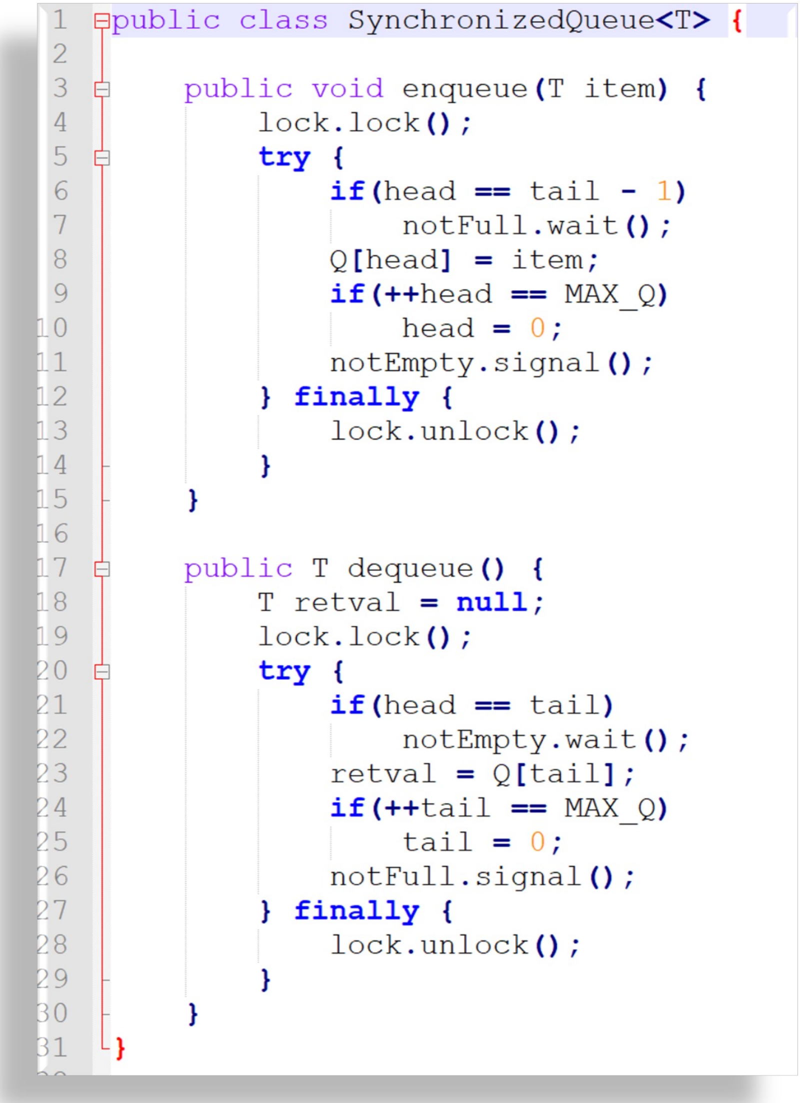

```
public class SynchronizedQueue<T> {

    public void enqueue(T item) {
        lock.lock();
        try {
            if(head == tail - 1)
                notFull.wait();
            Q[head] = item;
            if(++head == MAX_Q)
                head = 0;
            notEmpty.signal();
        } finally {
            lock.unlock();
        }
    }

    public T dequeue() {
        T retval = null;
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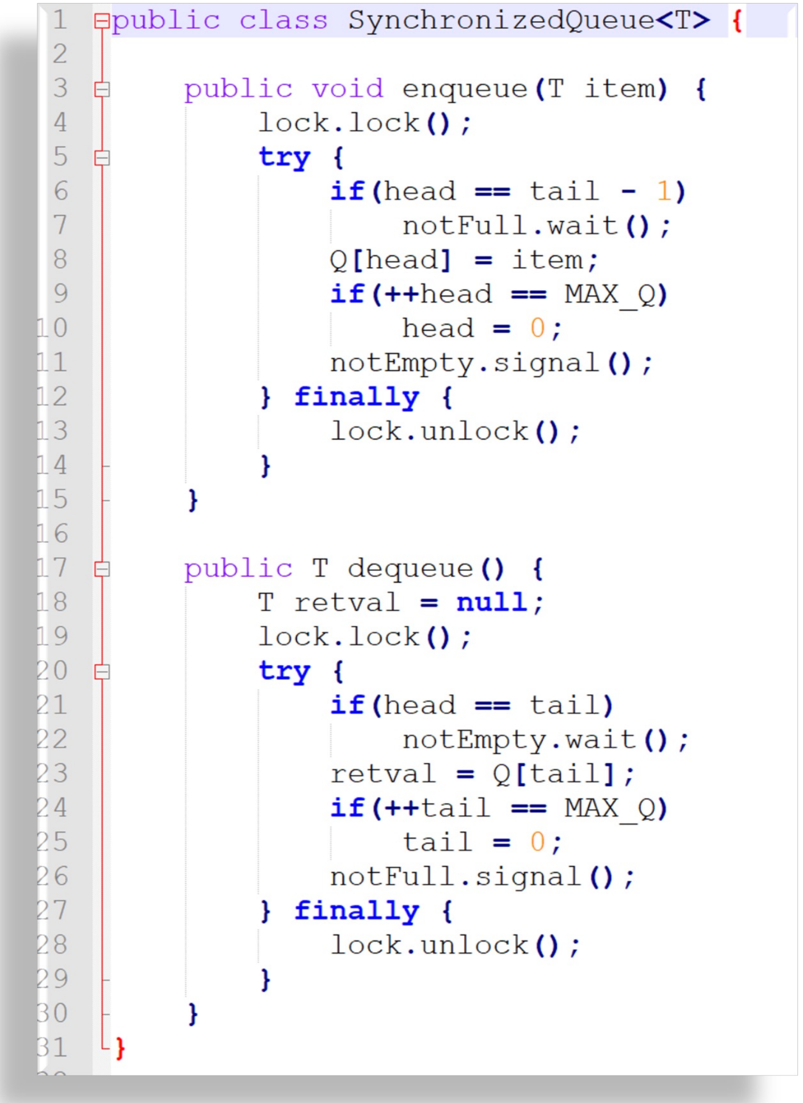

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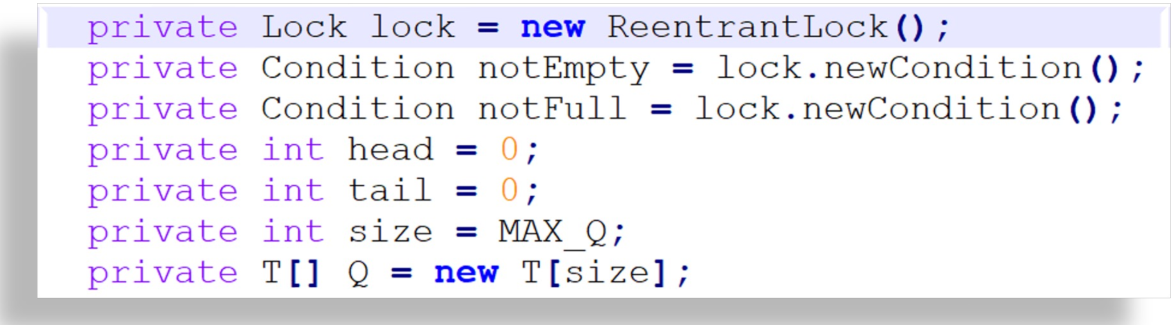

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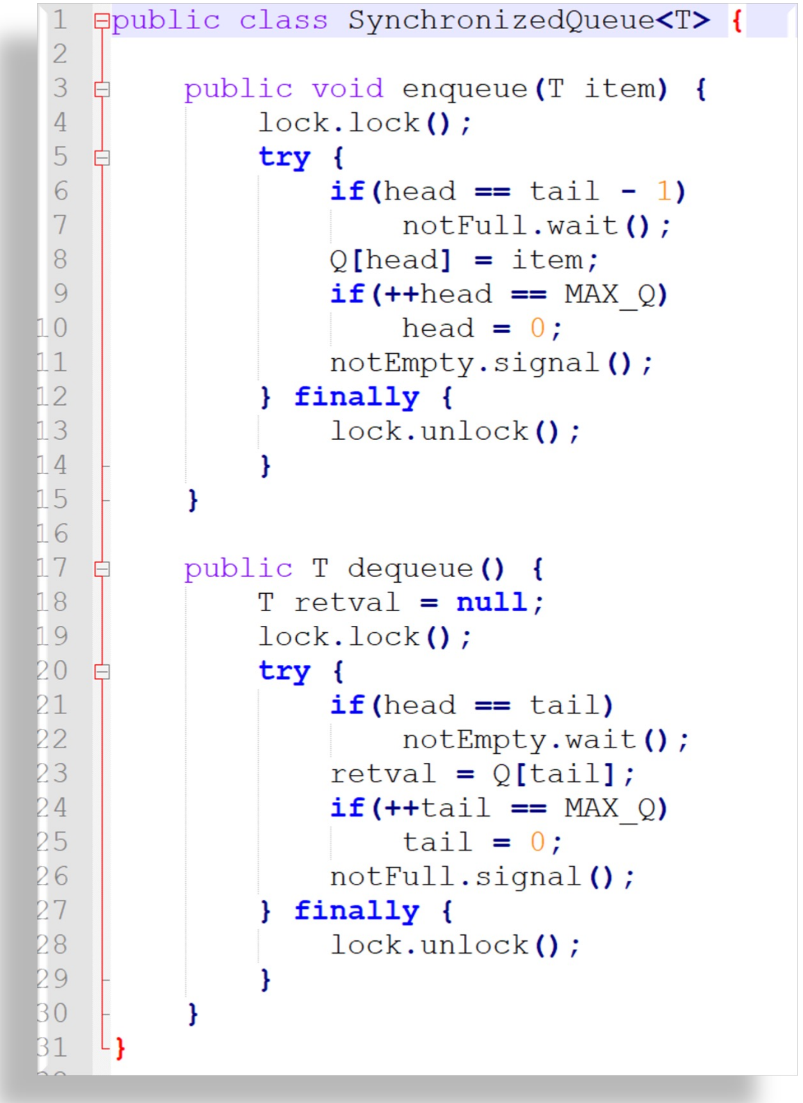

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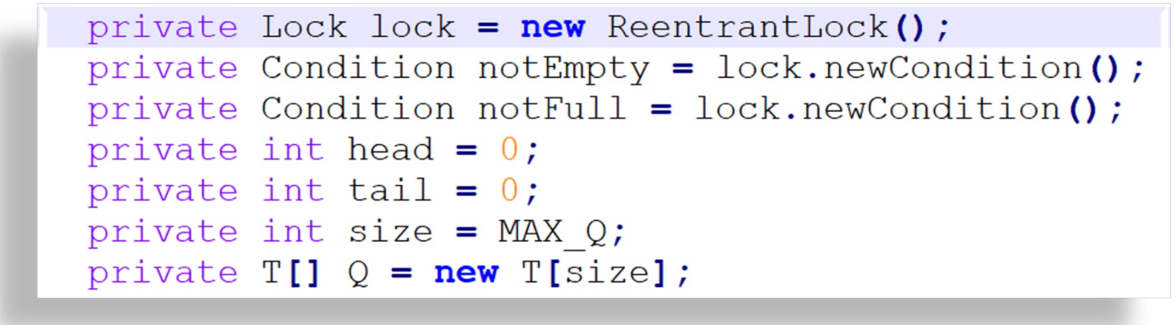

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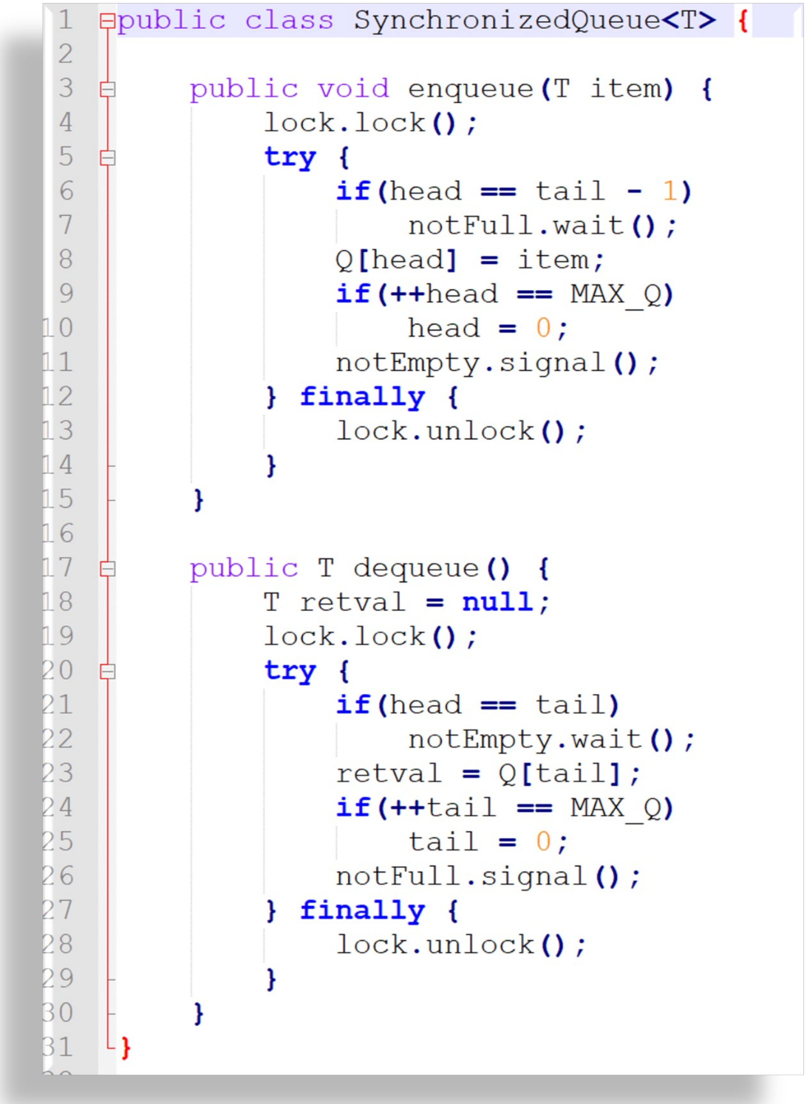

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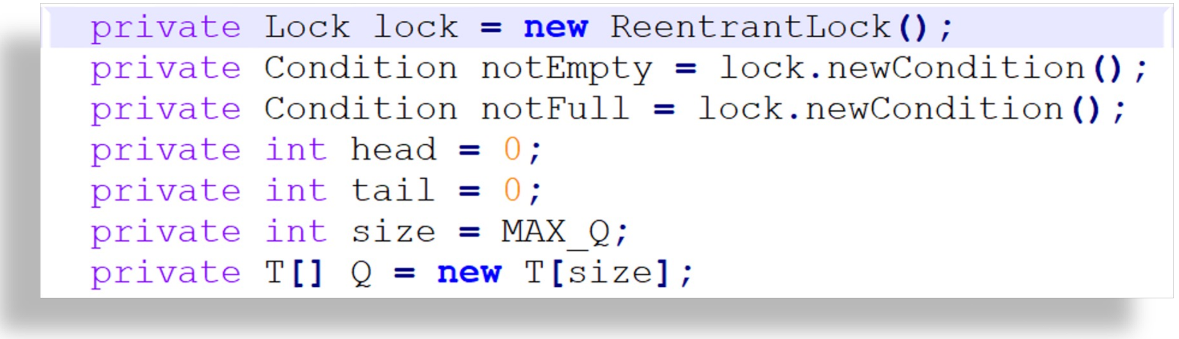

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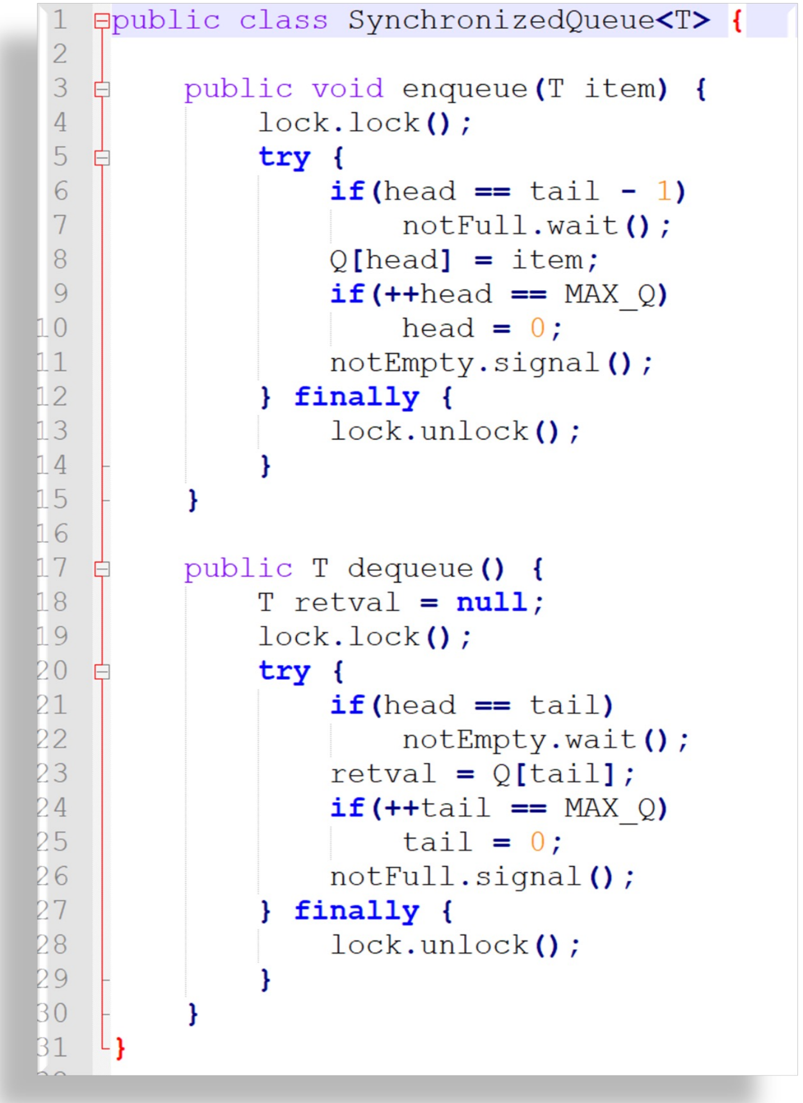
- Uses “if” to check invariants.
- Why doesn’t **if** work?

Does this code work?



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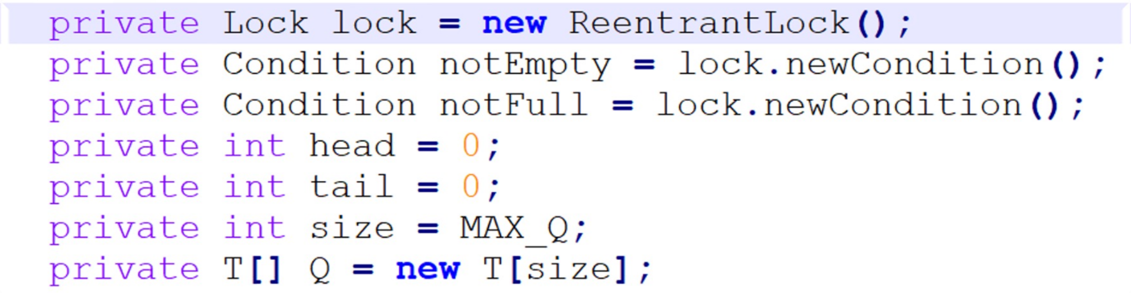

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- Uses “if” to check invariants.
- Why doesn’t **if** work?
- How could we MAKE it work?

Hoare-style Monitors

(aka blocking condition variables)

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Given entrance queue ‘e’, signal queue ‘s’, condition var ‘c’

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(aka blocking condition variables)

Given entrance queue ‘e’, signal queue ‘s’, condition var ‘c’

```
enter:
    if(locked):
       e.push_back(thread)
    else
      lock
```

Hoare-style Monitors

(aka blocking condition variables)

Given entrance queue ‘e’, signal queue ‘s’, condition var ‘c’

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enter:
    if(locked):
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    else
      lock
```

```
schedule:
    if s.any()
        t ← s.pop_first()
        t.run
    else if e.any()
        t ← e.pop_first()
        t. run
    else
        unlock // monitor unoccupied
```

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```
wait C:

  C.q.push_back(thread)
  schedule  // block this thread
```

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wait C:

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```
signal C :

   if (C.q.any())

       t = C.q.pop_front() // t → "the signaled thread"
       s.push_back(thread)
       t.run
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(aka blocking condition variables)

Given entrance queue ‘e’, signal queue ‘s’, condition var ‘c’

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- Signaler must wait, but gets priority over threads on entrance queue
- Lock only released by
  - Schedule (if no waiters)
  - Application
- Pros/Cons?

Hoare-style Monitors
(aka blocking condition variables)

Given entrance queue ‘e’, signal queue ‘s’, condition var ‘c’

```
enter:
    if(locked):
       e.push_back(thread)
    else
      lock
```

```
schedule:
    if s.any()
        t ← s.pop_first()
        t.run
    else if e.any()
        t ← e.pop_first()
        t. run
    else
        unlock // monitor unoccupied
```

```
wait C:

  C.q.push_back(thread)
  schedule   // block this thread
```

```
leave:
    schedule
```

```
signal C :

   if (C.q.any())

       t = C.q.pop_front() // t → "the signaled thread"
       s.push_back(thread)
       t.run
```

- Signaler must wait, but gets priority over threads on entrance queue
- Lock only released by
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Must run signaled thread immediately

- Signaler must wait, but gets priority over threads on entrance queue
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Must run signaled thread immediately
Options for signaler:

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```

- Signaler must wait, but gets priority over threads on entrance queue
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- Pros/Cons?

Must run signaled thread immediately
Options for signaler:

- Switch out (go on s queue)

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schedule:
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        t ← s.pop_first()
        t.run
    else if e.any()
        t ← e.pop_first()
        t. run
    else
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```
wait C:

  C.q.push_back(thread)
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```

```
leave:
    schedule
```

```
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      t = C.q.pop_front() // t → "the signaled thread"
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      t.run
```

- Signaler must wait, but gets priority over threads on entrance queue
- Lock only released by
  - Schedule (if no waiters)
  - Application
- Pros/Cons?

Must run signaled thread immediately
Options for signaler:

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- Exit (Hansen monitors)

Hoare-style Monitors

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        t ← e.pop_first()
        t. run
    else
        unlock // monitor unoccupied
```

```
wait C:

  C.q.push_back(thread)
  schedule  // block this thread
```

```
leave:
    schedule
```

```
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   if (C.q.any())
        t = C.q.pop_front() // t → "the signaled thread"
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```

- Signaler must wait, but gets priority over threads on entrance queue
- Lock only released by
  - Schedule (if no waiters)
  - Application
- Pros/Cons?

Must run signaled thread immediately
Options for signaler:

- Switch out (go on s queue)
- Exit (Hansen monitors)
- Continue executing?

Mesa-style monitors

(aka non-blocking condition variables)

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        t ← e.pop_front
        t. run
    else
        unlock
```

```
notify C:

    if C.q.any()

         t ← C.q.pop_front() // t is "notified ‟
         e.push_back(t)
```

Mesa-style monitors

(aka non-blocking condition variables)

```
enter:
    if locked:
        e.push_back(thread)
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    else
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        t. run
    else
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```

```
notify C:

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         t ← C.q.pop_front() // t is "notified “
         e.push_back(t)
```

```
wait C:

  C.q.push_back(thread)
  schedule
  block
```

Mesa-style monitors

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    else
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```
schedule:
    if e.any()
        t ← e.pop_front
        t. run
    else
        unlock
```

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notify C:

    if C.q.any()

         t ← C.q.pop_front() // t is "notified ‟
         e.push_back(t)
```

```
wait C:

  C.q.push_back(thread)
  schedule
  block
```

- Leave still calls schedule

Mesa-style monitors

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        e.push_back(thread)
        block
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```
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```
wait C:

  C.q.push_back(thread)
  schedule
  block
```

- Leave still calls schedule
- No signal queue

Mesa-style monitors

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wait C:

  C.q.push_back(thread)
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```

- Leave still calls schedule
- No signal queue
- Extendable with more queues for priority

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    else
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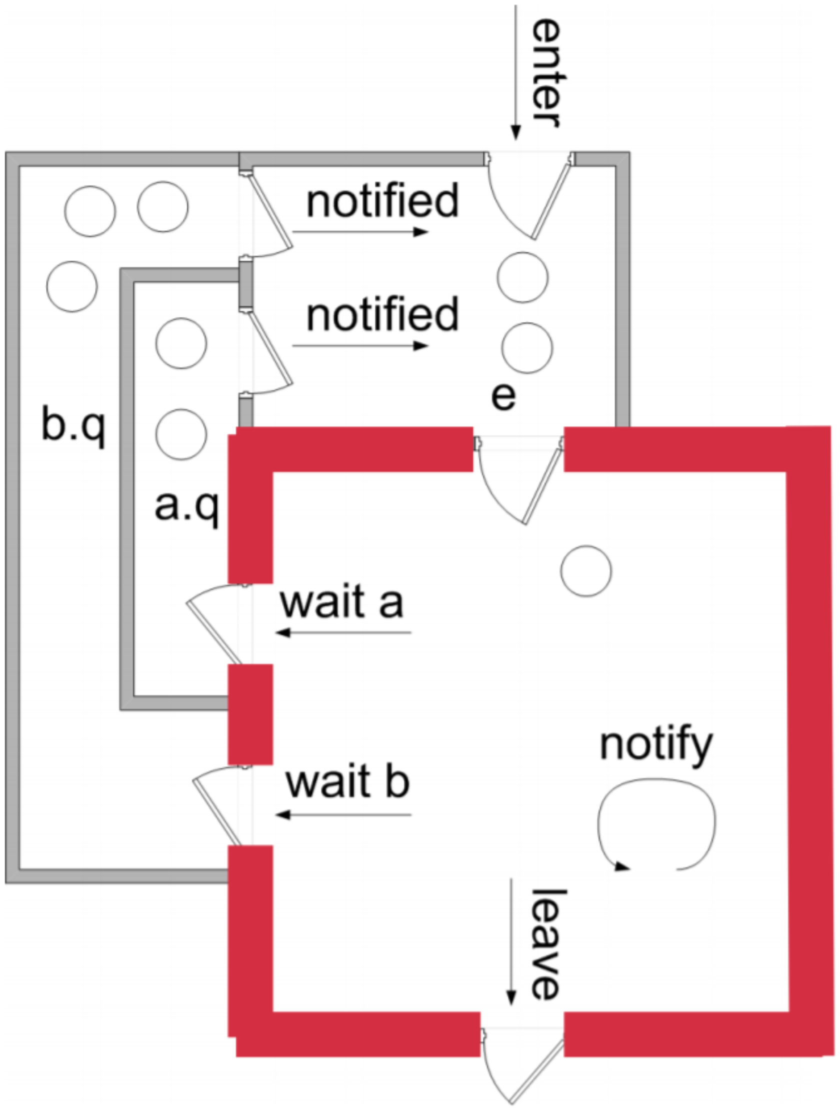
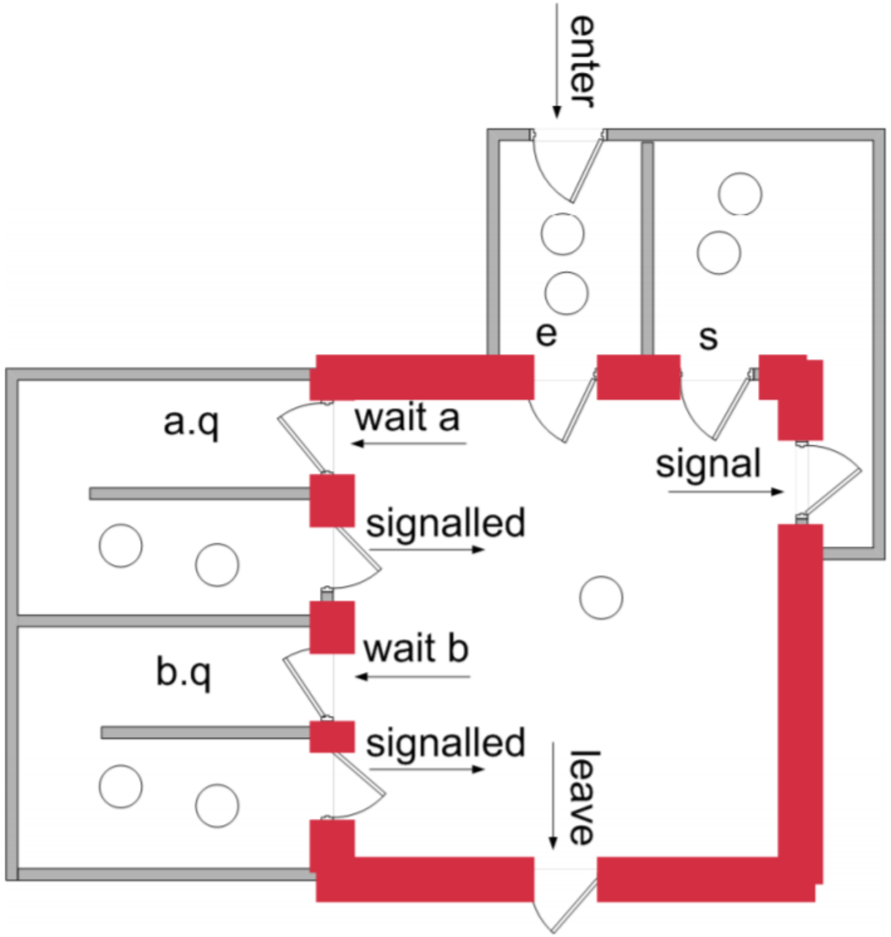
```
wait C:

  C.q.push_back(thread)
  schedule
  block
```

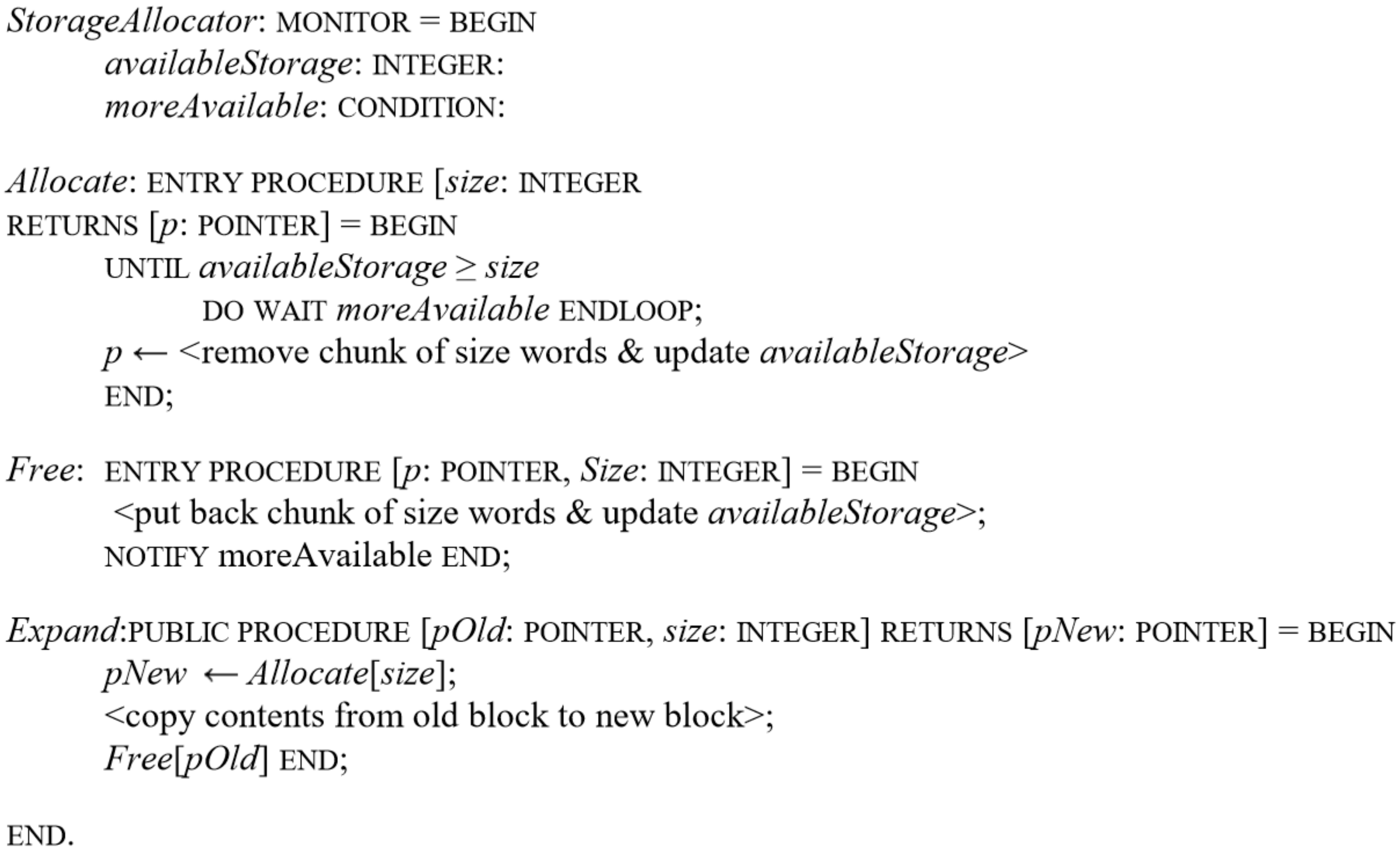
- Leave still calls schedule
- No signal queue
- Extendable with more queues for priority
- What are the differences/pros/cons?

Mesa, Hansen, Hoare





Example: anyone see a bug?



```
StorageAllocator: MONITOR = BEGIN
        availableStorage: INTEGER:
        moreAvailable: CONDITION:

Allocate: ENTRY PROCEDURE [size: INTEGER
RETURNS [p: POINTER] = BEGIN
        UNTIL availableStorage ≥ size
                DO WAIT moreAvailable ENDLOOP;
        p ← <remove chunk of size words & update availableStorage>
        END;

Free:   ENTRY PROCEDURE [p: POINTER, Size: INTEGER] = BEGIN
         <put back chunk of size words & update availableStorage>;
        NOTIFY moreAvailable END;

Expand:PUBLIC PROCEDURE [pOld: POINTER, size: INTEGER] RETURNS [pNew: POINTER] = BEGIN
        pNew  ← Allocate[size];
        <copy contents from old block to new block>;
        Free[pOld] END;

END.
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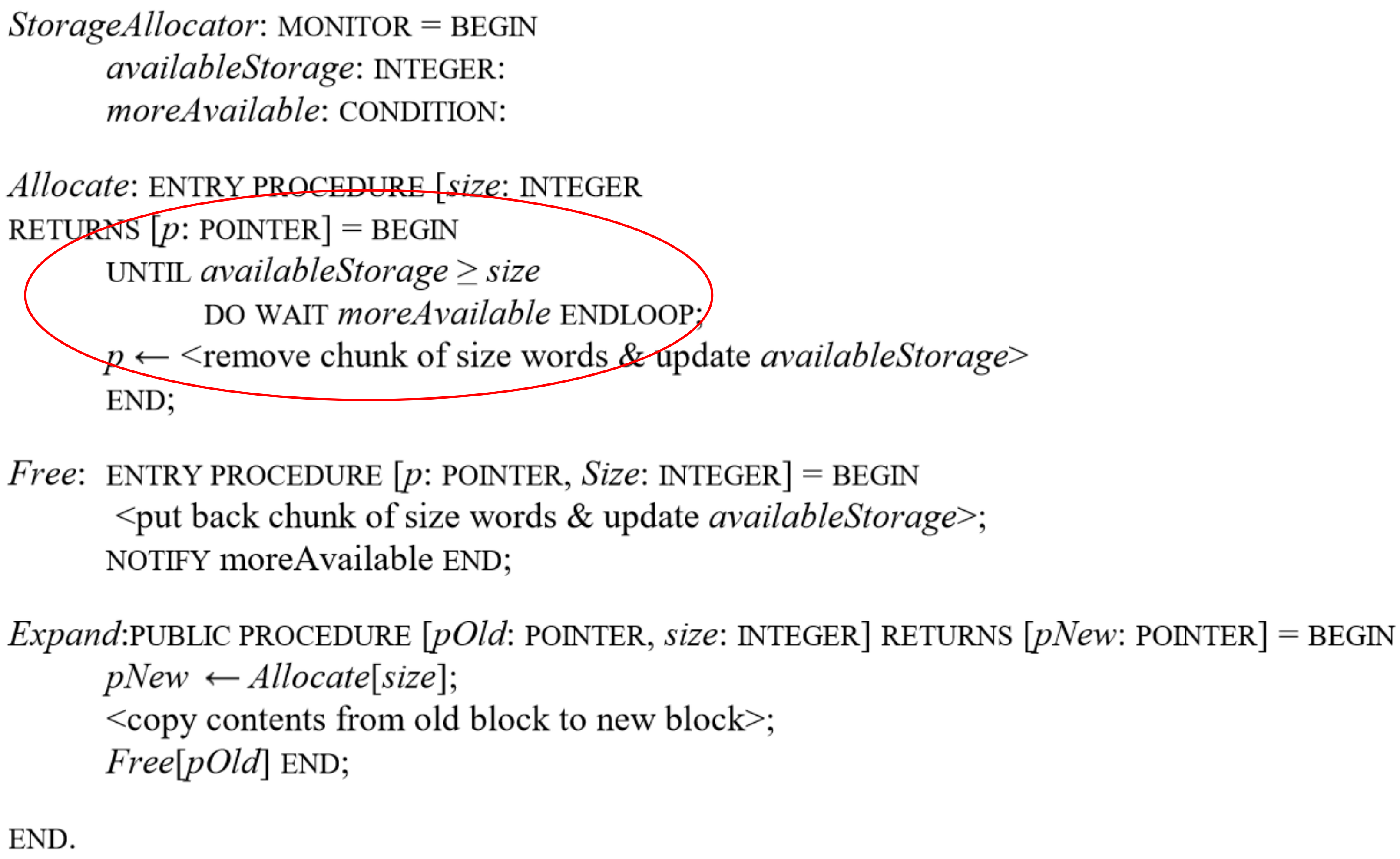

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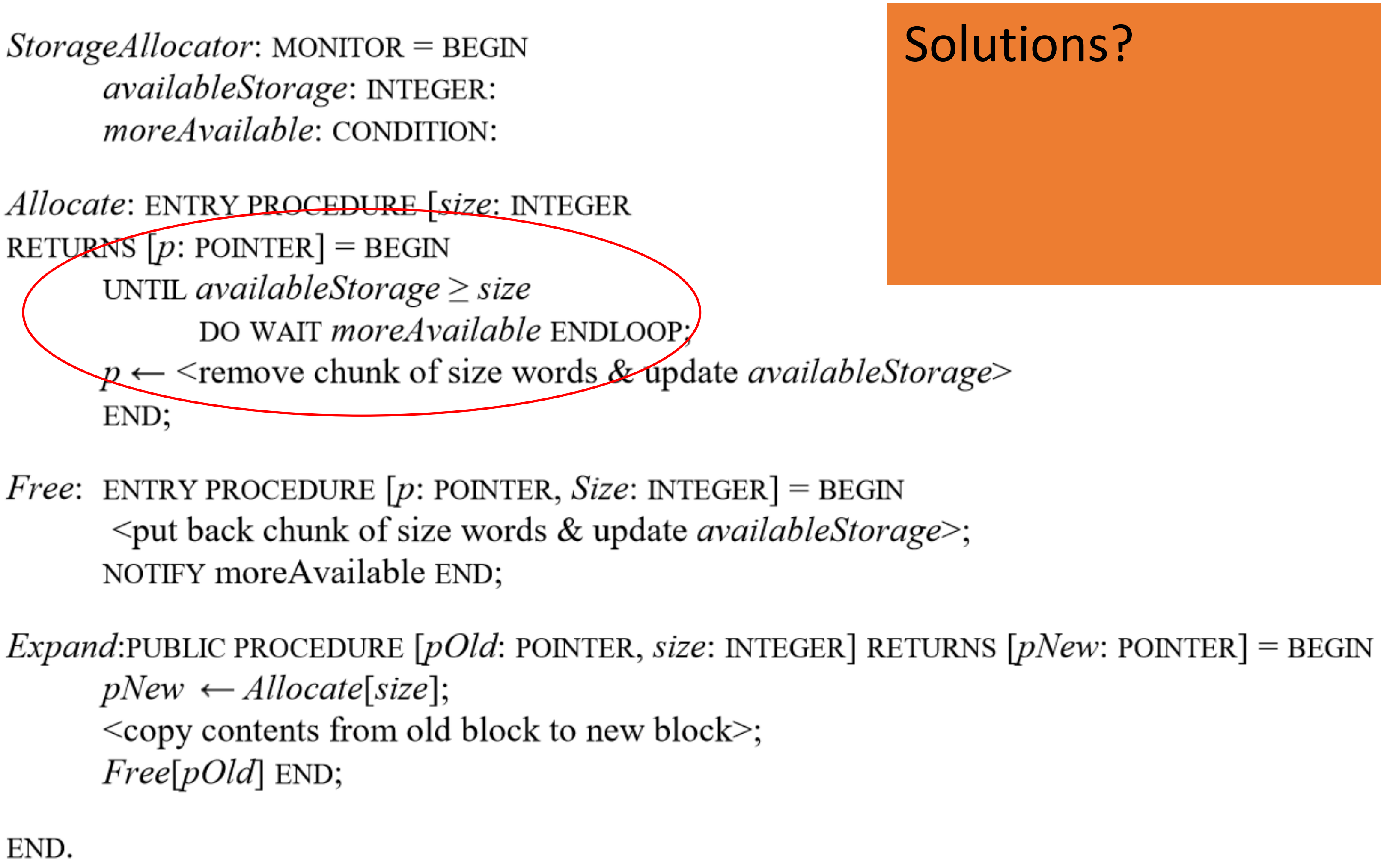

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Solutions?

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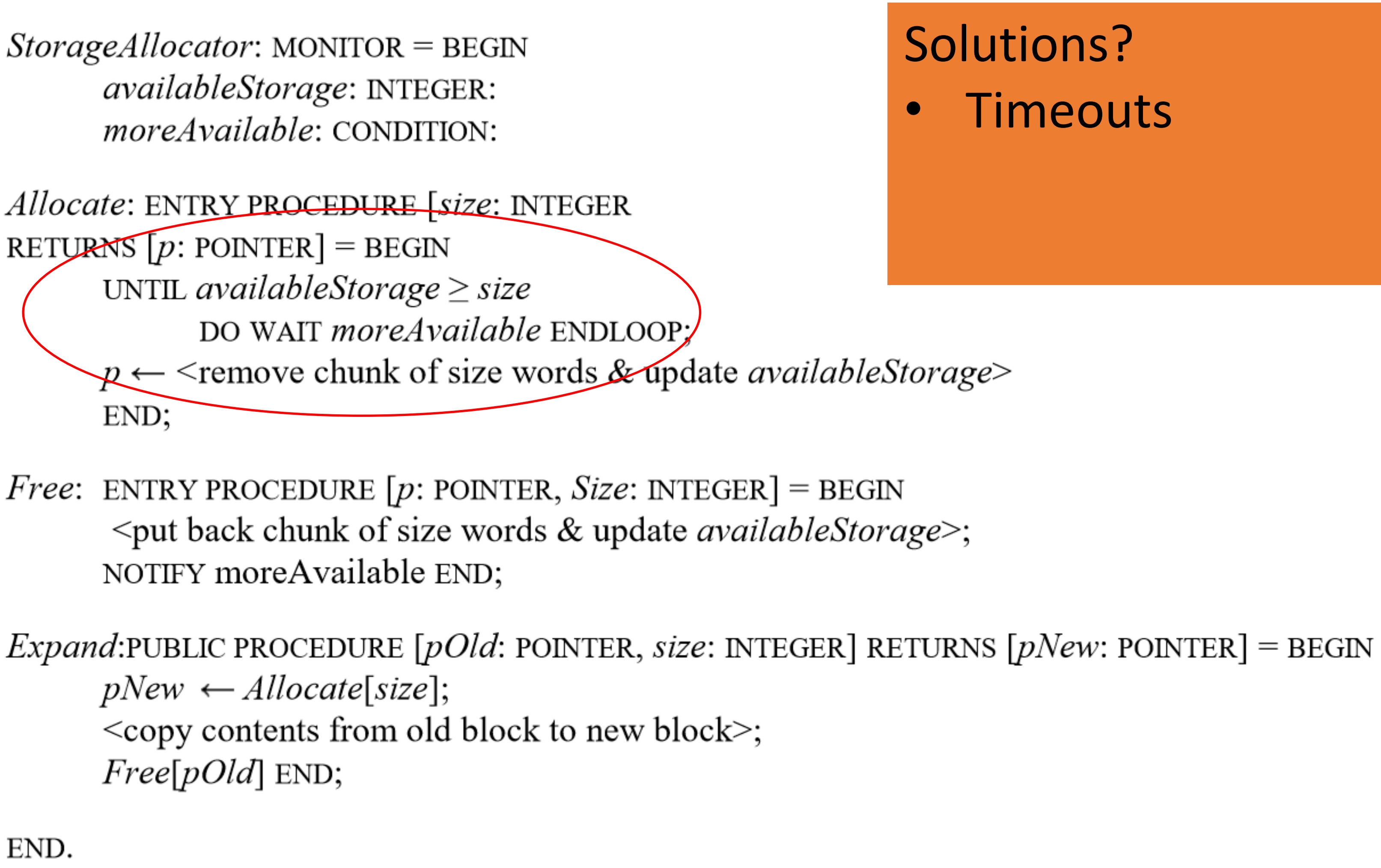

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Solutions?

- Timeouts

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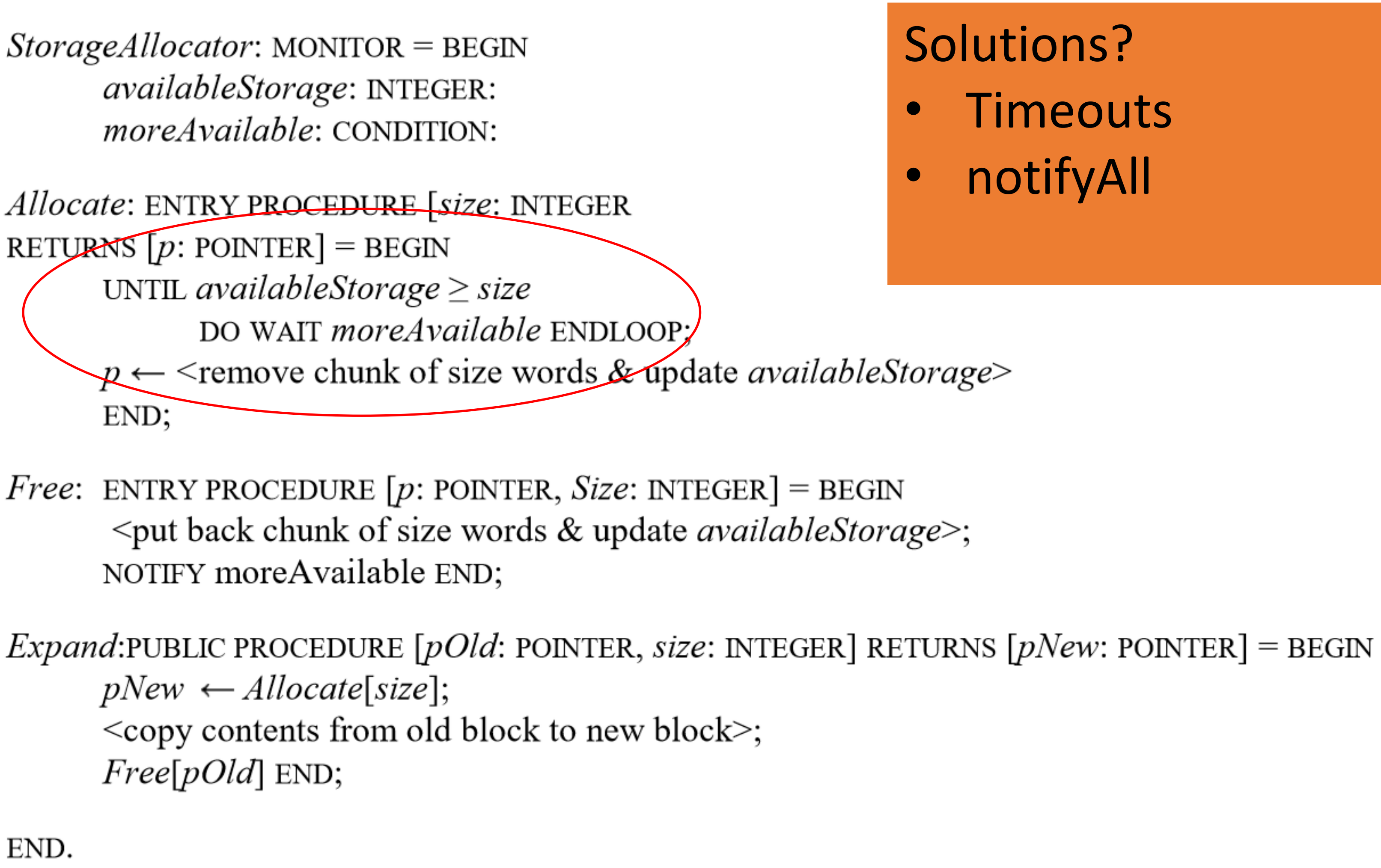

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Solutions?

- Timeouts
- notifyAll

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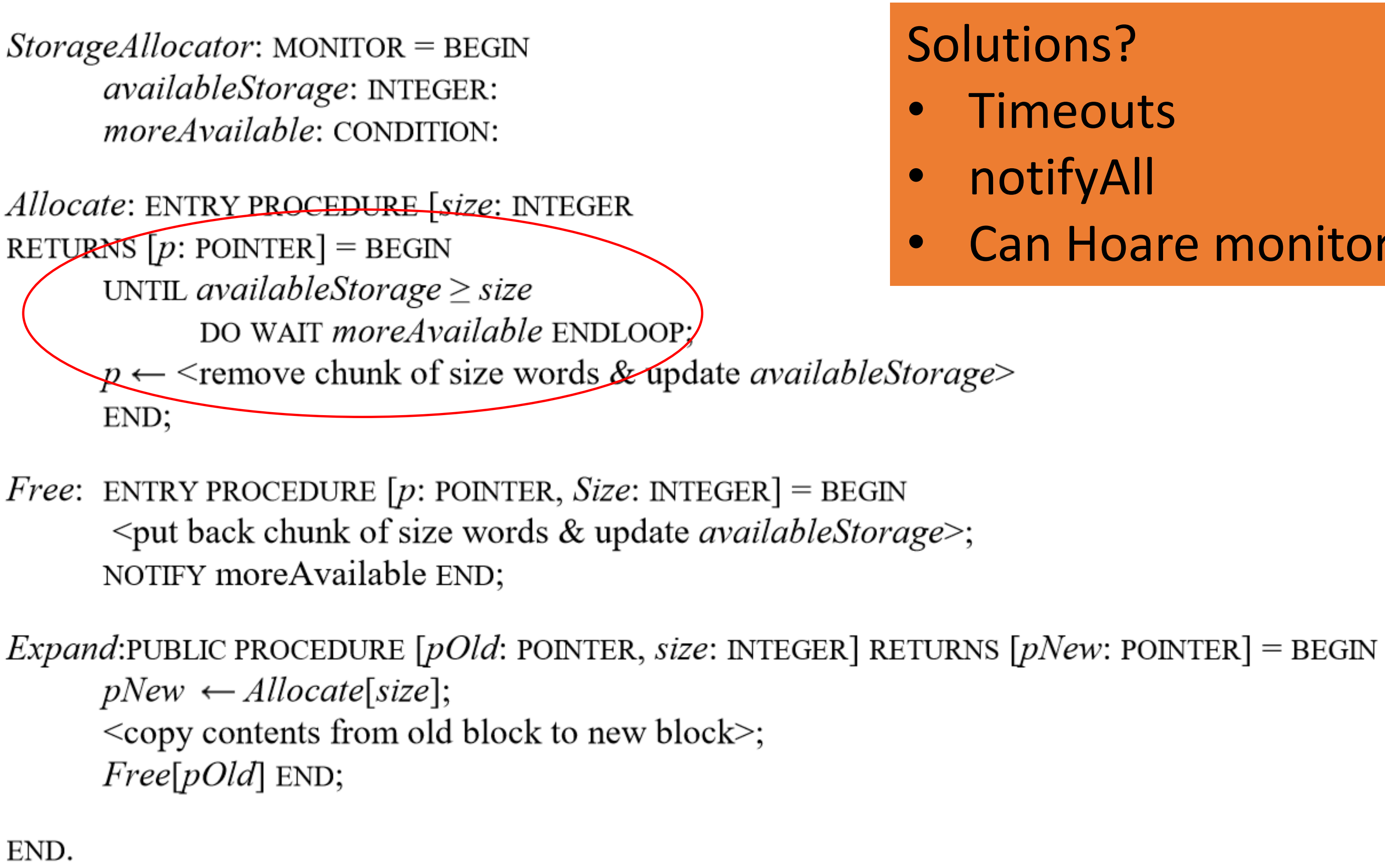

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Solutions?

- Timeouts
- notifyAll
- Can Hoare monitors support notifyAll?

Barriers

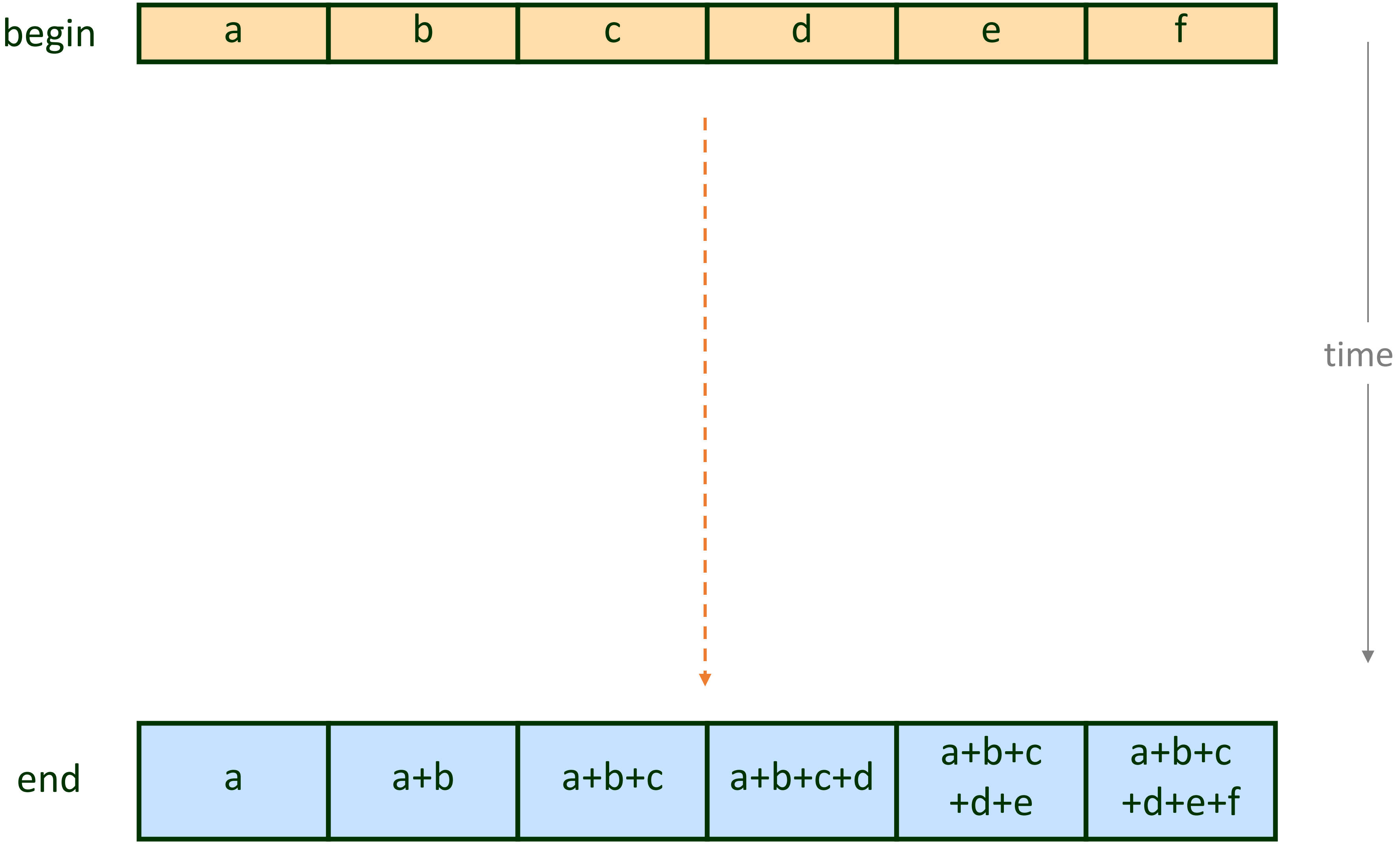
Barriers

Prefix Sum

Prefix Sum

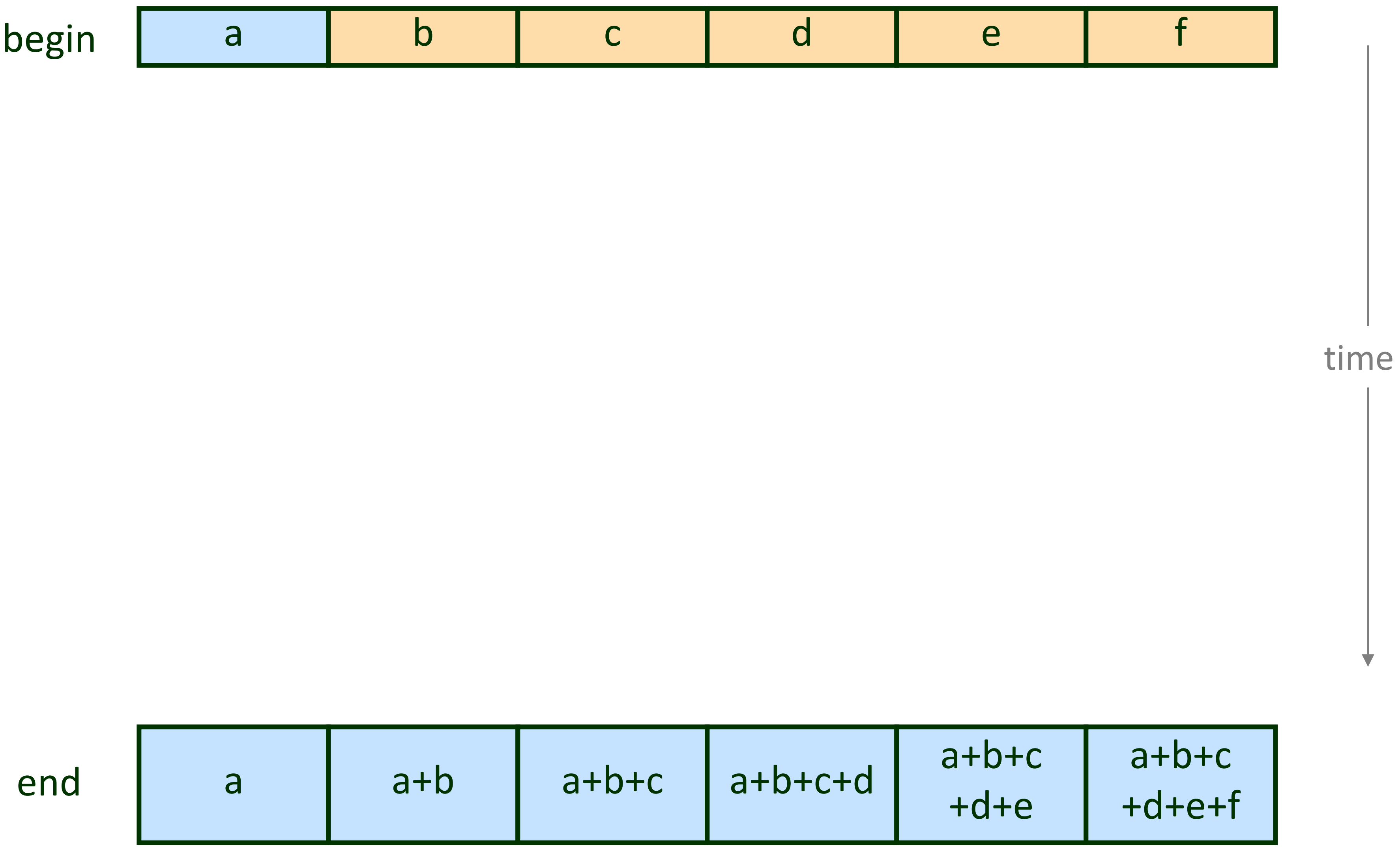
begin	a	b	c	d	e	f

Prefix Sum



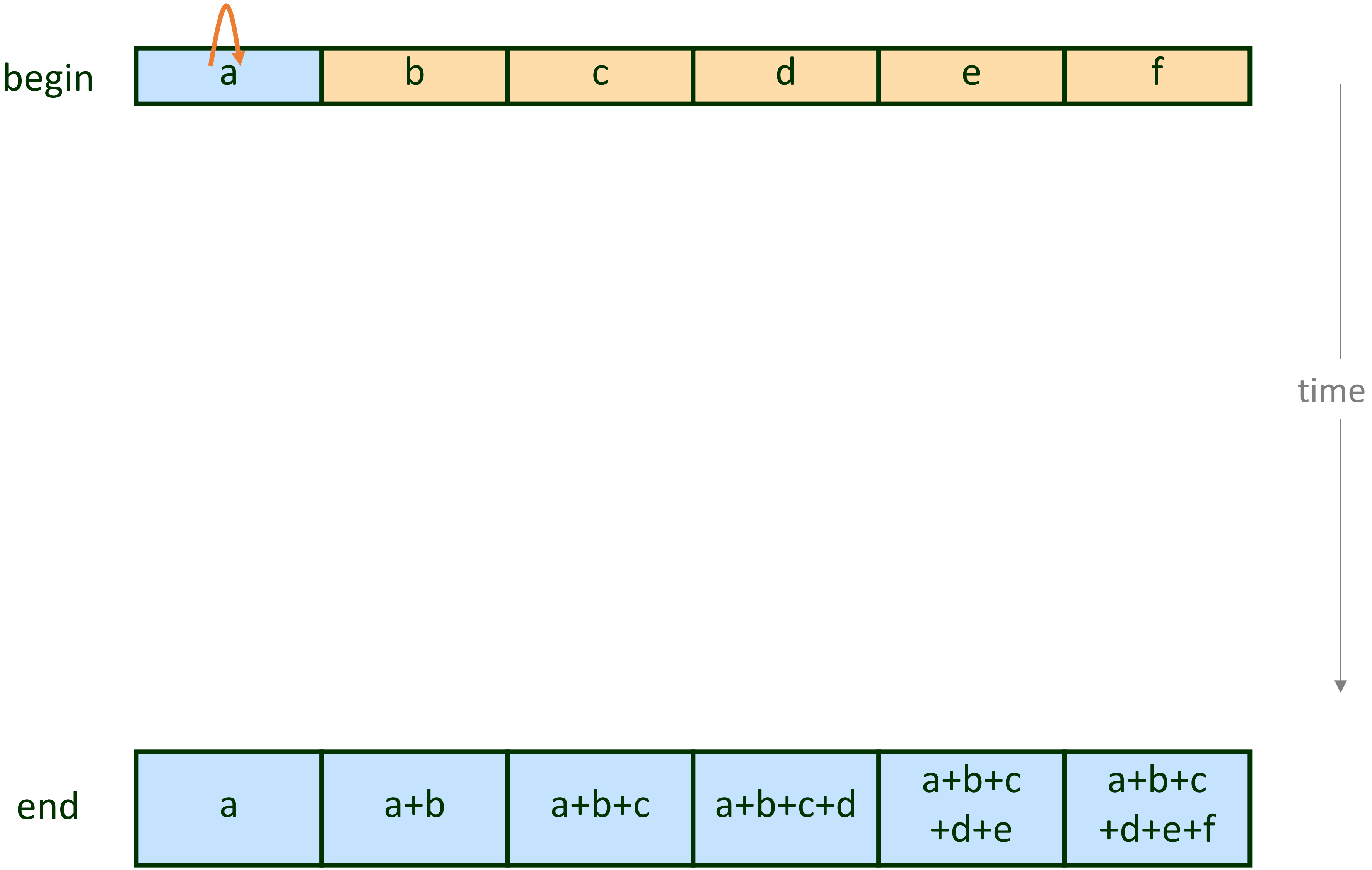
begin	a	b	c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



begin	a	b	c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



begin

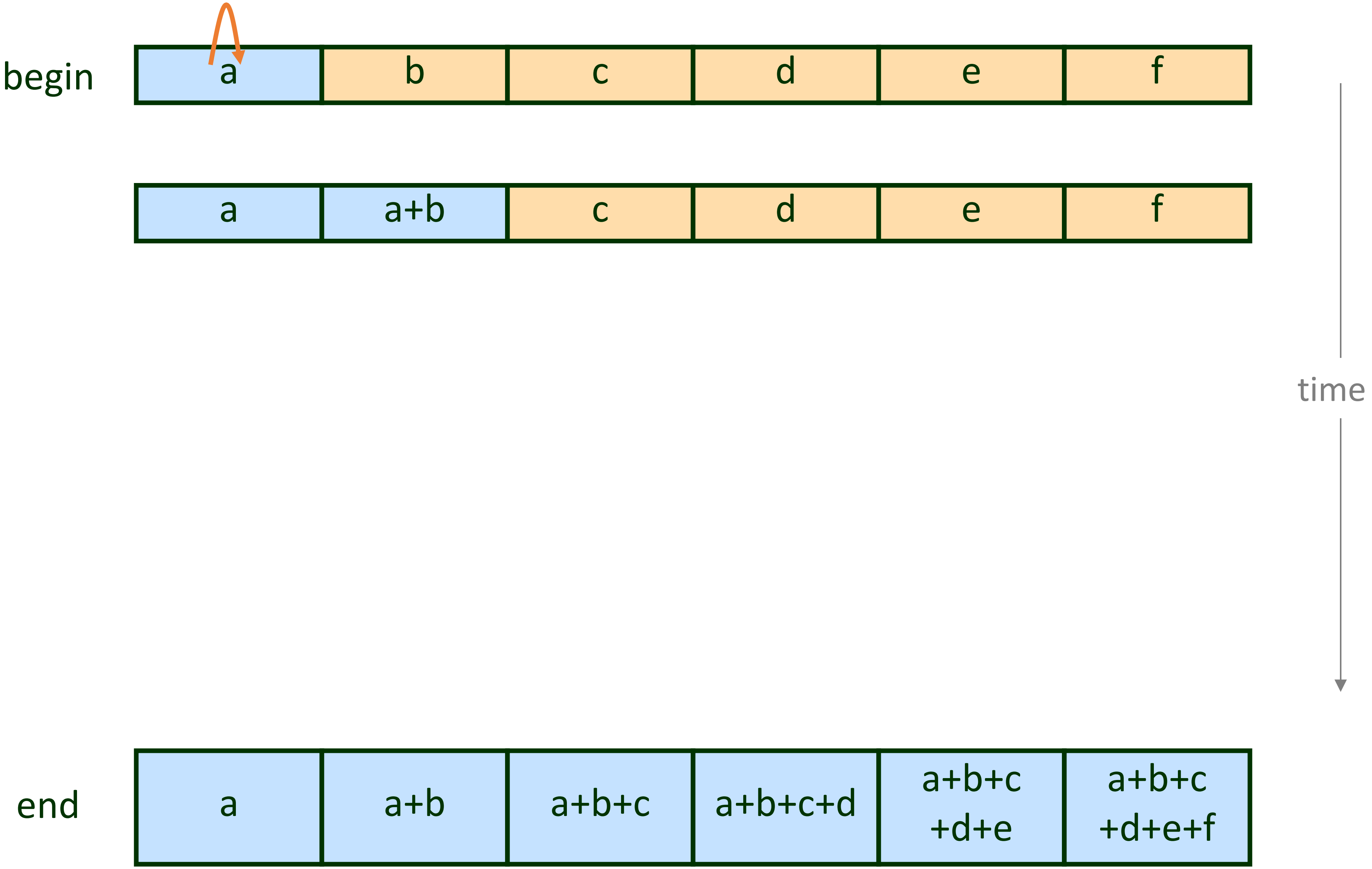
a	b	c	d	e	f

time

end

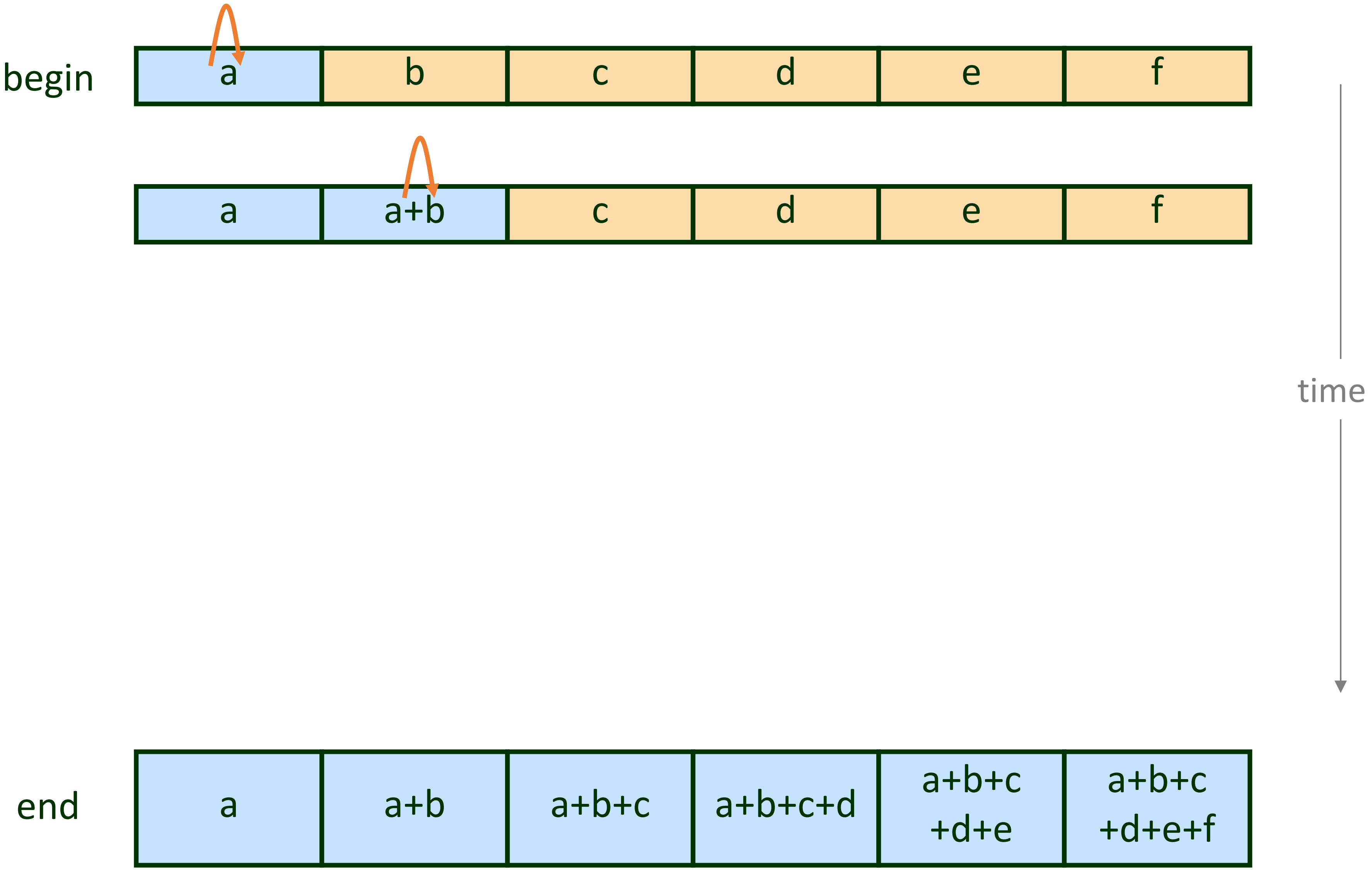
a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum

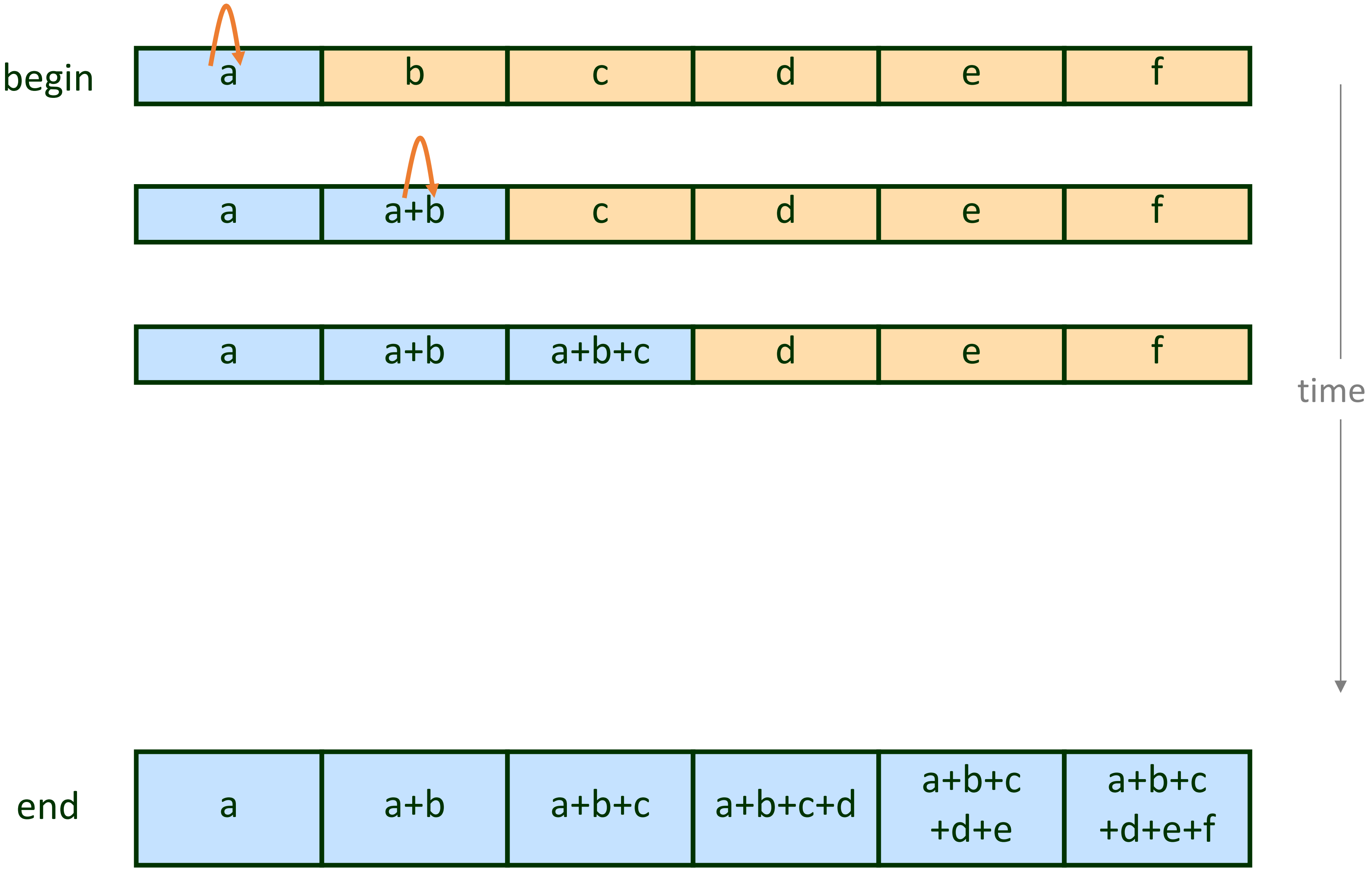


begin | a | b | c | d | e | f

| a | a+b | c | d | e | f

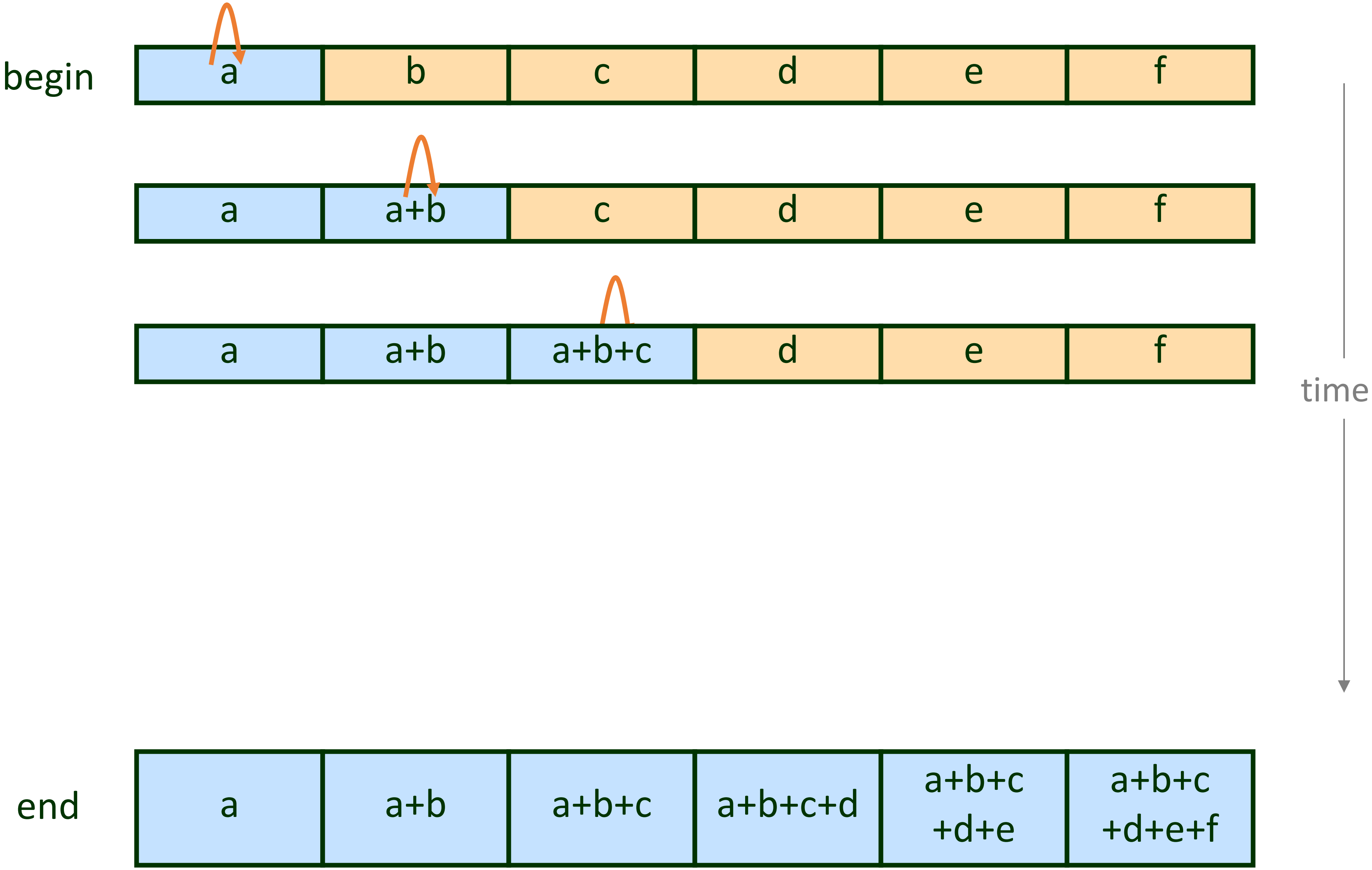
end | a | a+b | a+b+c | a+b+c+d | a+b+c +d+e | a+b+c +d+e+f

Prefix Sum



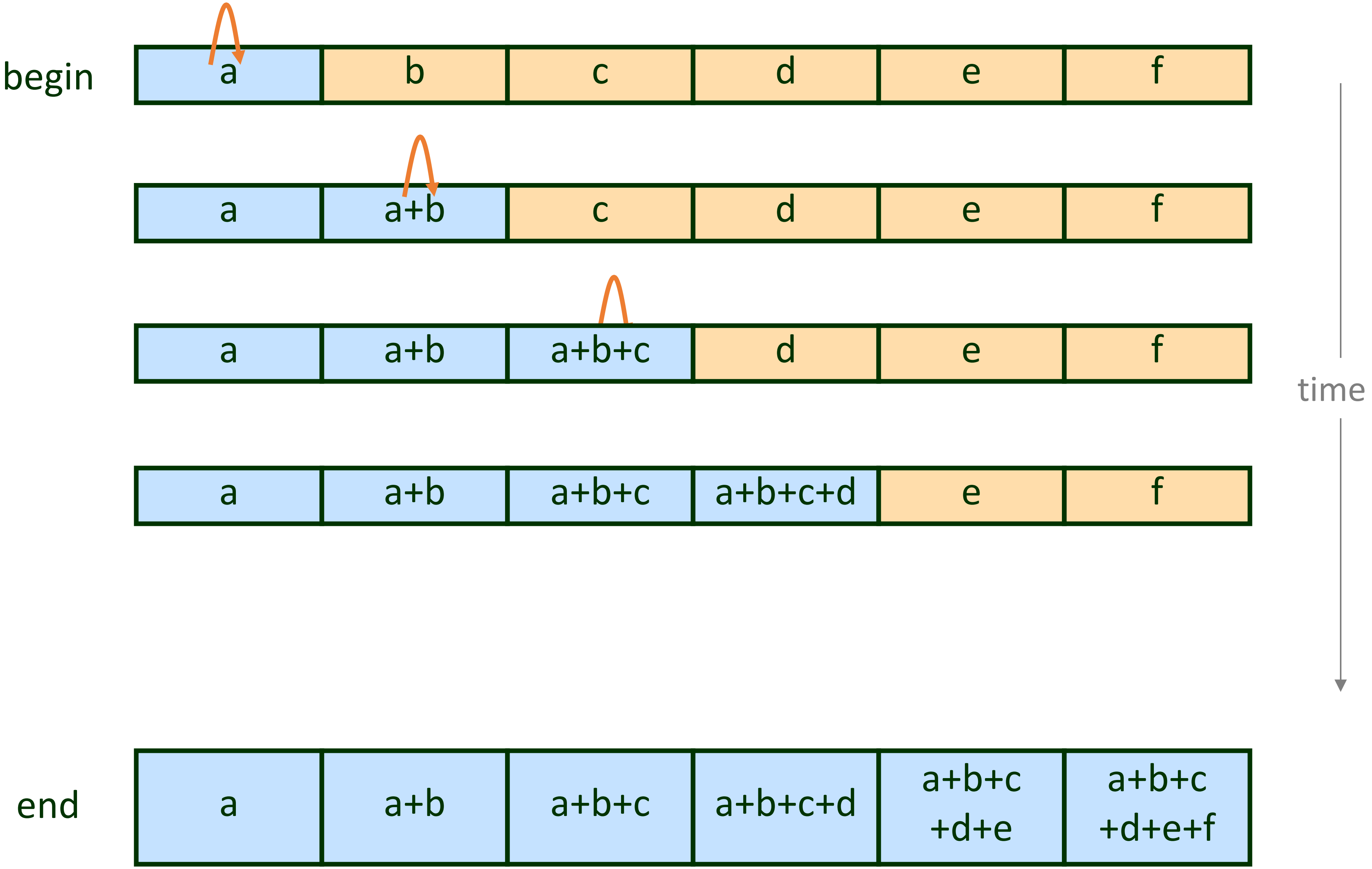
begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
	a	a+b	a+b+c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



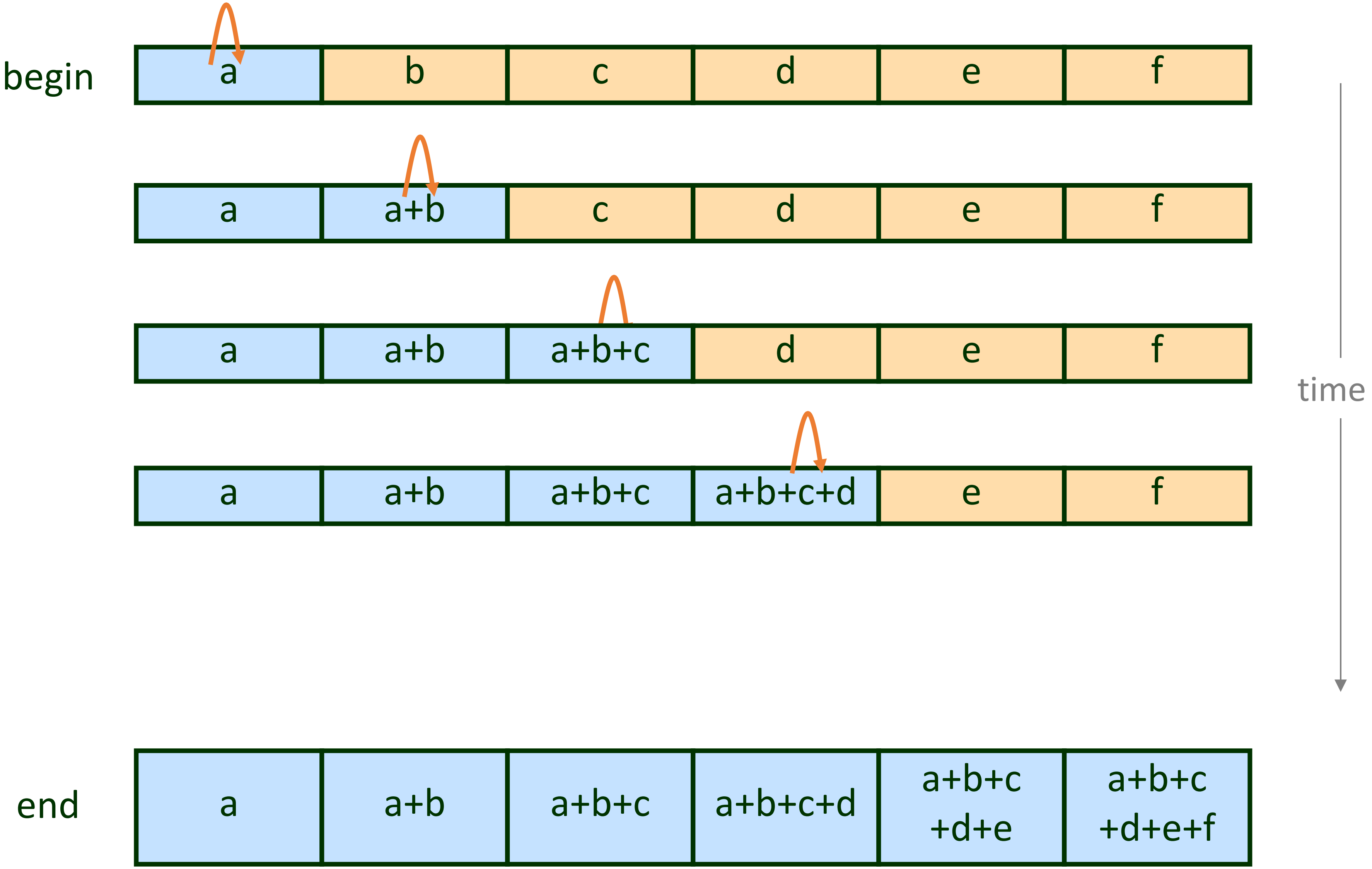
begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
	a	a+b	a+b+c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
	a	a+b	a+b+c	d	e	f
	a	a+b	a+b+c	a+b+c+d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
	a	a+b	a+b+c	d	e	f
	a	a+b	a+b+c	a+b+c+d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

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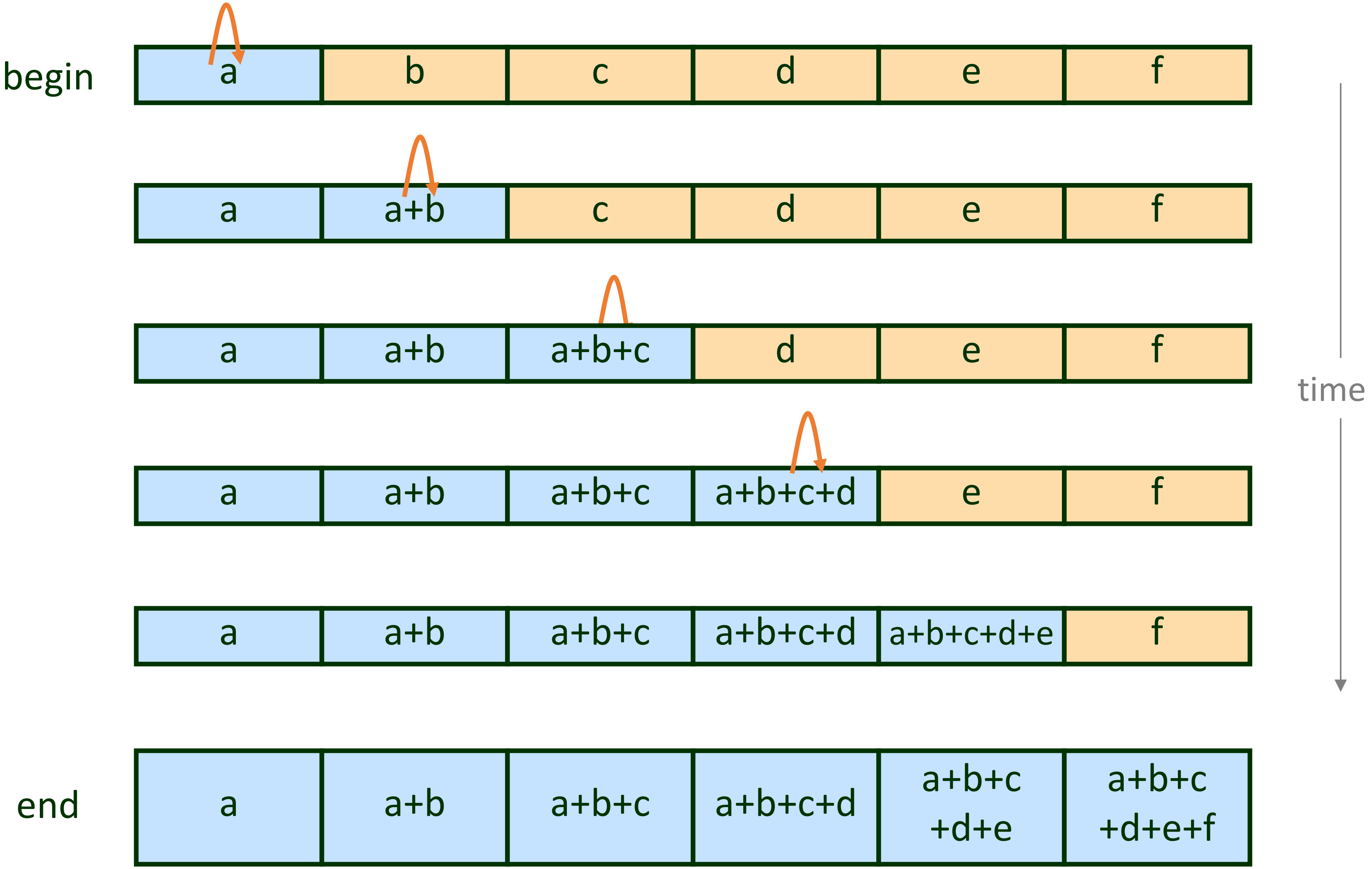
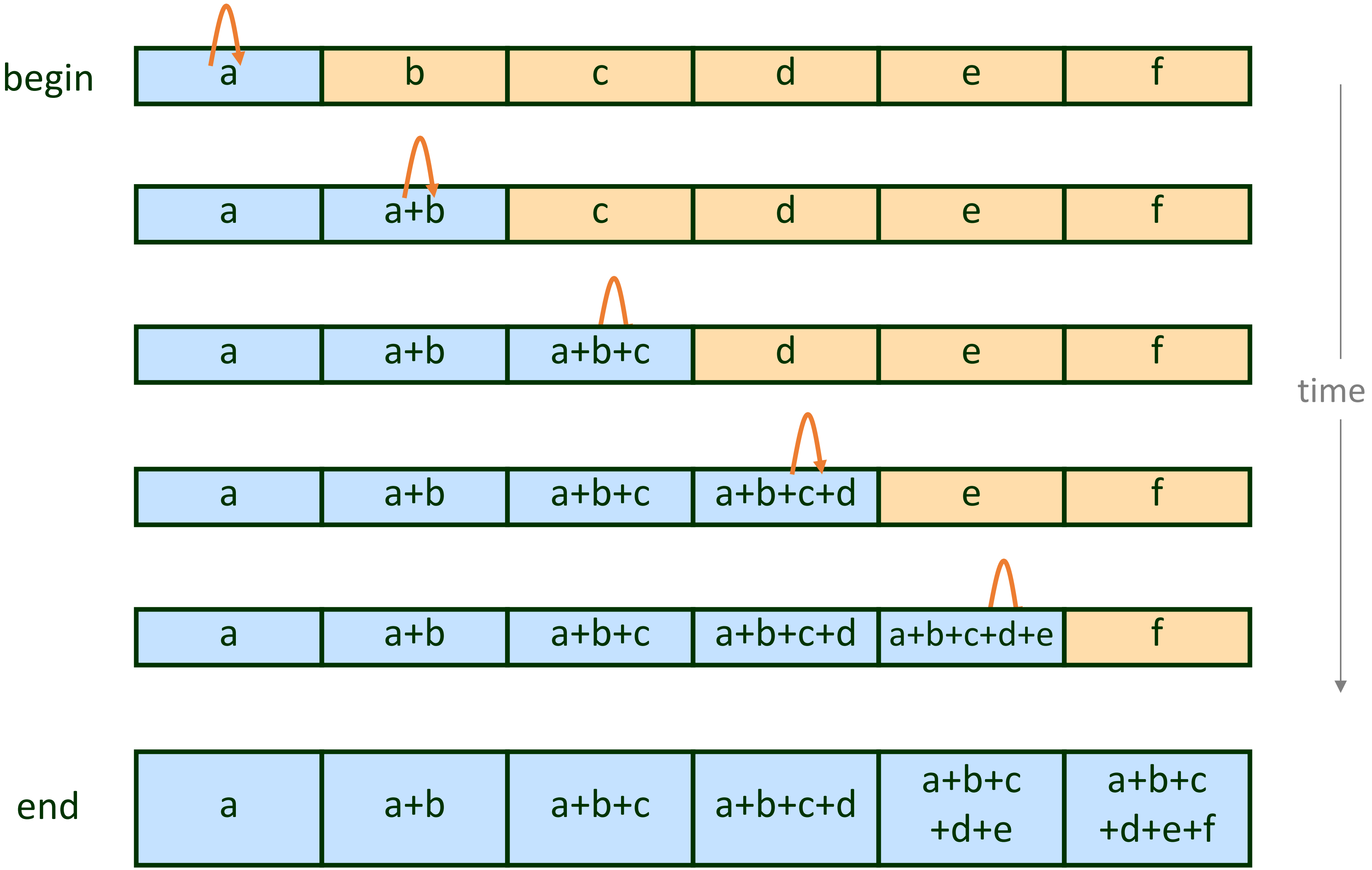


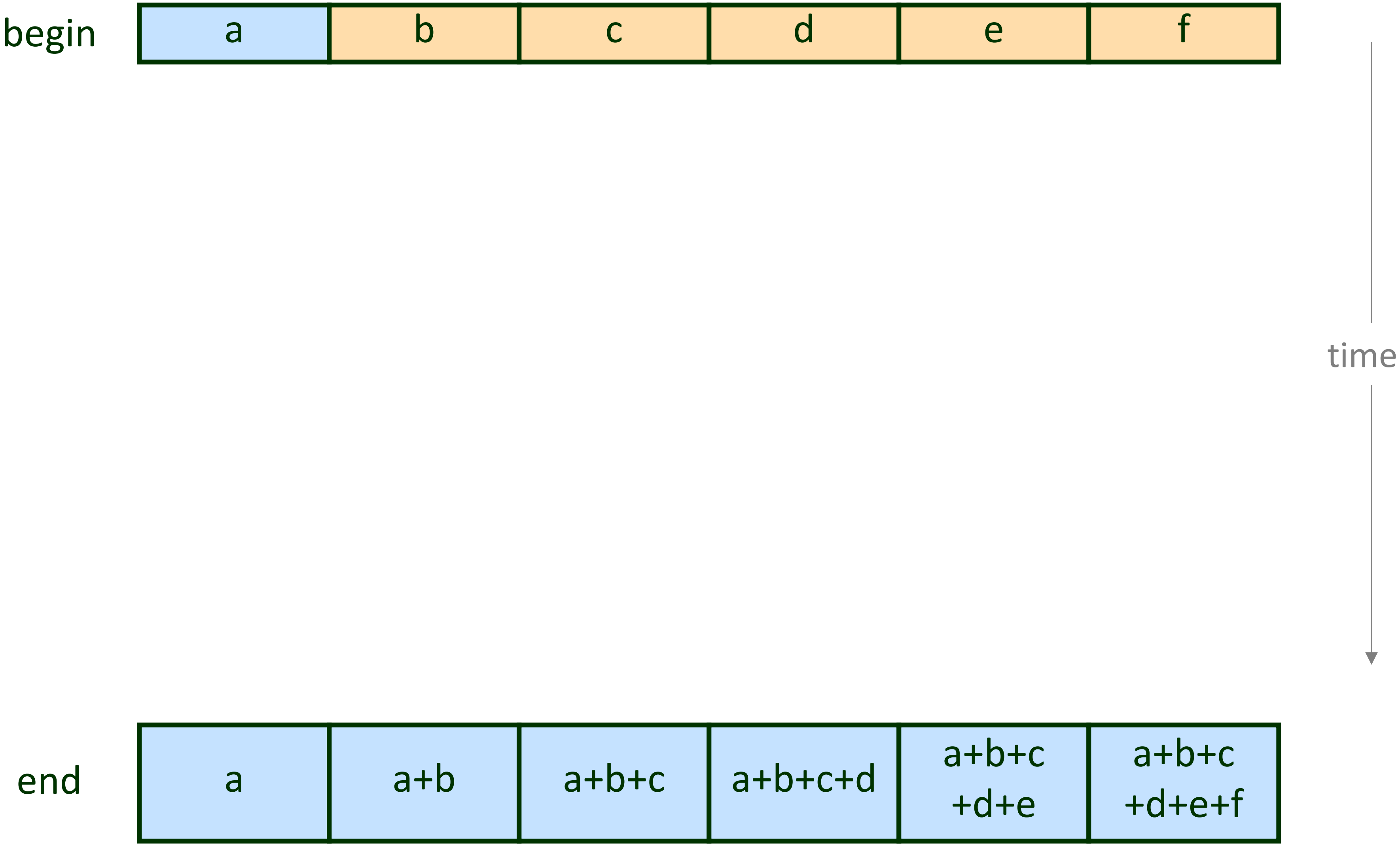
begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
	a	a+b	a+b+c	d	e	f
	a	a+b	a+b+c	a+b+c+d	e	f
	a	a+b	a+b+c	a+b+c+d	a+b+c+d+e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Prefix Sum



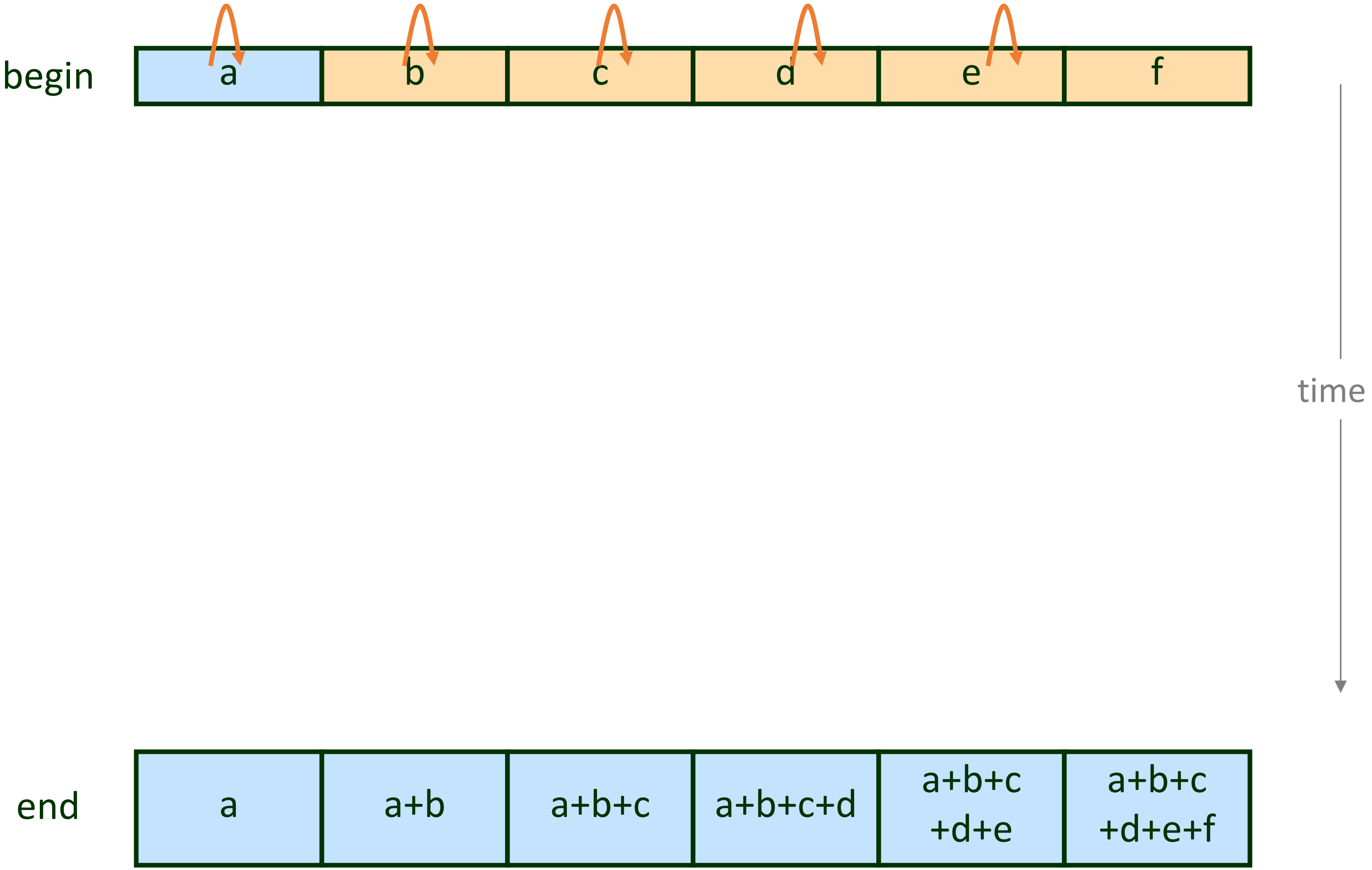
begin	a	b	c	d	e	f
	a	a+b	c	d	e	f
	a	a+b	a+b+c	d	e	f
	a	a+b	a+b+c	a+b+c+d	e	f
	a	a+b	a+b+c	a+b+c+d	a+b+c+d+e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Parallel Prefix Sum



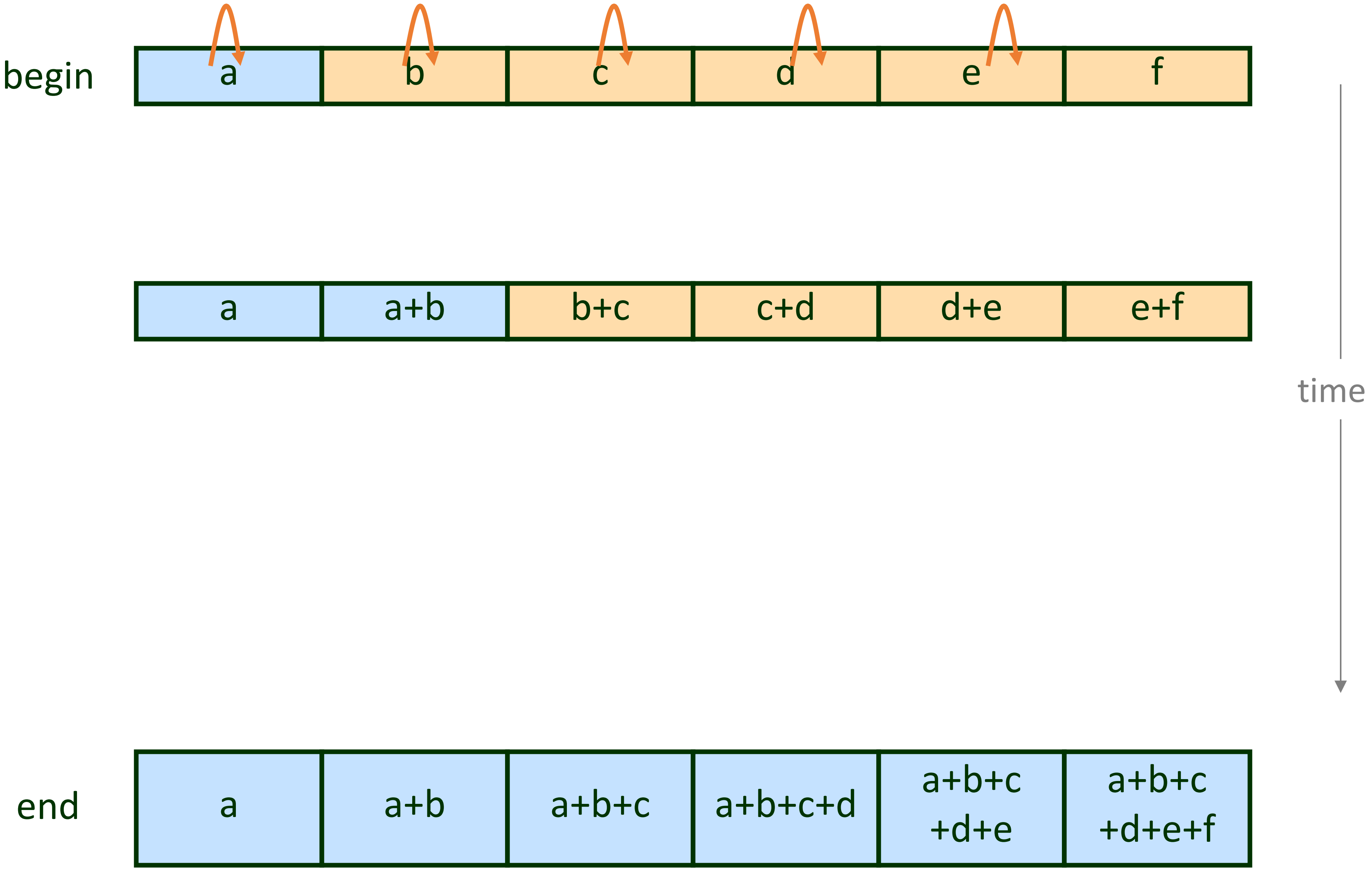
begin	a	b	c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Parallel Prefix Sum



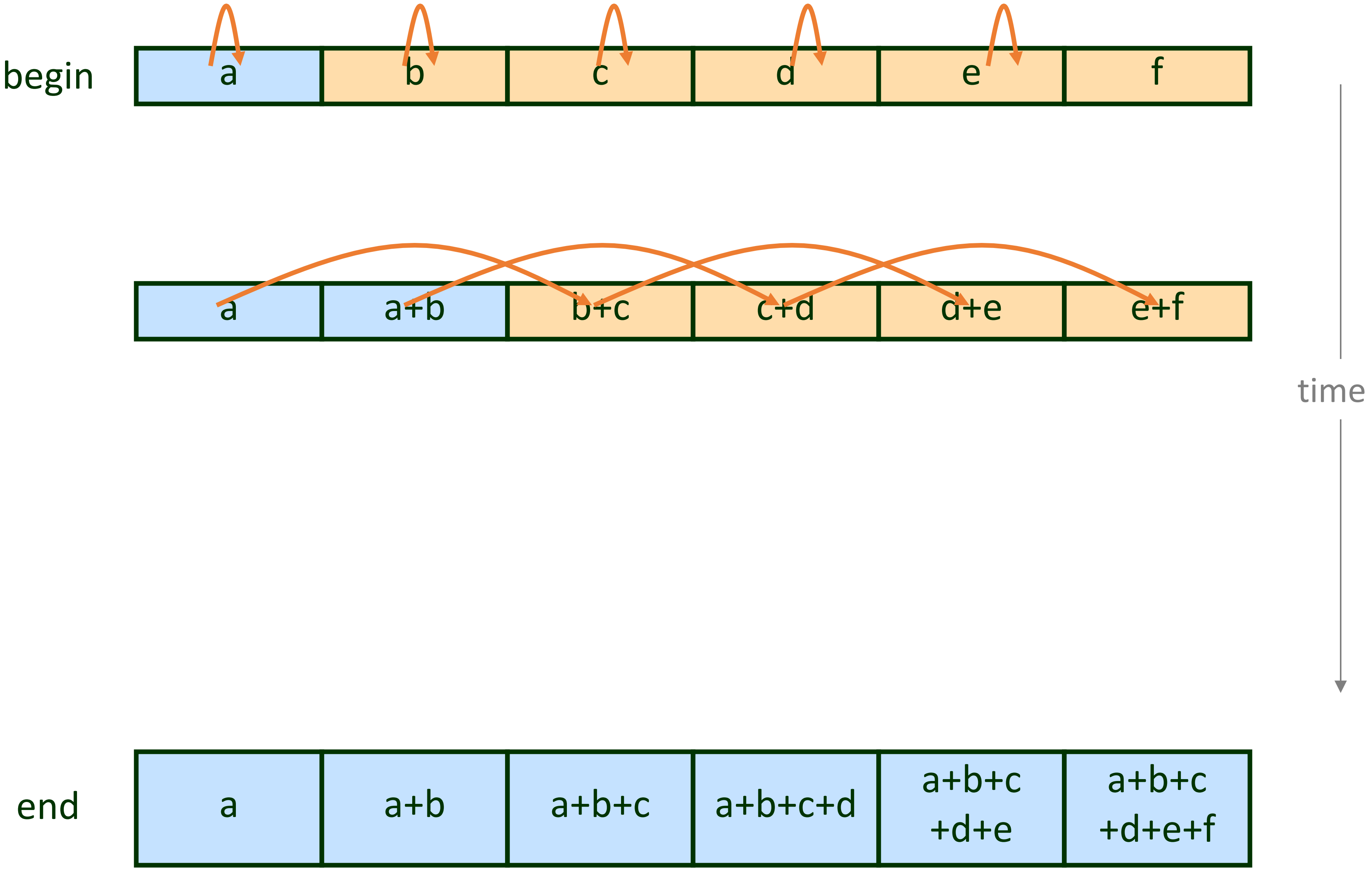
begin	a	b	c	d	e	f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Parallel Prefix Sum



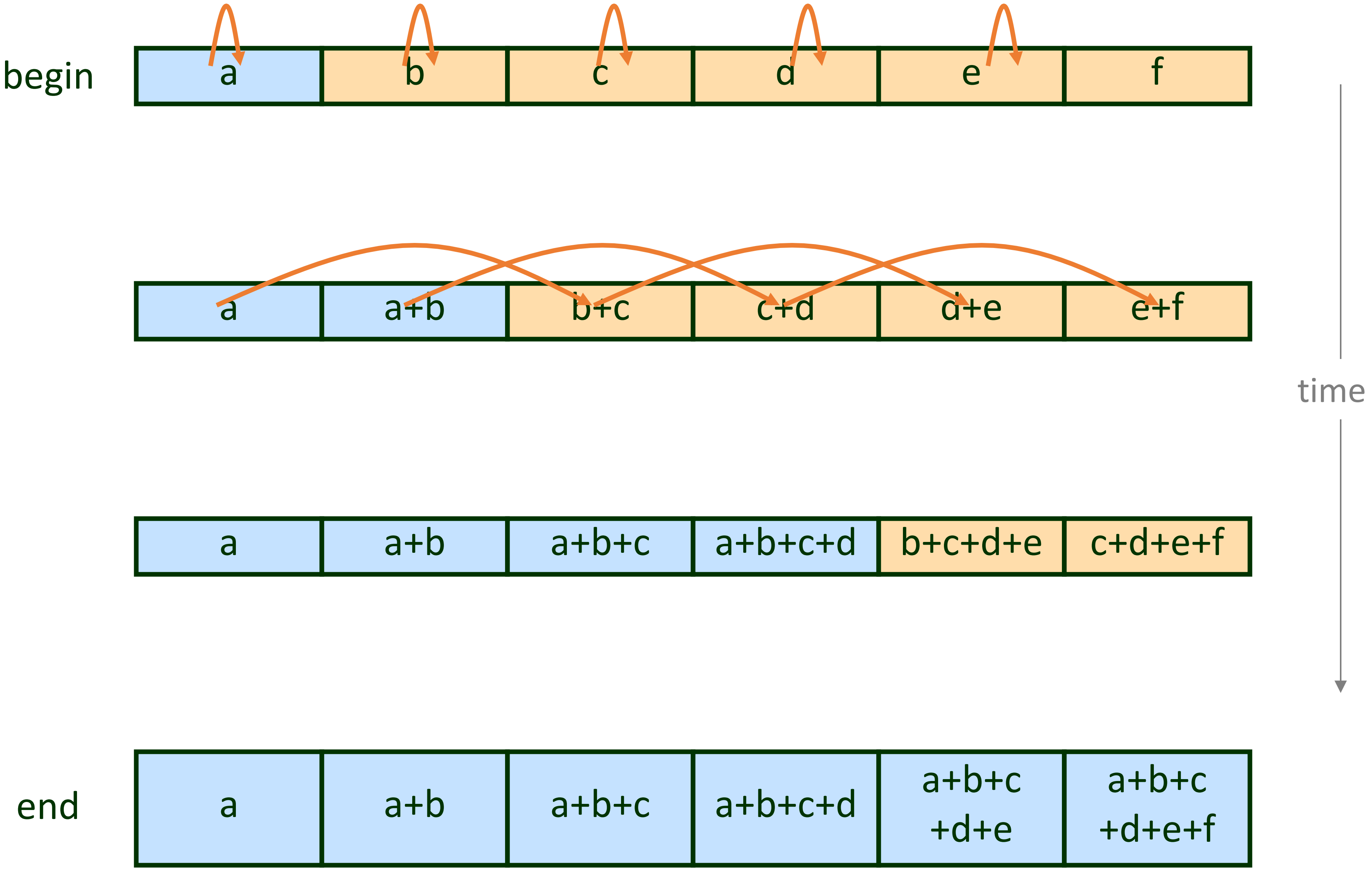
begin	a	b	c	d	e	f
	a	a+b	b+c	c+d	d+e	e+f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Parallel Prefix Sum



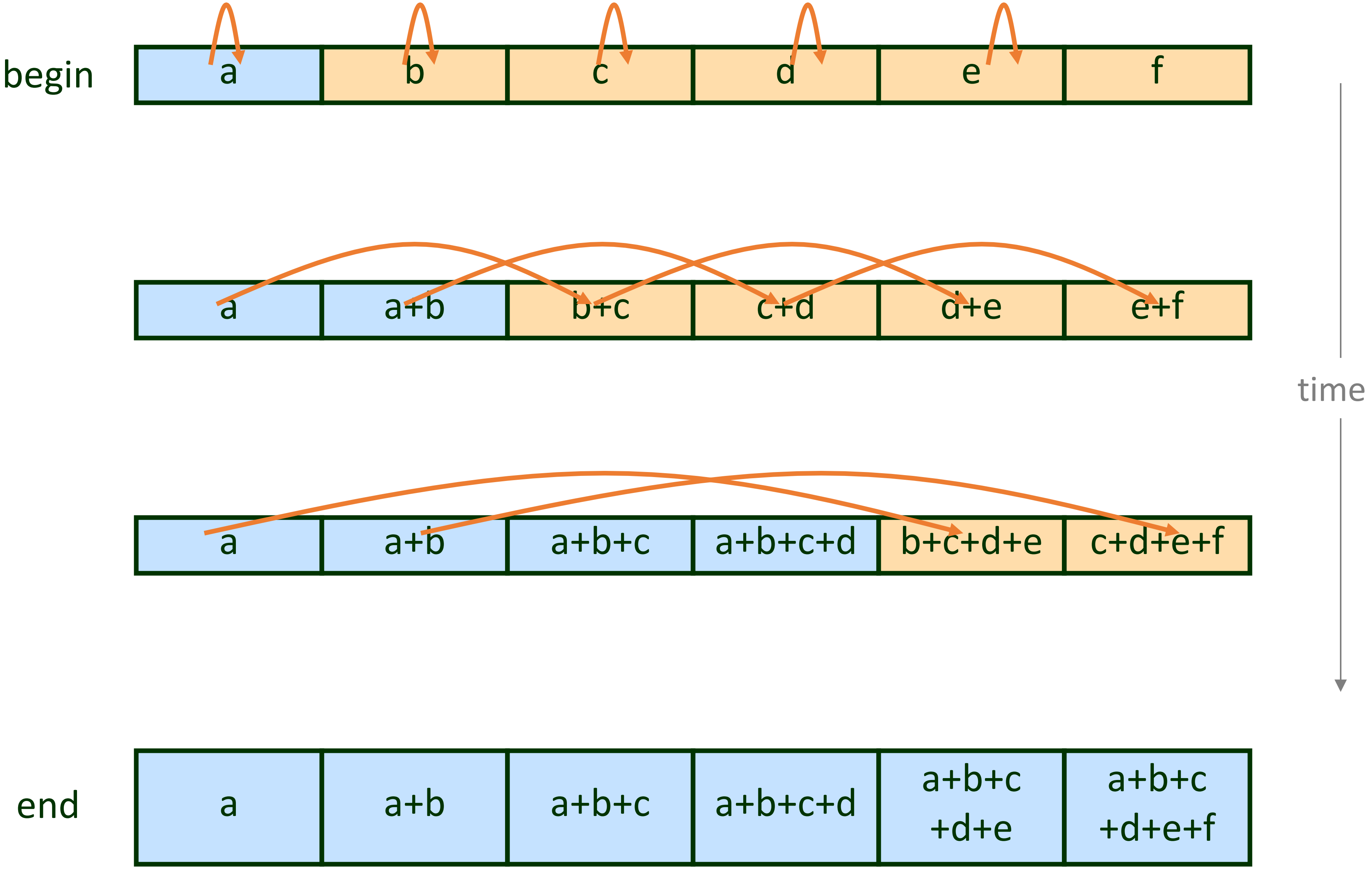
begin	a	b	c	d	e	f
	a	a+b	b+c	c+d	d+e	e+f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Parallel Prefix Sum



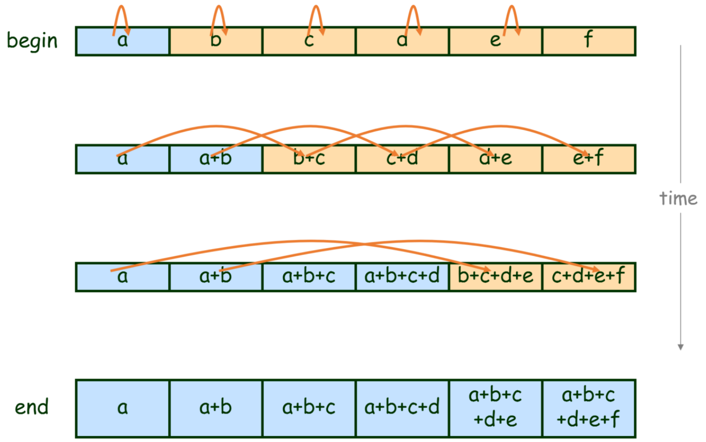
begin	a	b	c	d	e	f
	a	a+b	b+c	c+d	d+e	e+f
	a	a+b	a+b+c	a+b+c+d	b+c+d+e	c+d+e+f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Parallel Prefix Sum



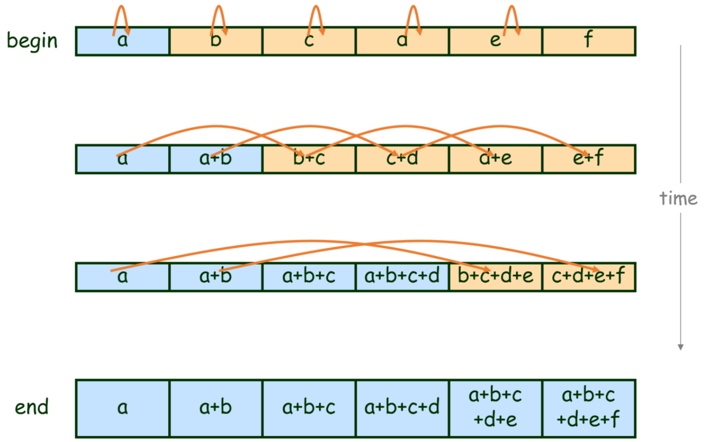
begin	a	b	c	d	e	f
	a	a+b	b+c	c+d	d+e	e+f
	a	a+b	a+b+c	a+b+c+d	b+c+d+e	c+d+e+f
end	a	a+b	a+b+c	a+b+c+d	a+b+c +d+e	a+b+c +d+e+f

Pthreads Parallel Prefix Sum


begin
a b c d e f
a a+b b+c c+d d+e e+f
a a+b a+b+c a+b+c+d b+c+d+e c+d+e+f
end
a a+b a+b+c a+b+c+d a+b+c +d+e a+b+c +d+e+f
time

```
int g_values[N] = { a, b, c, d, e, f };

void prefix_sum_thread(void * param) {

  int i;
  int id = *((int*)param);
  int stride = 0;

  for(stride=1; stride<=N/2; stride<<1) {
    g_values[id+stride] += g_values[id];
  }

}
```

Pthreads Parallel Prefix Sum



begin | a | b | c | d | e | f

| a | a+b | b+c | c+d | d+e | e+f

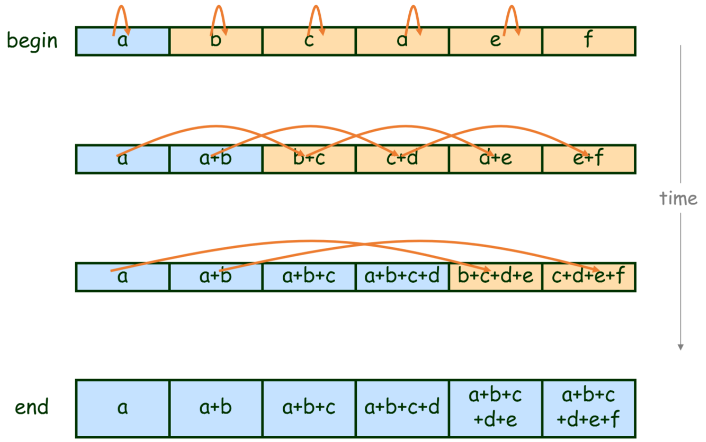
| a | a+b | a+b+c | a+b+c+d | b+c+d+e | c+d+e+f

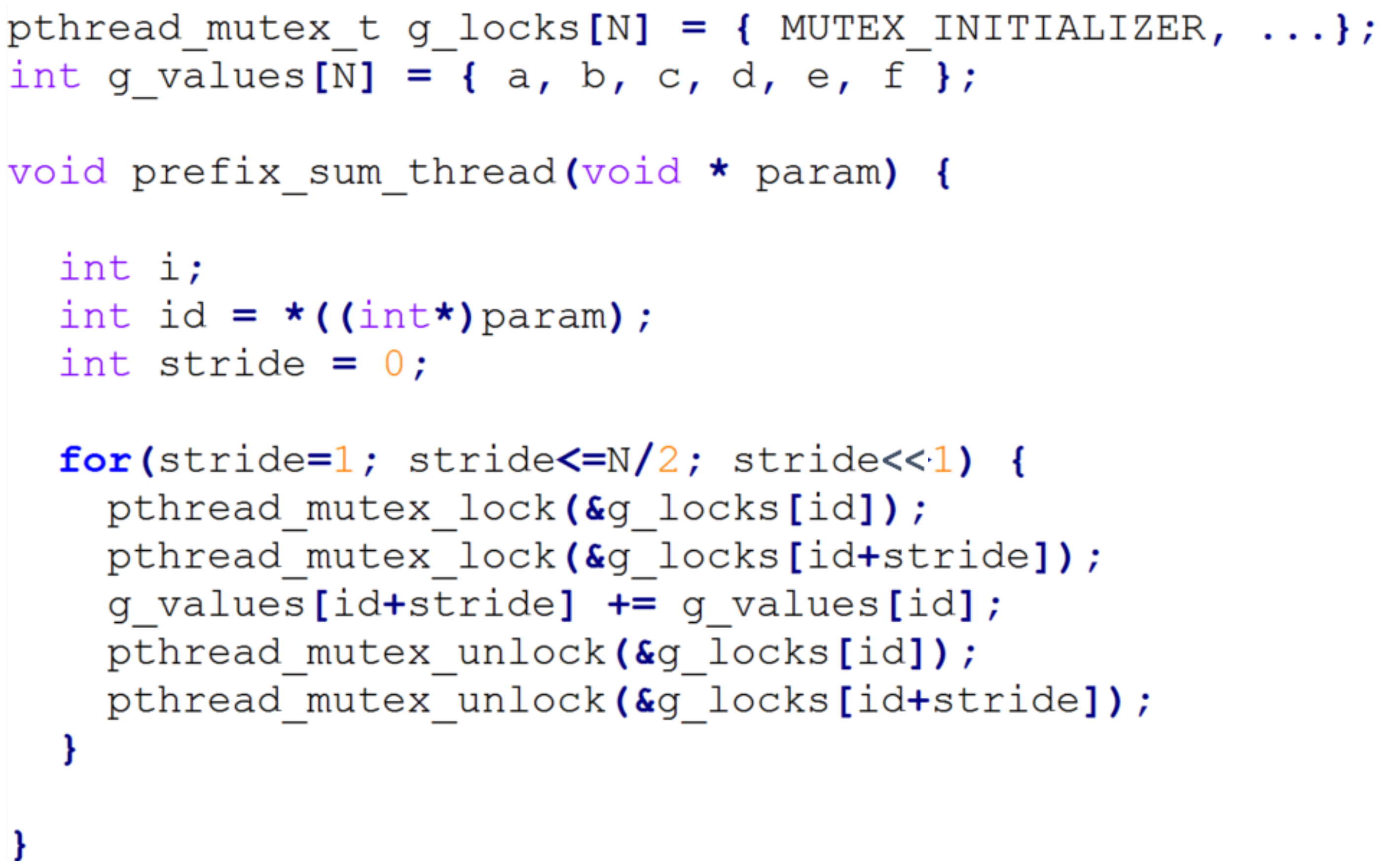
end | a | a+b | a+b+c | a+b+c+d | a+b+c+d+e | a+b+c+d+e+f

```
int g_values[N] = { a, b, c, d, e, f };

void prefix_sum_thread(void * param) {

  int i;
  int id = *((int*)param);
  int stride = 0;

  for(stride=1; stride<=N/2; stride<<1) {
    g_values[id+stride] += g_values[id];
  }

}
```

Will this work?

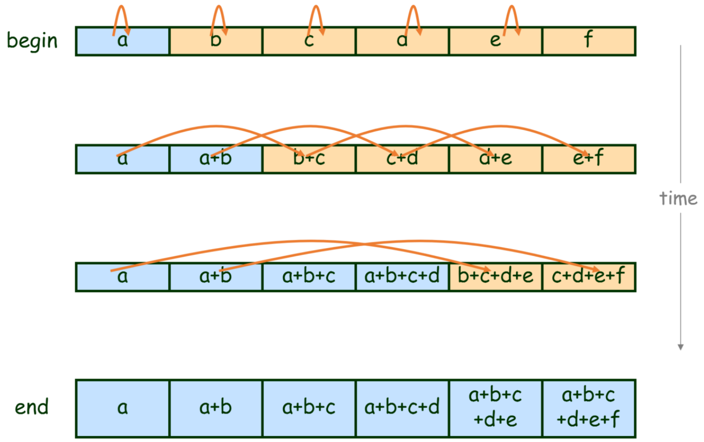
Pthreads Parallel Prefix Sum





```
pthread_mutex_t g_locks[N] = { MUTEX_INITIALIZER, ...};
int g_values[N] = { a, b, c, d, e, f };

void prefix_sum_thread(void * param) {

  int i;
  int id = *((int*)param);
  int stride = 0;

  for(stride=1; stride<=N/2; stride<<1) {
    pthread_mutex_lock(&g_locks[id]);
    pthread_mutex_lock(&g_locks[id+stride]);
    g_values[id+stride] += g_values[id];
    pthread_mutex_unlock(&g_locks[id]);
    pthread_mutex_unlock(&g_locks[id+stride]);
  }

}
```

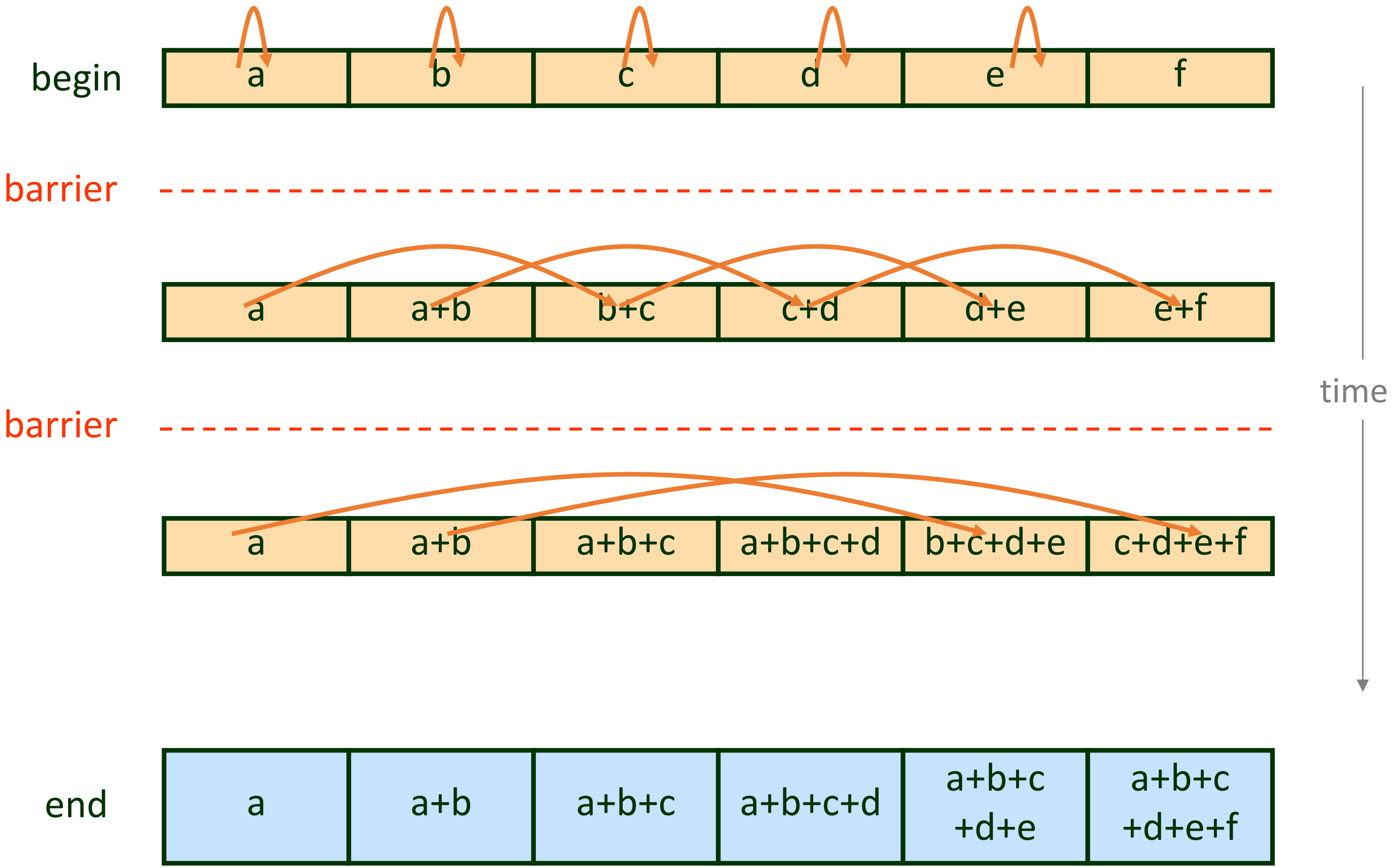
Pthreads Parallel Prefix Sum


begin
a | b | c | d | e | f
a | a+b | b+c | c+d | d+e | e+f
a | a+b | a+b+c | a+b+c+d | b+c+d+e | c+d+e+f
end
a | a+b | a+b+c | a+b+c+d | a+b+c +d+e | a+b+c +d+e+f
time

```
pthread_mutex_t g_locks[N] = { MUTEX_INITIALIZER, ...};
int g_values[N] = { a, b, c, d, e, f };

void prefix_sum_thread(void * param) {

  int i;
  int id = *((int*)param);
  int stride = 0;

  for(stride=1; stride<=N/2; stride<<1) {
    pthread_mutex_lock(&g_locks[id]);
    pthread_mutex_lock(&g_locks[id+stride]);
    g_values[id+stride] += g_values[id];
    pthread_mutex_unlock(&g_locks[id]);
    pthread_mutex_unlock(&g_locks[id+stride]);
  }

}
```

fixed?

Parallel Prefix Sum



begin: a | b | c | d | e | f

barrier

a | a+b | b+c | c+d | d+e | e+f

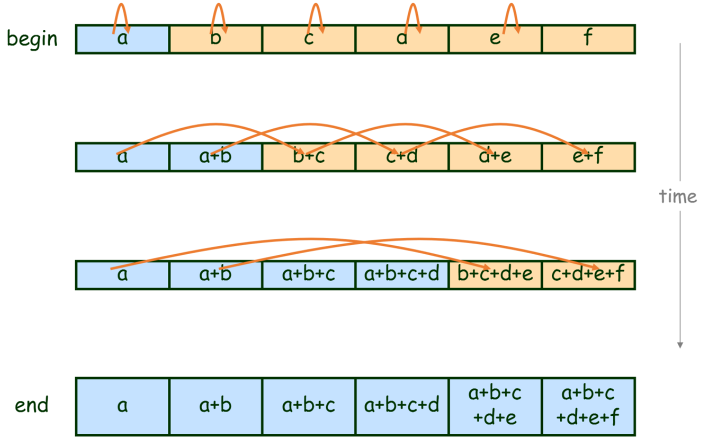
barrier

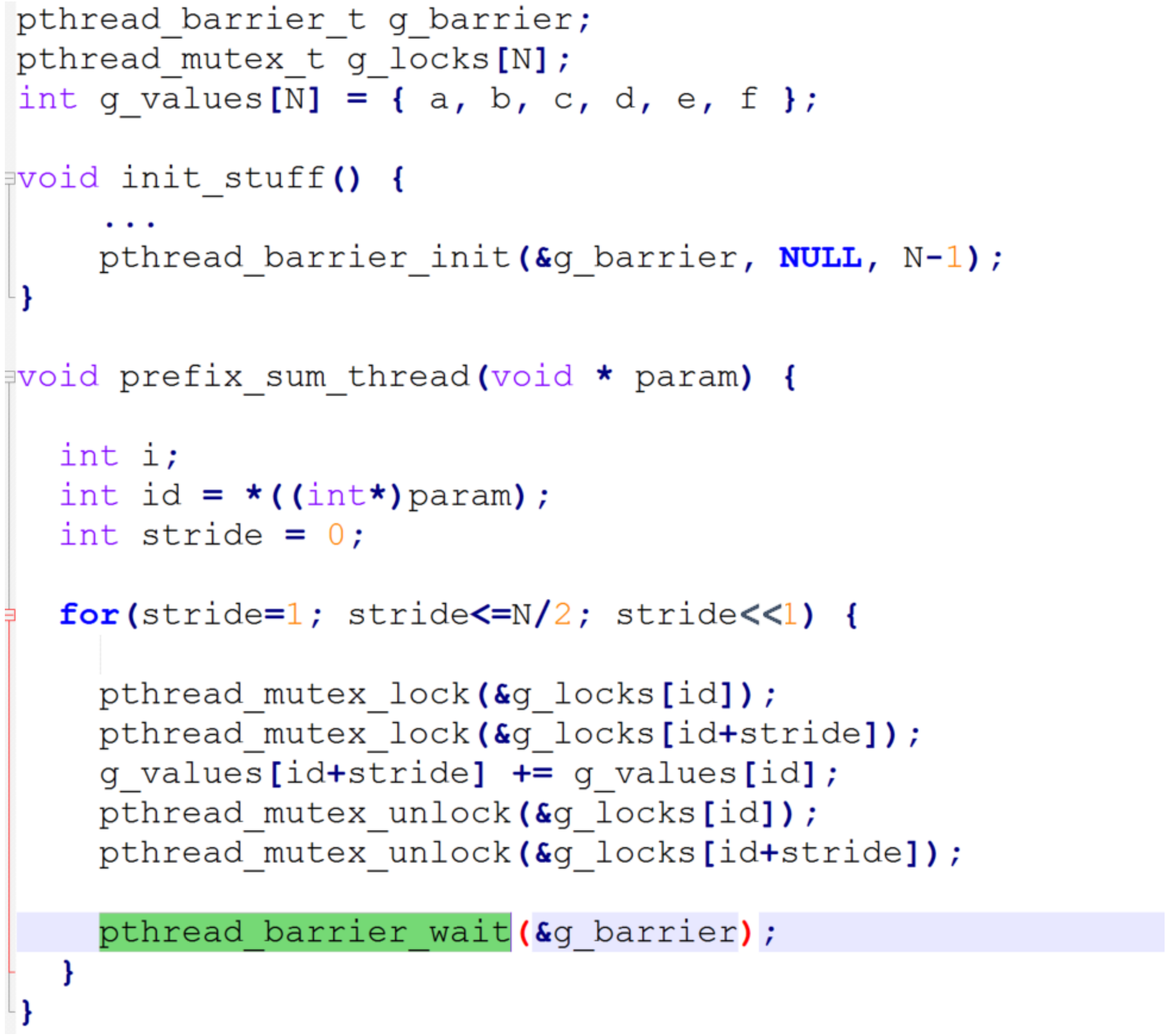
a | a+b | a+b+c | a+b+c+d | b+c+d+e | c+d+e+f

end: a | a+b | a+b+c | a+b+c+d | a+b+c +d+e | a+b+c +d+e+f

time

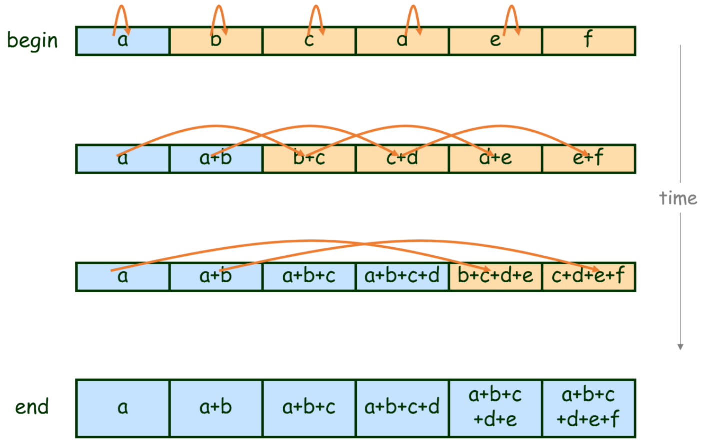
Pthreads Parallel Prefix Sum

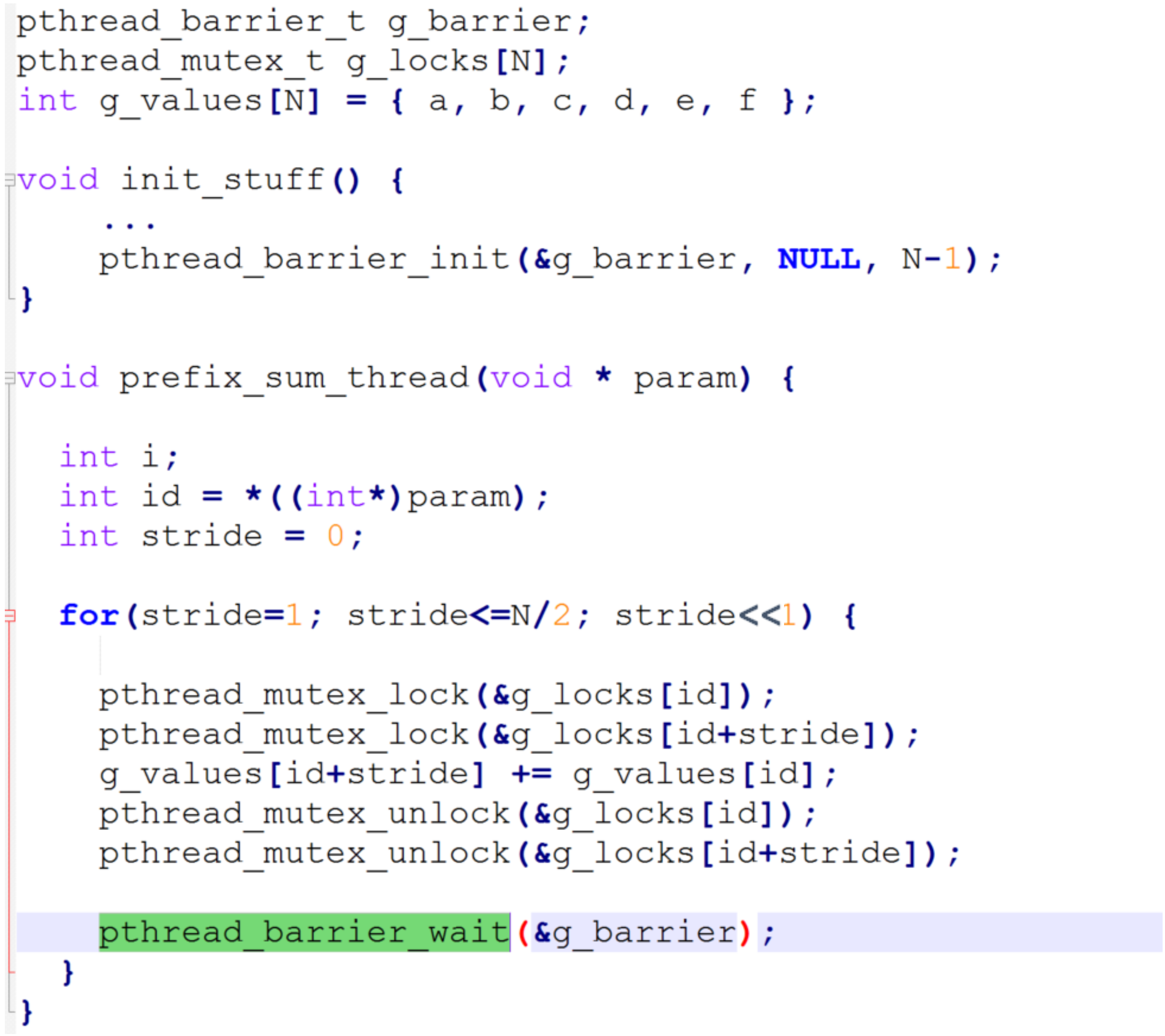




```
pthread_barrier_t g_barrier;
pthread_mutex_t g_locks[N];
int g_values[N] = { a, b, c, d, e, f };

void init_stuff() {
    ...
    pthread_barrier_init(&g_barrier, NULL, N-1);
}

void prefix_sum_thread(void * param) {

  int i;
  int id = *((int*)param);
  int stride = 0;

  for(stride=1; stride<=N/2; stride<<1) {

    pthread_mutex_lock(&g_locks[id]);
    pthread_mutex_lock(&g_locks[id+stride]);
    g_values[id+stride] += g_values[id];
    pthread_mutex_unlock(&g_locks[id]);
    pthread_mutex_unlock(&g_locks[id+stride]);

    pthread_barrier_wait(&g_barrier);
  }
}
```

Pthreads Parallel Prefix Sum





```
pthread_barrier_t g_barrier;
pthread_mutex_t g_locks[N];
int g_values[N] = { a, b, c, d, e, f };

void init_stuff() {
    ...
    pthread_barrier_init(&g_barrier, NULL, N-1);
}

void prefix_sum_thread(void * param) {

  int i;
  int id = *((int*)param);
  int stride = 0;

  for(stride=1; stride<=N/2; stride<<1) {

    pthread_mutex_lock(&g_locks[id]);
    pthread_mutex_lock(&g_locks[id+stride]);
    g_values[id+stride] += g_values[id];
    pthread_mutex_unlock(&g_locks[id]);
    pthread_mutex_unlock(&g_locks[id+stride]);

    pthread_barrier_wait(&g_barrier);
  }
}
```

fixed?

Barrier Goals

Desirable barrier properties:

Barrier Goals

Desirable barrier properties:

- Low shared memory space complexity

Barrier Goals

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- Low contention on shared objects

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- Symmetric: same amount of work for all processes

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- Algorithm simplicity

Barrier Goals

Desirable barrier properties:

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- Low contention on shared objects
- Low shared memory references per process
- No need for shared memory initialization
- Symmetric: same amount of work for all processes
- Algorithm simplicity
- Simple basic primitive

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Desirable barrier properties:

- Low shared memory space complexity
- Low contention on shared objects
- Low shared memory references per process
- No need for shared memory initialization
- Symmetric: same amount of work for all processes
- Algorithm simplicity
- Simple basic primitive
- Minimal propagation time

Barrier Goals

Desirable barrier properties:

- Low shared memory space complexity
- Low contention on shared objects
- Low shared memory references per process
- No need for shared memory initialization
- Symmetric: same amount of work for all processes
- Algorithm simplicity
- Simple basic primitive
- Minimal propagation time
- Reusability of the barrier (must!)

Barrier Building Blocks

- Conditions
- Semaphores
- Atomic Bit
- Atomic Register
- Fetch-and-increment register
- Test and set bits
- Read-Modify-Write register

Barrier with Semaphores

Barrier using Semaphores

Algorithm for N threads

Barrier using Semaphores

Algorithm for N threads

Barrier using Semaphores

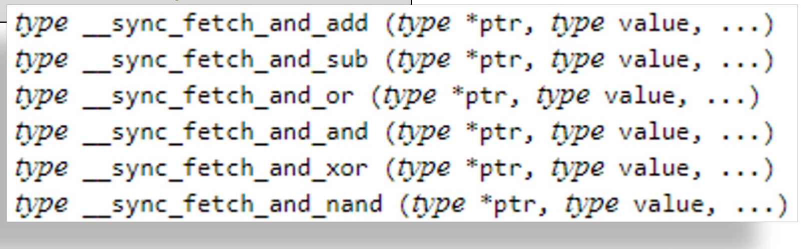
Algorithm for N threads

```
shared    sem_t arrival = 1;            // sem_init(&arrival, NULL, 1)
          sem_t departure = 0;          // sem_init(&departure, NULL, 0)
          atomic int counter = 0;       // (gcc intrinsics are verbose)
```

Barrier using Semaphores

Algorithm for N threads

```
shared    sem_t arrival = 1;            // sem_init(&arrival, NULL, 1)
          sem_t departure = 0;          // sem_init(&departure, NULL, 0)
          atomic int counter = 0;       // (gcc intrinsics are verbose)
```



```
type __sync_fetch_and_add (type *ptr, type value, ...)
type __sync_fetch_and_sub (type *ptr, type value, ...)
type __sync_fetch_and_or (type *ptr, type value, ...)
type __sync_fetch_and_and (type *ptr, type value, ...)
type __sync_fetch_and_xor (type *ptr, type value, ...)
type __sync_fetch_and_nand (type *ptr, type value, ...)
```

Barrier using Semaphores

Algorithm for N threads

```
shared    sem_t arrival = 1;           // sem_init(&arrival, NULL, 1)
          sem_t departure = 0;         // sem_init(&departure, NULL, 0)
          atomic int counter = 0;      // (gcc intrinsics are verbose)
```

Barrier using Semaphores

Algorithm for N threads

```
shared    sem_t arrival = 1;          // sem_init(&arrival, NULL, 1)
          sem_t departure = 0;        // sem_init(&departure, NULL, 0)
          atomic int counter = 0;     // (gcc intrinsics are verbose)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Barrier using Semaphores

Algorithm for N threads

```
shared      sem_t arrival = 1;            // sem_init(&arrival, NULL, 1)
            sem_t departure = 0;          // sem_init(&departure, NULL, 0)
            atomic int counter = 0;       // (gcc intrinsics are verbose)
```

Phase I

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
```

Phase II

```
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Barrier using Semaphores

Algorithm for N threads

```
shared    sem_t arrival = 1;           // sem_init(&arrival, NULL, 1)
          sem_t departure = 0;         // sem_init(&departure, NULL, 0)
          atomic int counter = 0;      // (gcc intrinsics are verbose)
```

Phase I

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
```

Phase II

```
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

First N-1 threads post on arrival, wait on departure

Barrier using Semaphores

Algorithm for N threads

```
shared     sem_t arrival = 1;             // sem_init(&arrival, NULL, 1)
           sem_t departure = 0;           // sem_init(&departure, NULL, 0)
           atomic int counter = 0;        // (gcc intrinsics are verbose)
```

Phase I

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
```

Phase II

```
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

First N-1 threads post on arrival, wait on departure

Nth thread post on departure, releasing threads into phase II (what is value of arrival?)

Barrier using Semaphores

Algorithm for N threads

```
shared    sem_t arrival = 1;           // sem_init(&arrival, NULL, 1)
          sem_t departure = 0;         // sem_init(&departure, NULL, 0)
          atomic int counter = 0;      // (gcc intrinsics are verbose)
```

Phase I

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
```

Phase II

```
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

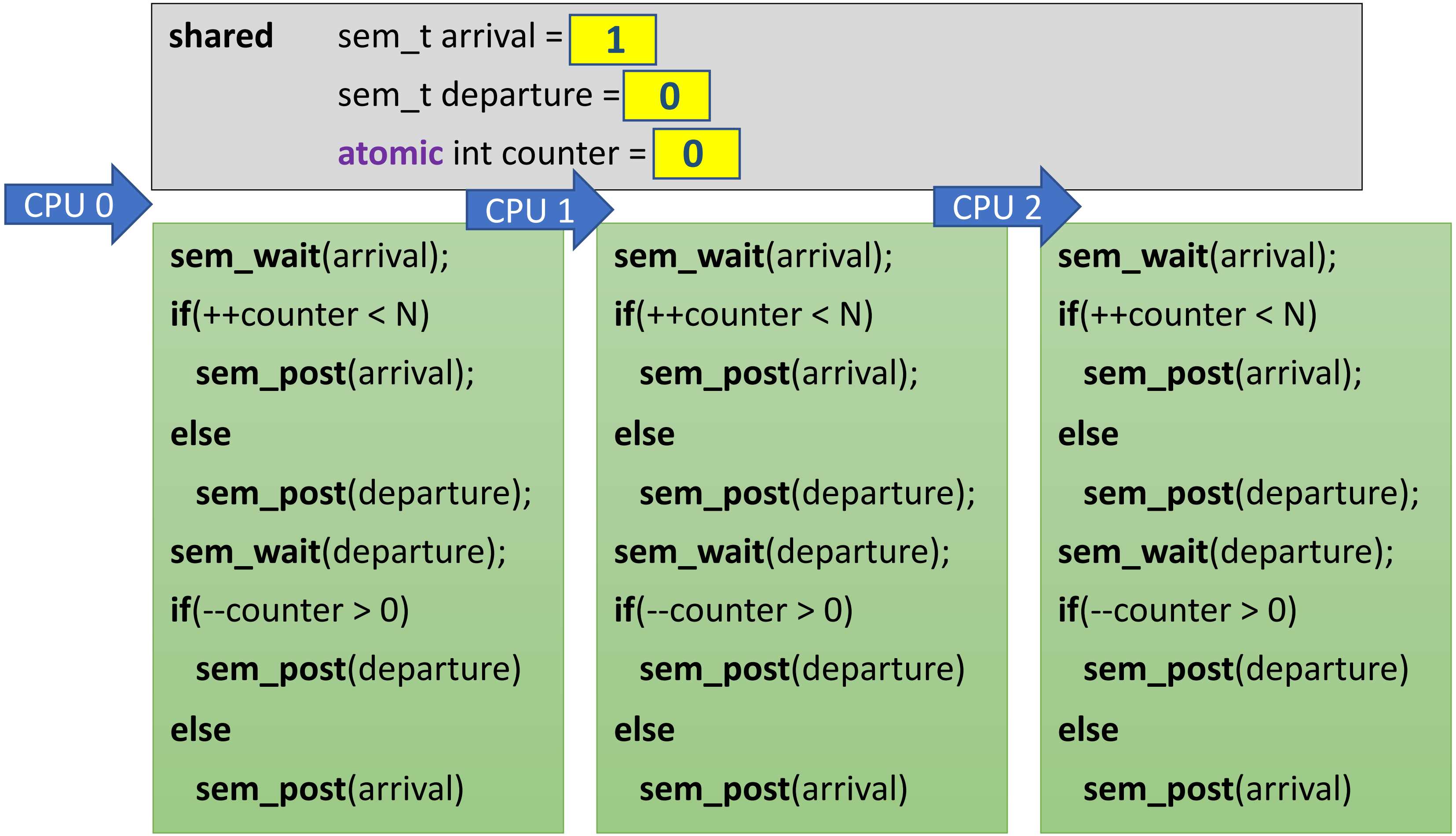
First N-1 threads post on arrival, wait on departure

Nth thread post on departure, releasing threads into phase II (what is value of arrival?)

First N-1 threads post on departure, last posts arrival

Semaphore Barrier Action Zone

N == 3



shared sem_t arrival = **1**

sem_t departure = **0**

atomic int counter = **0**

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

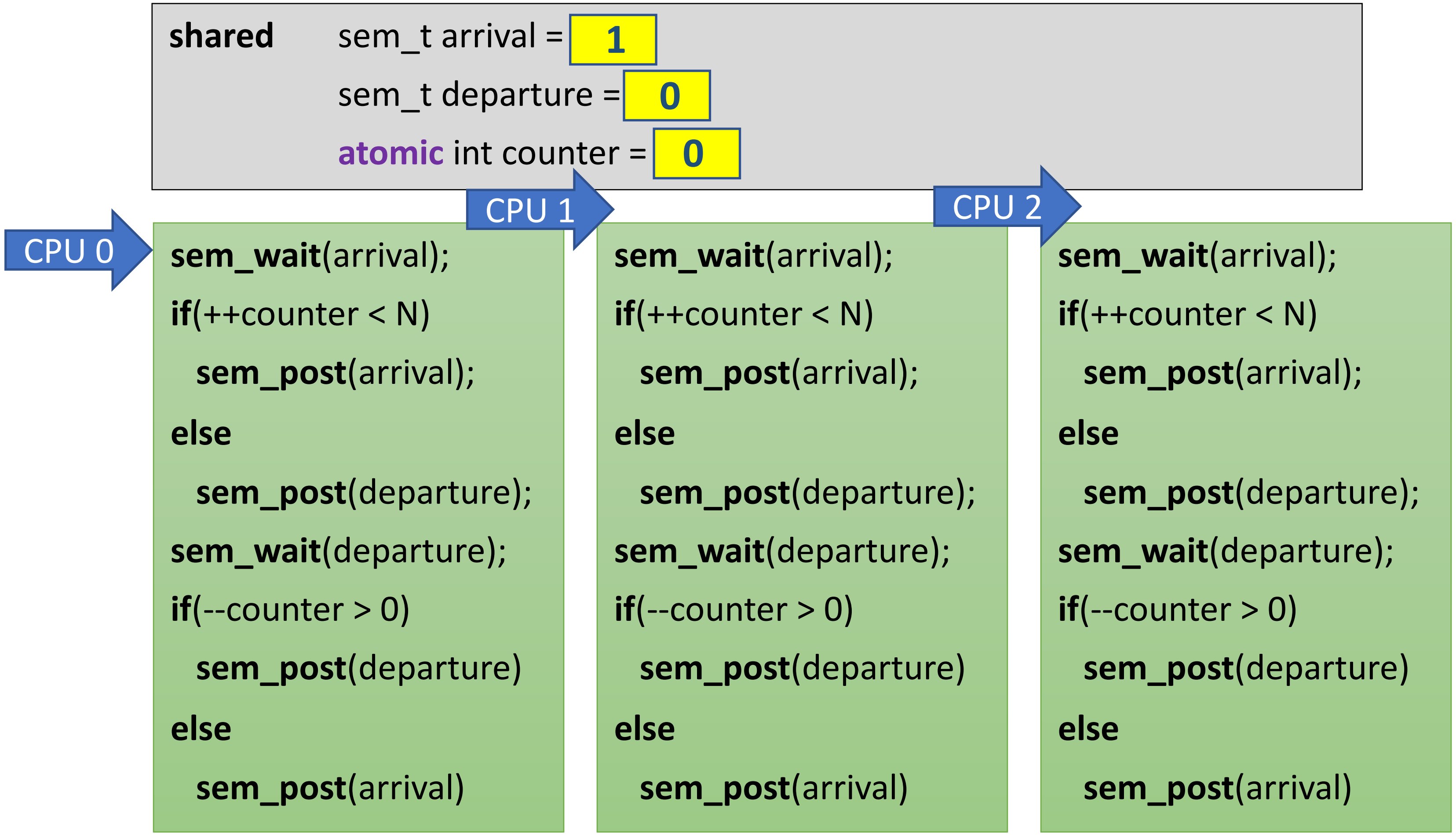
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared      sem_t arrival = 1
            sem_t departure = 0
            atomic int counter = 0
```

CPU 0

CPU 1

CPU 2

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

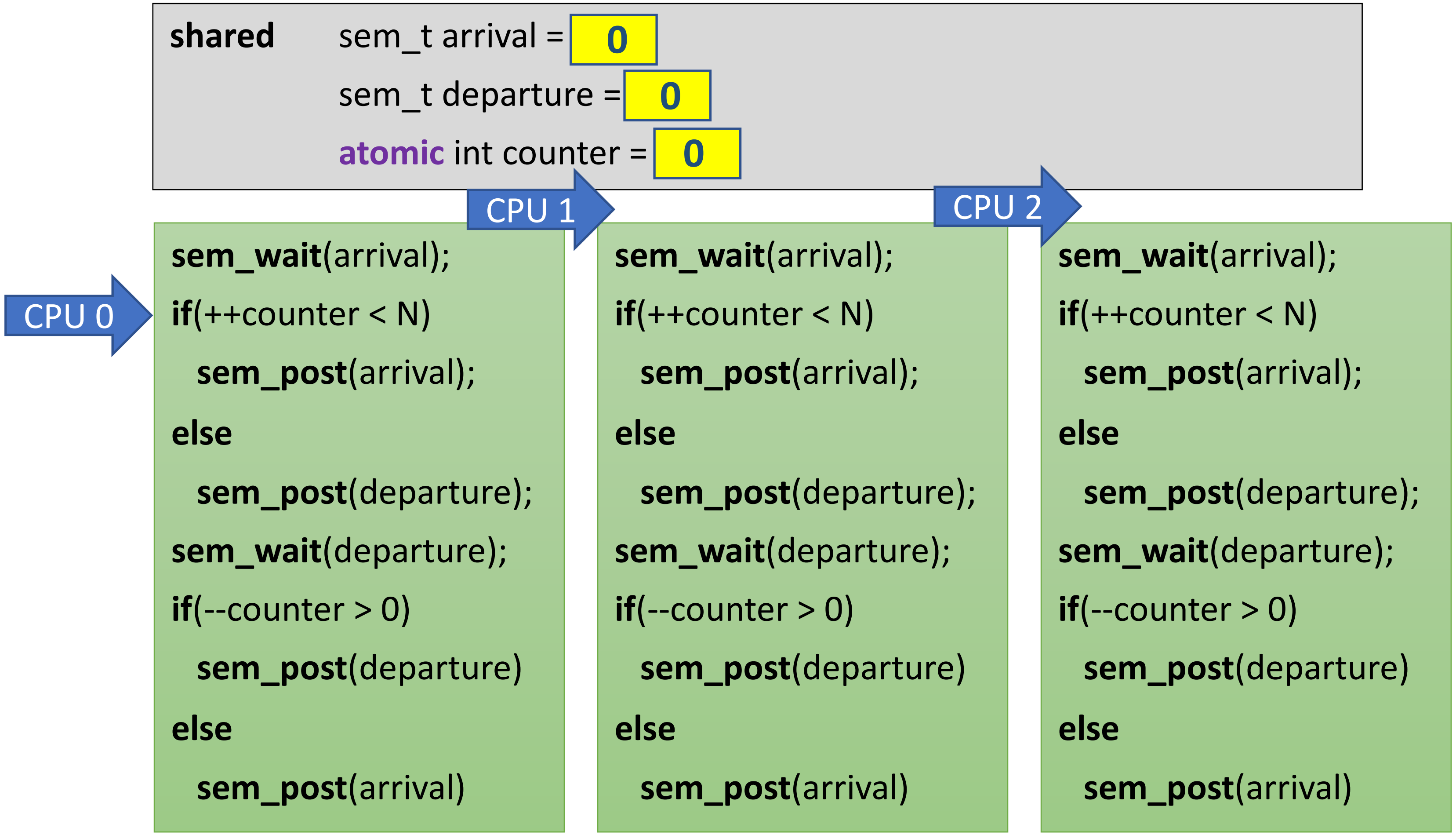
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 0
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3

```
shared      sem_t arrival = 0
            sem_t departure = 0
            atomic int counter = 0
```

CPU 0

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 1

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 2

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3

shared sem_t arrival = **0**

sem_t departure = **0**

atomic int counter = **0**

1

CPU 0

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 1

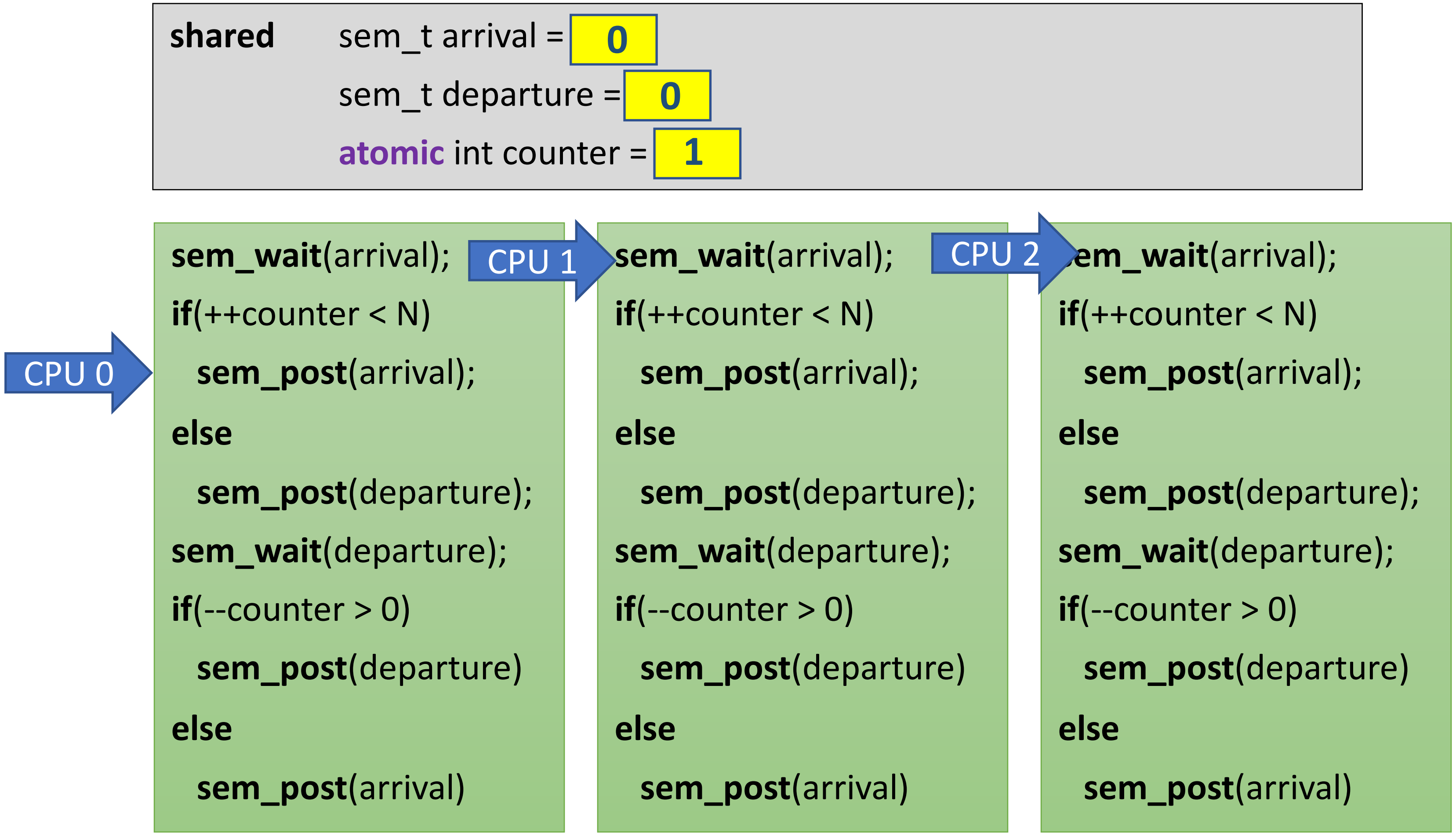
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 2

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 1
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

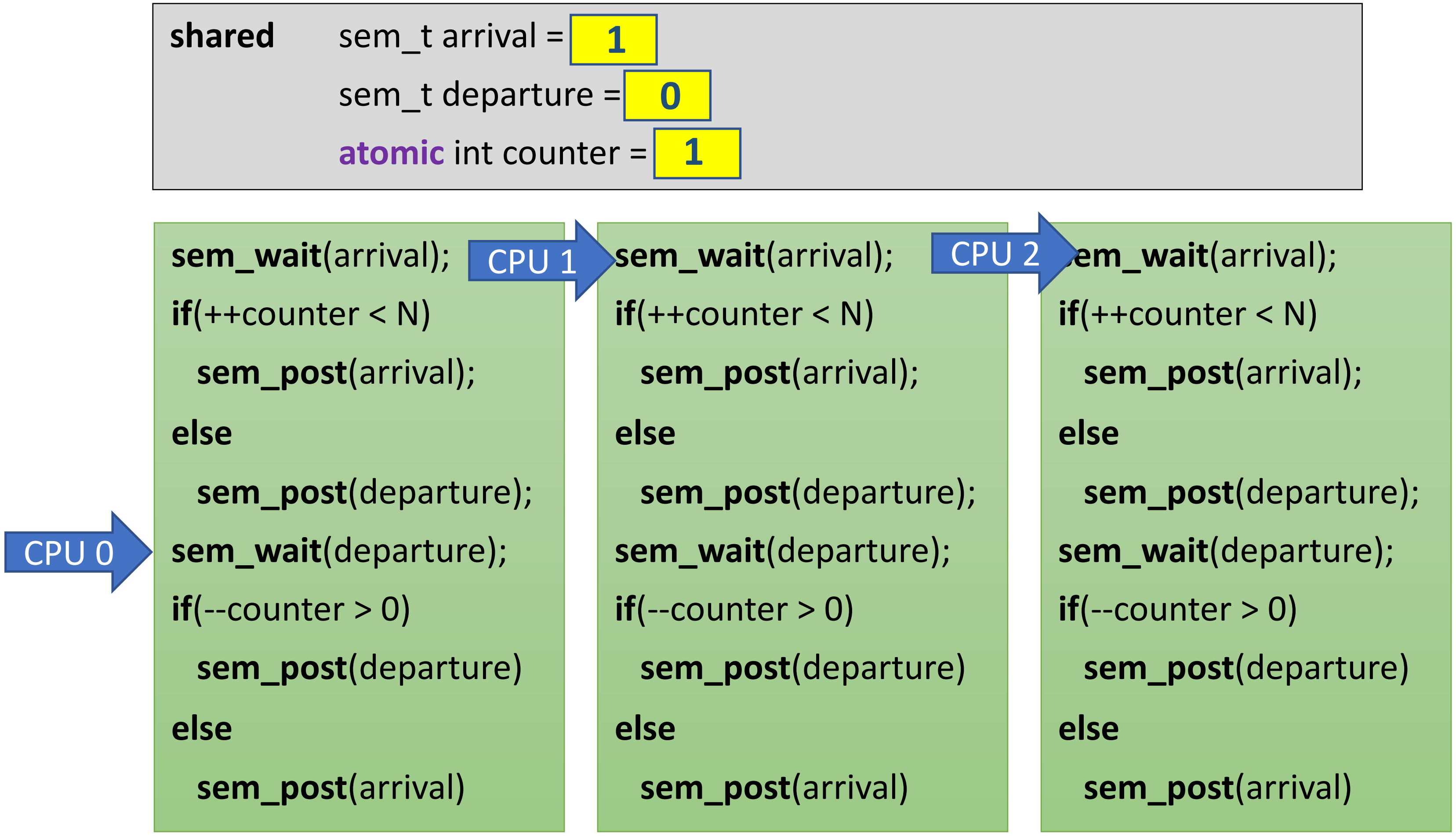
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 1
          sem_t departure = 0
          atomic int counter = 1
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

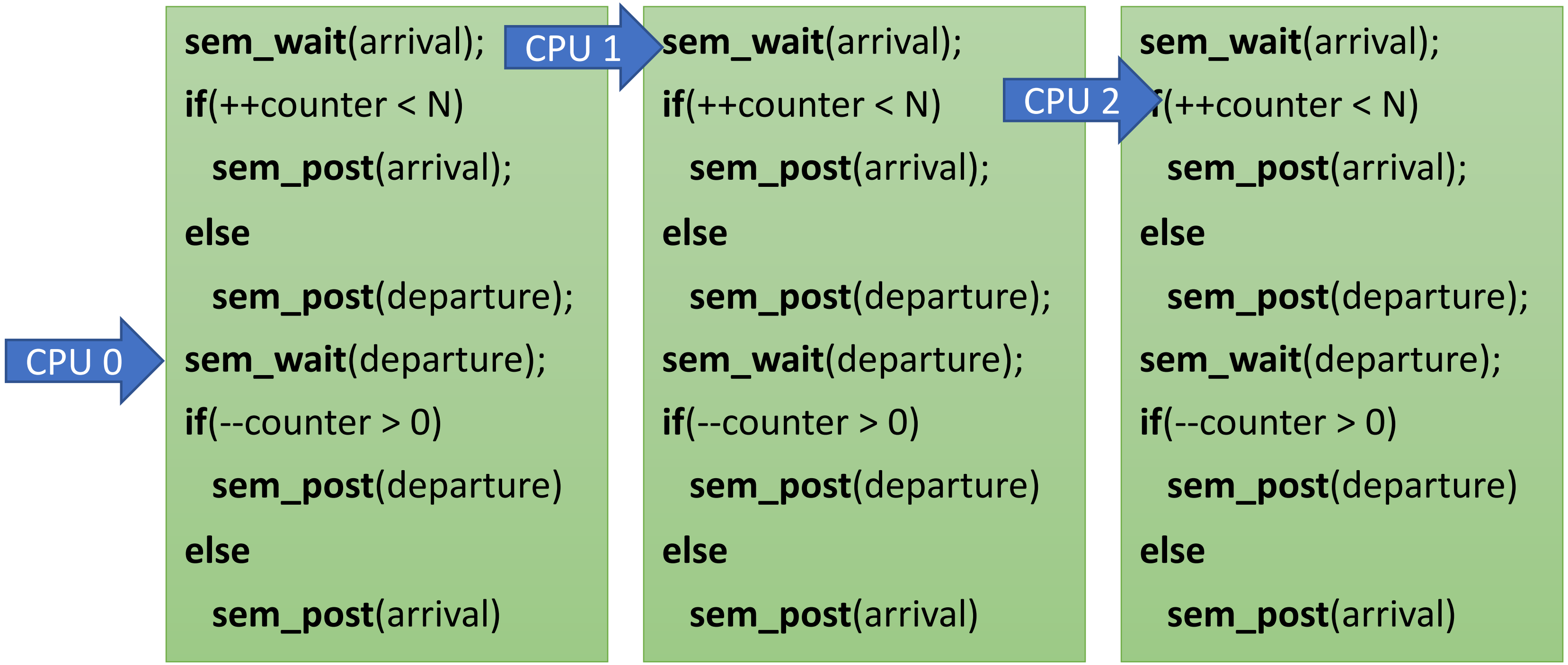
N == 3

shared sem_t arrival = **0**

sem_t departure = **0**

atomic int counter = **1**

1



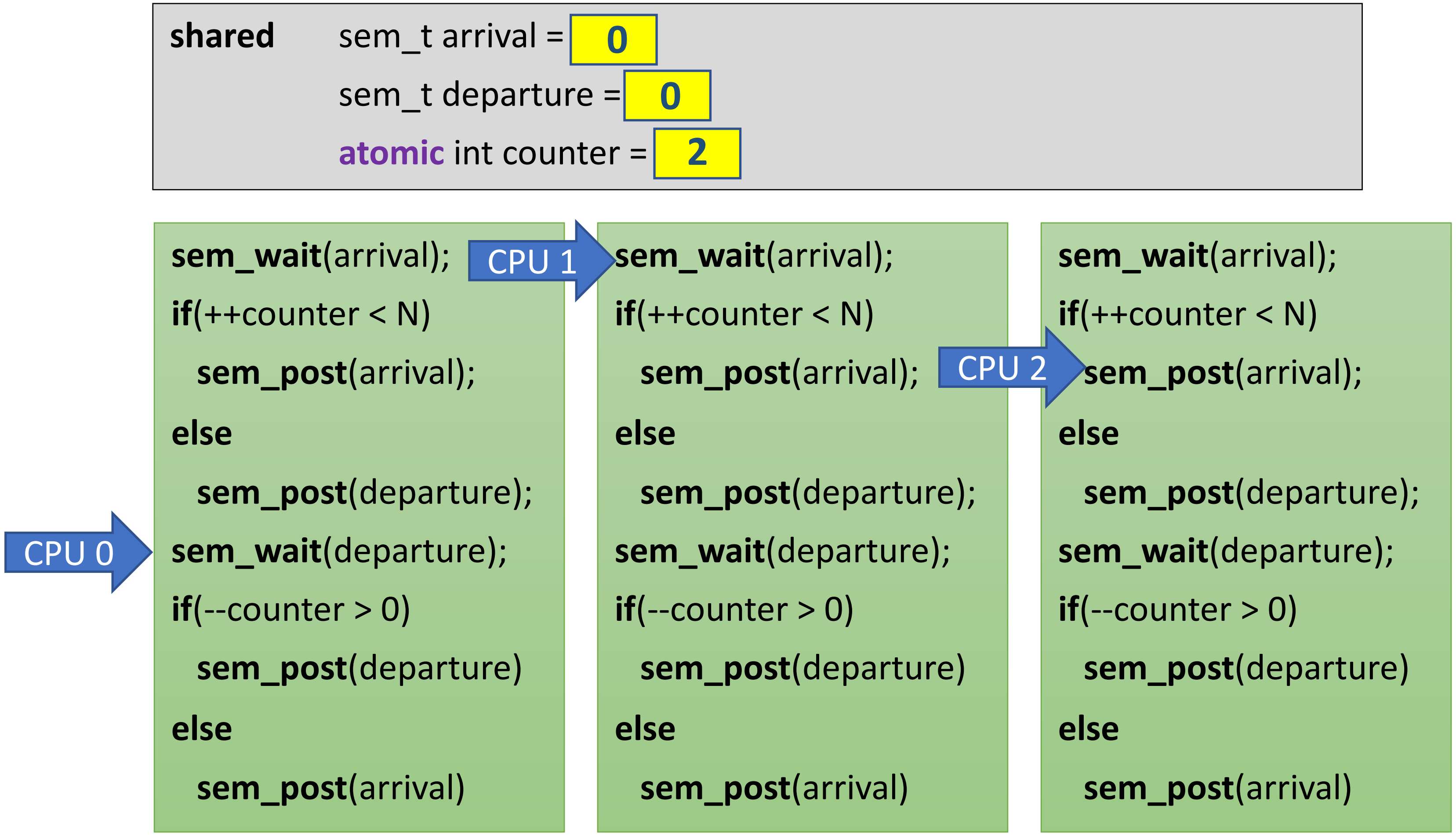
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3



shared sem_t arrival = **0**

sem_t departure = **0**

atomic int counter = **2**

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

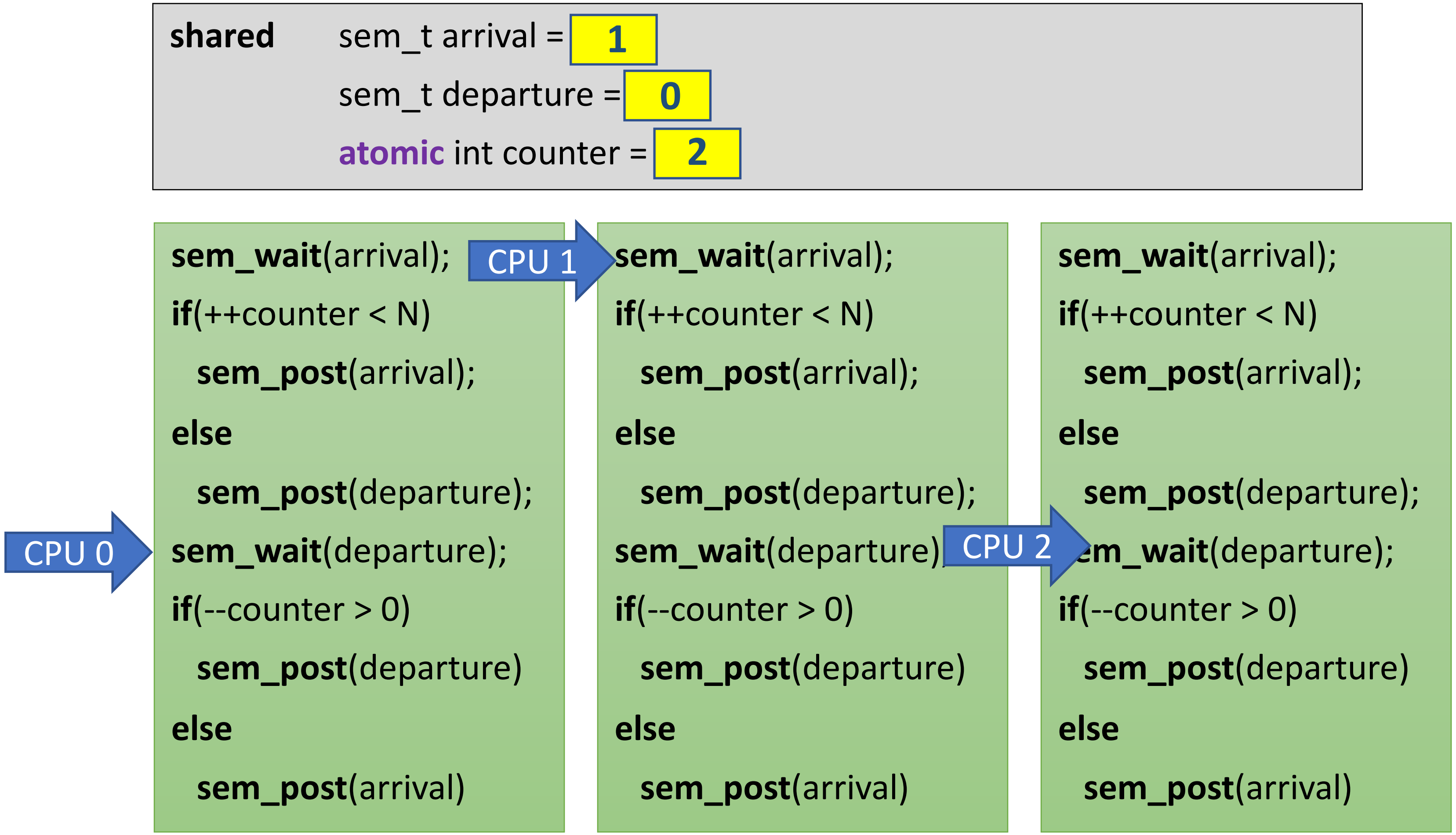
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 1
          sem_t departure = 0
          atomic int counter = 2
```

1

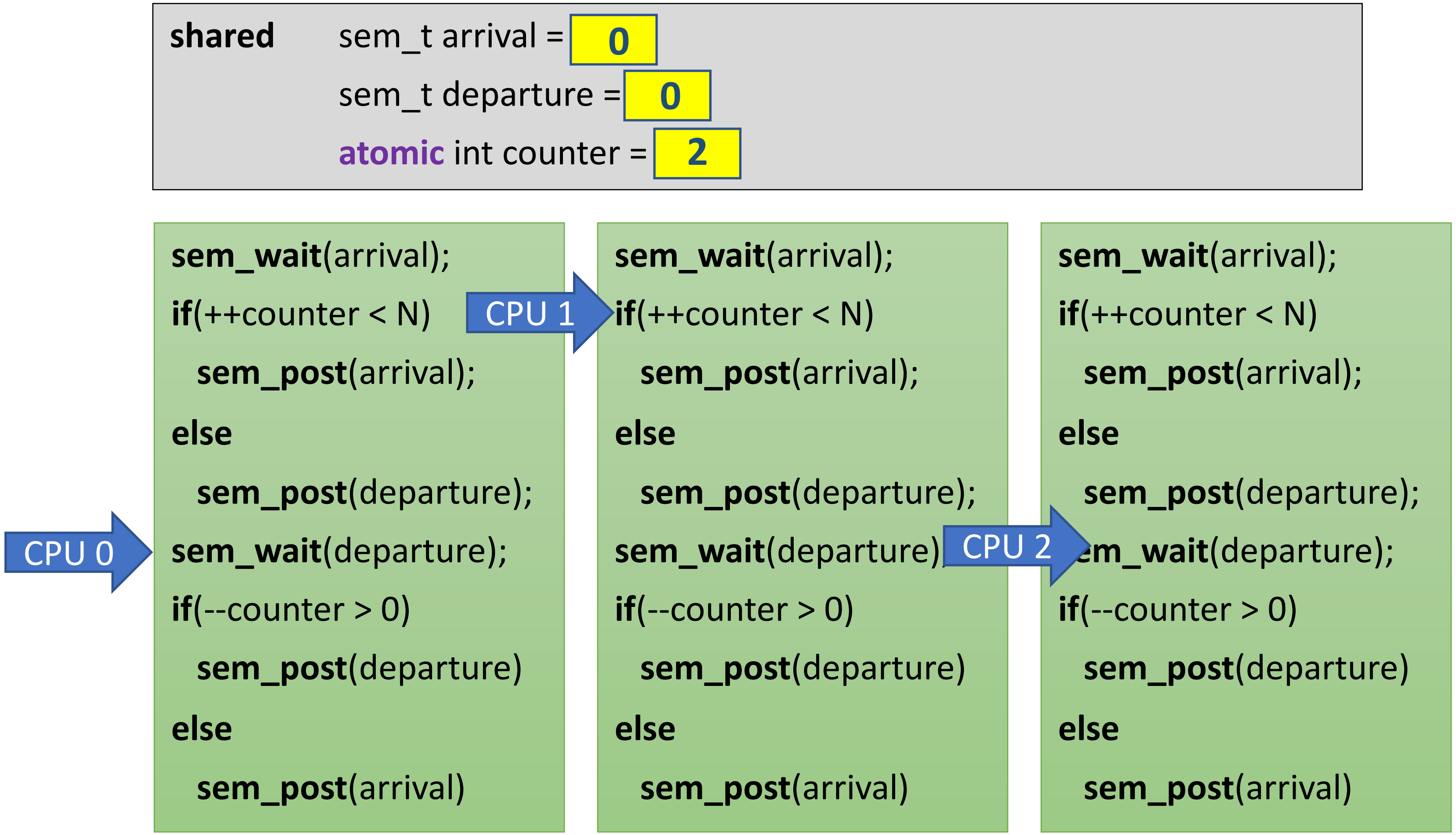
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 2
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

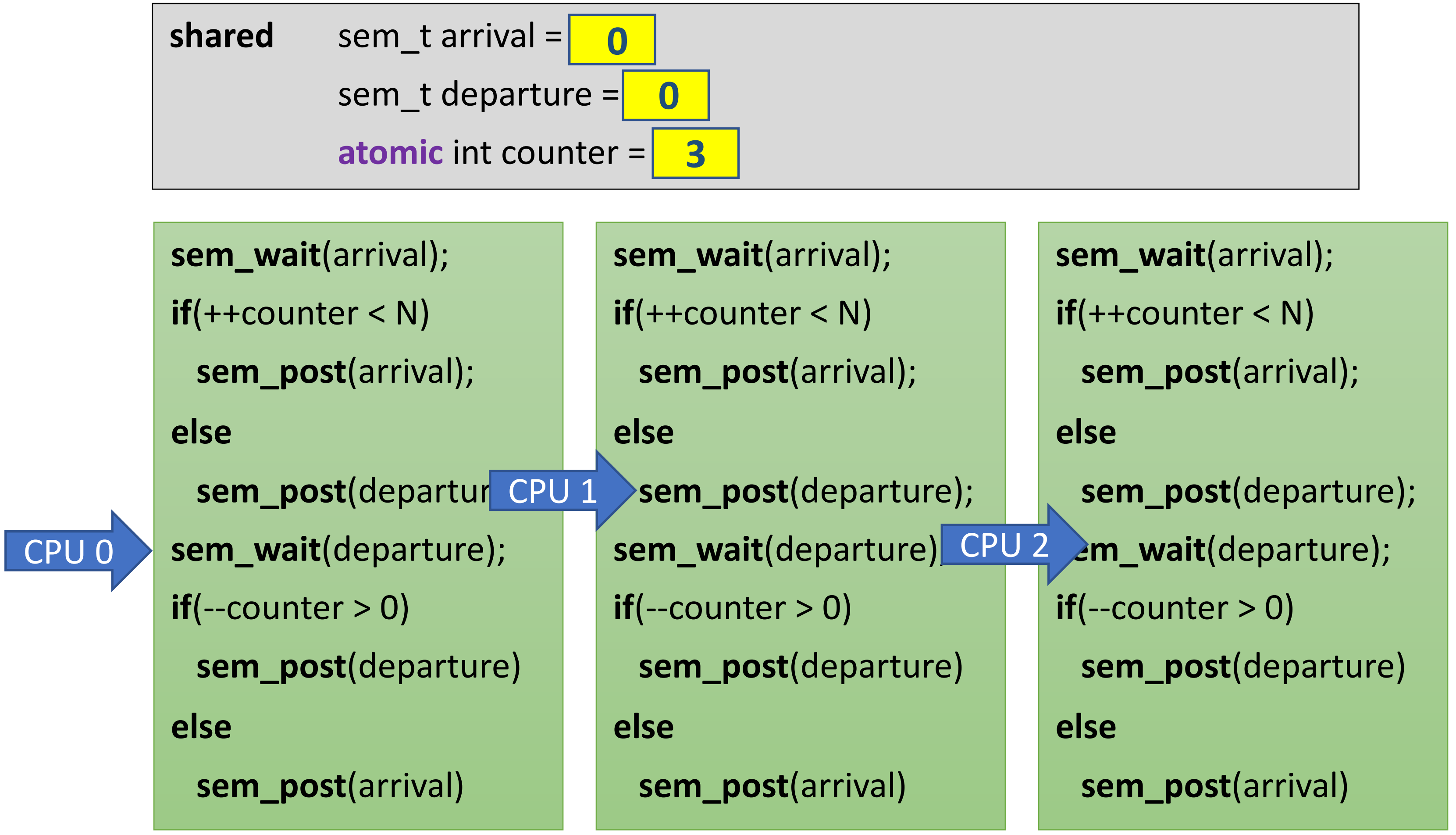
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared     sem_t arrival = 0
           sem_t departure = 0
           atomic int counter = 3
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departur
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure)
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
em_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

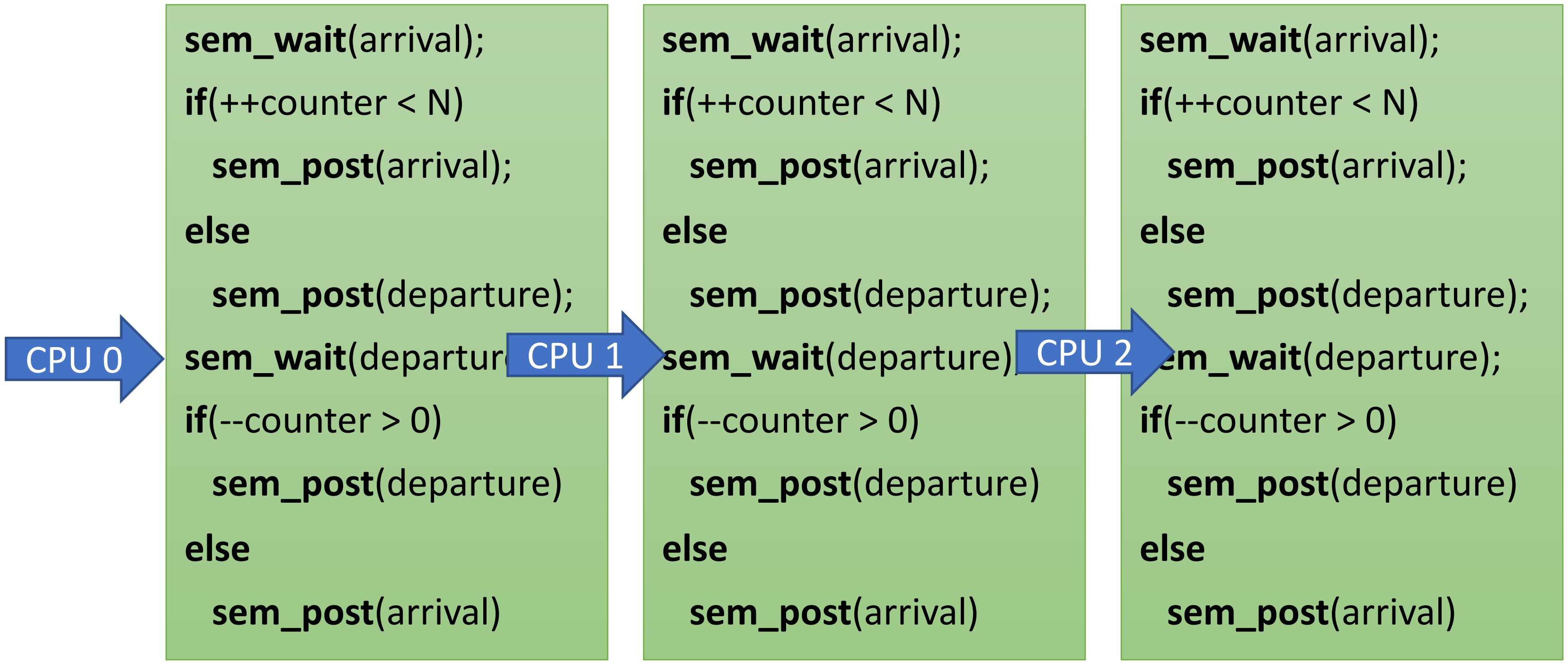
1

Semaphore Barrier Action Zone

N == 3

```
shared      sem_t arrival = 0
            sem_t departure = 1
            atomic int counter = 3
```

1



```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departur
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure)
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
em_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3

```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 3
```

1

CPU 0

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 1

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 2

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

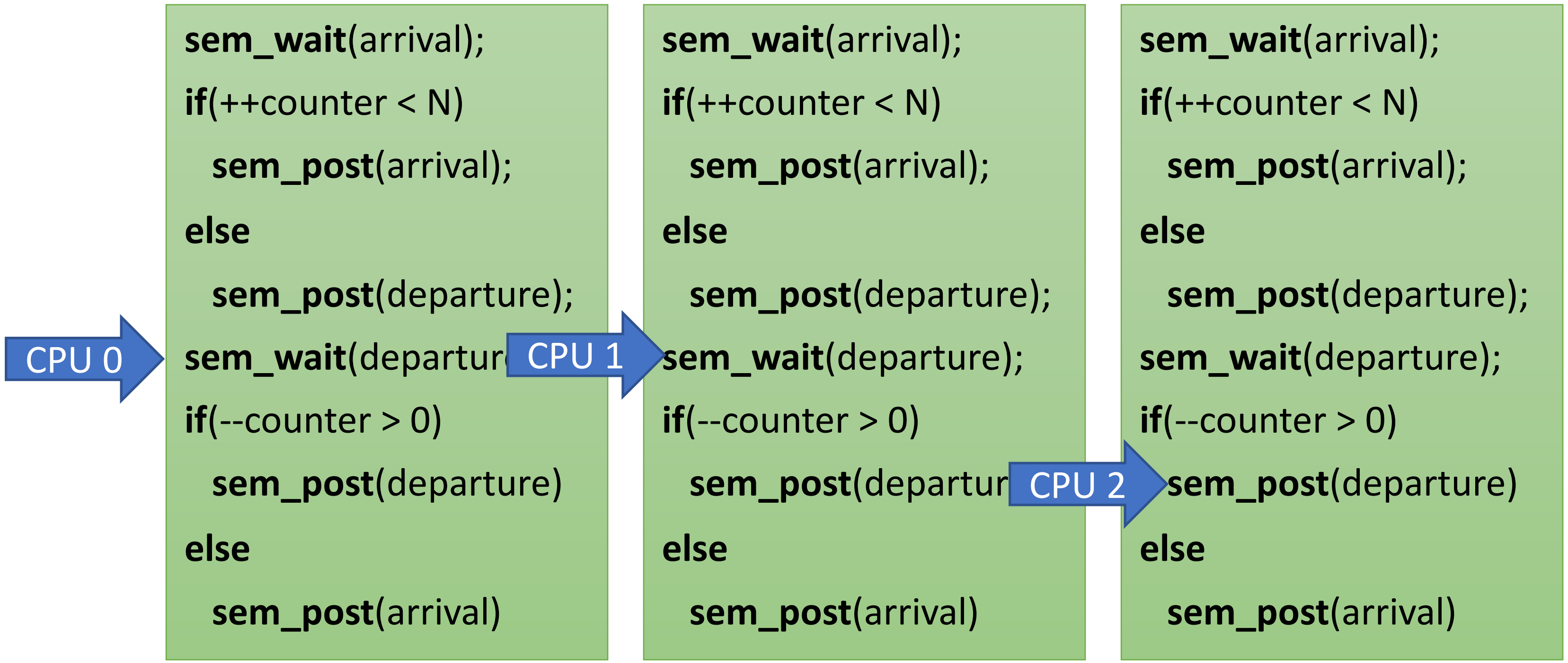
N == 3

shared sem_t arrival = **0**

sem_t departure = **0**

atomic int counter = **2**

1



```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

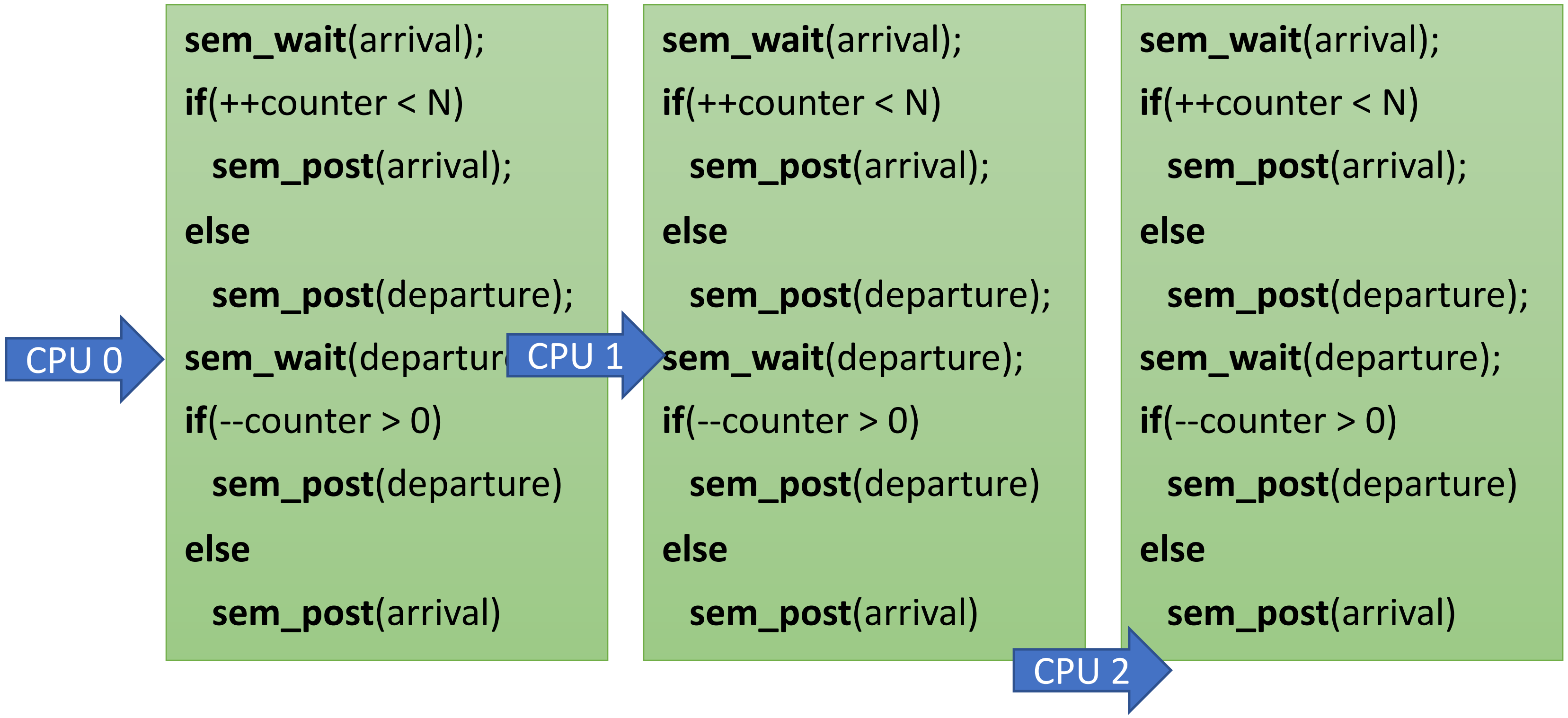
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3

```
shared      sem_t arrival = 0
            sem_t departure = 1
            atomic int counter = 2
```

1



```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

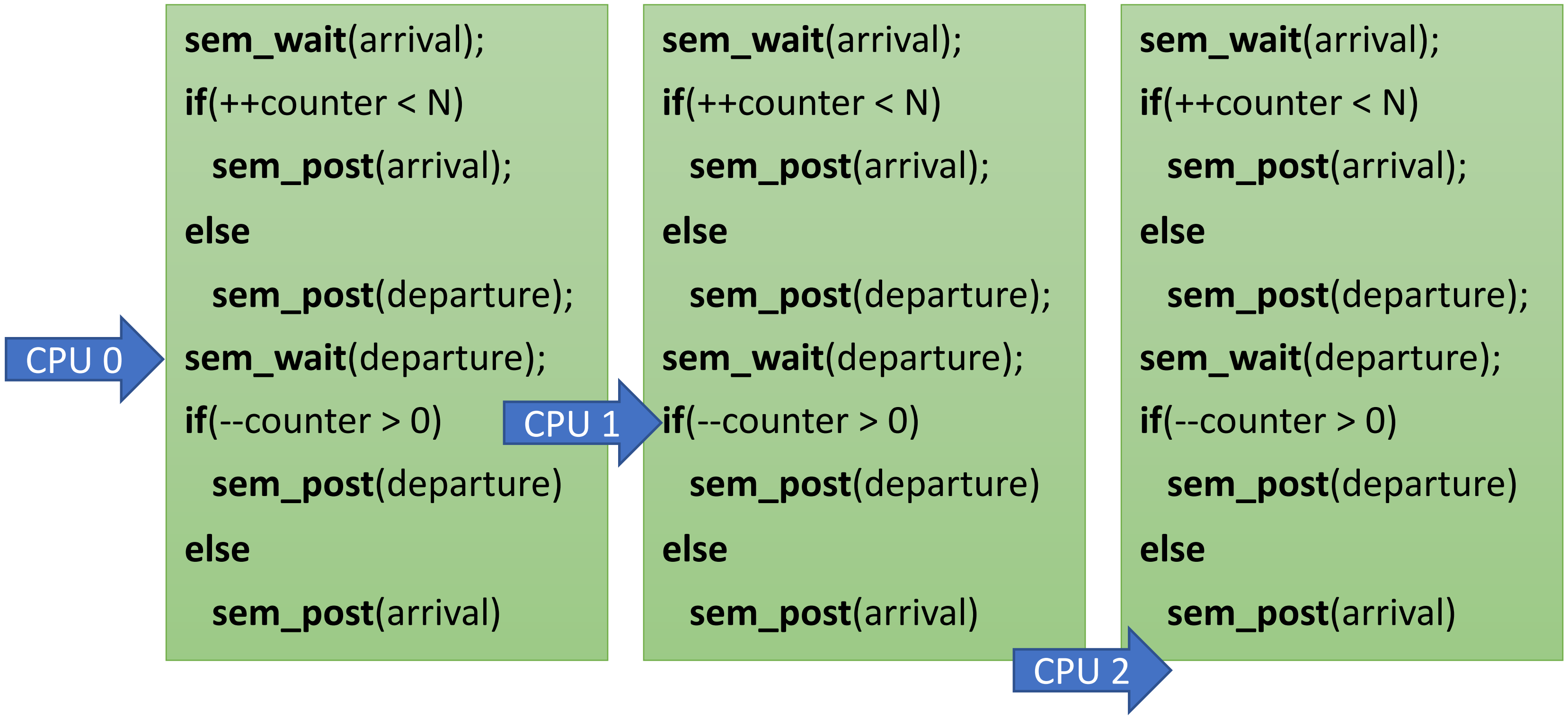
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3

```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 2
```

1



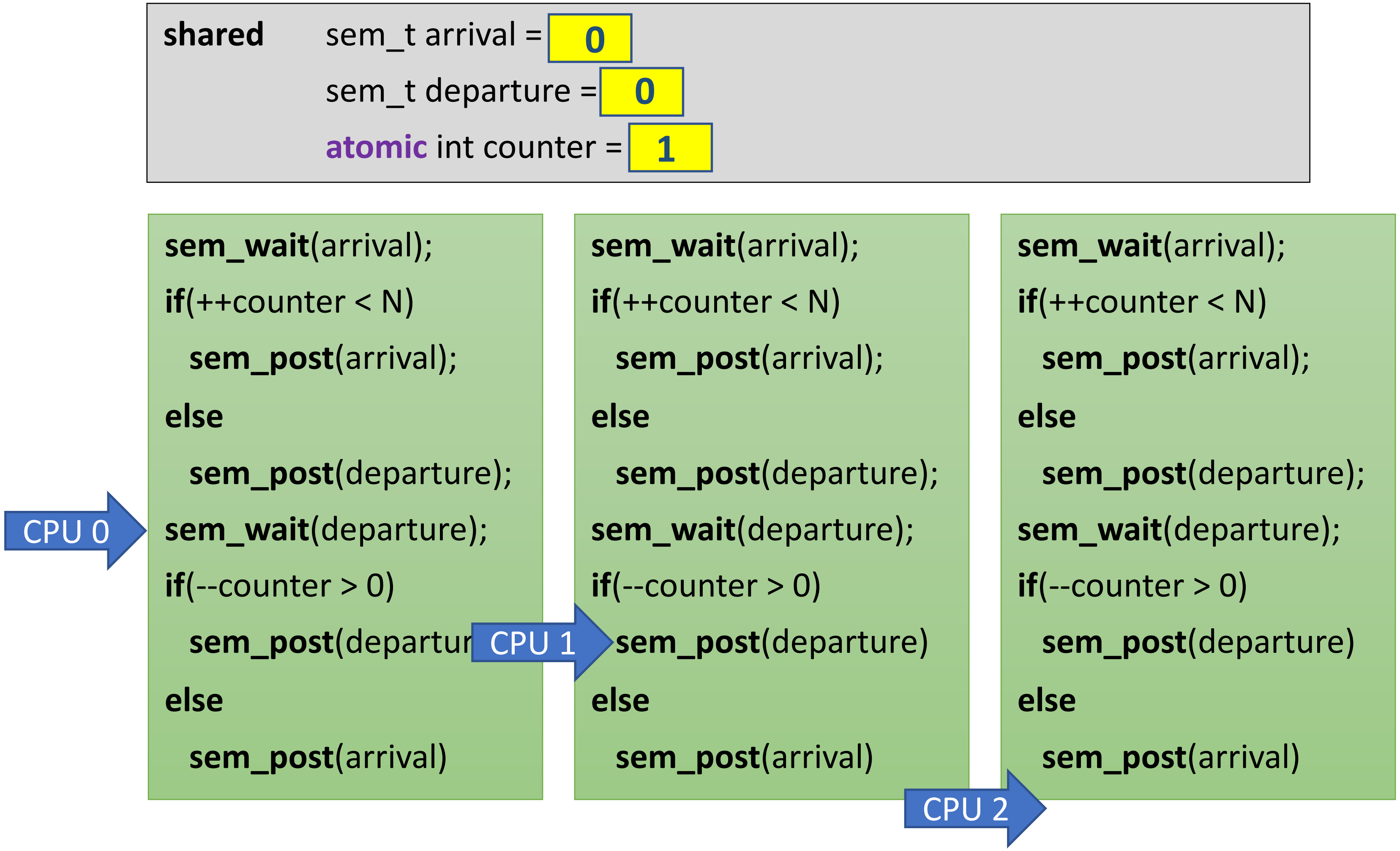
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);        <- CPU 0
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)           <- CPU 1
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
                            <- CPU 2
```

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 1
```

1

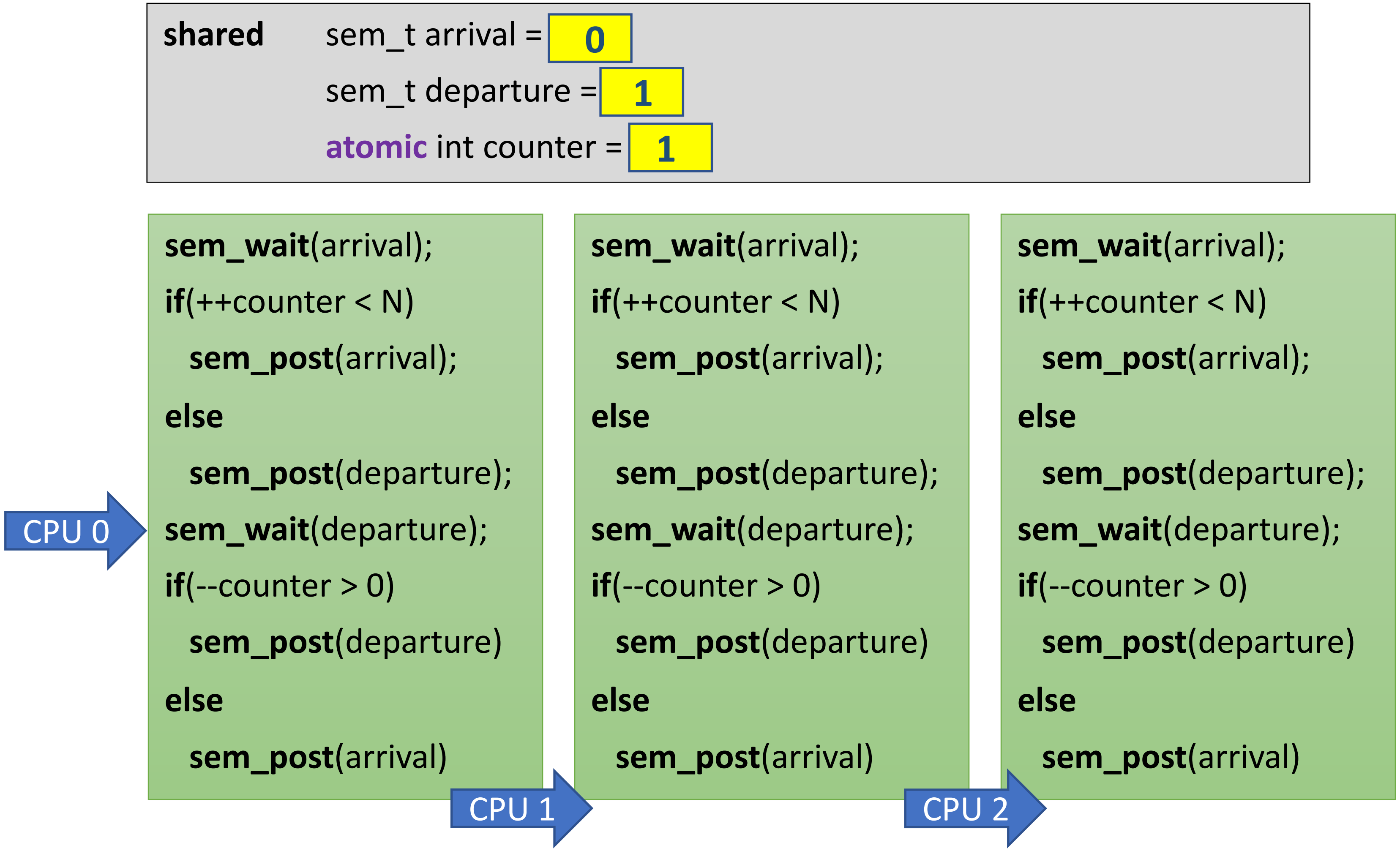
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);      <- CPU 0
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)     <- CPU 1
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
                          <- CPU 2
```

Semaphore Barrier Action Zone

N == 3



shared sem_t arrival = **0**

sem_t departure = **1**

atomic int counter = **1**

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

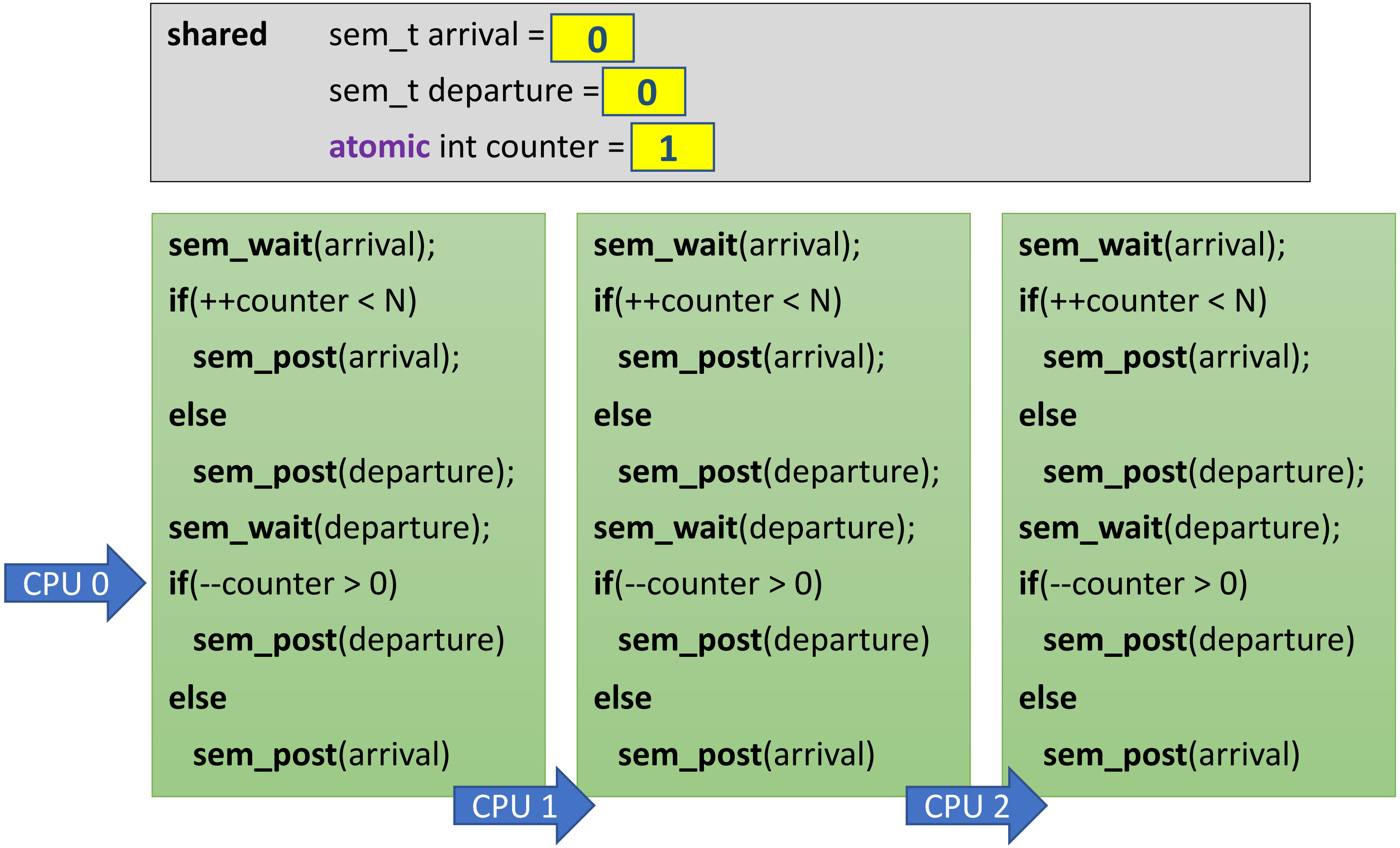
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 1
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

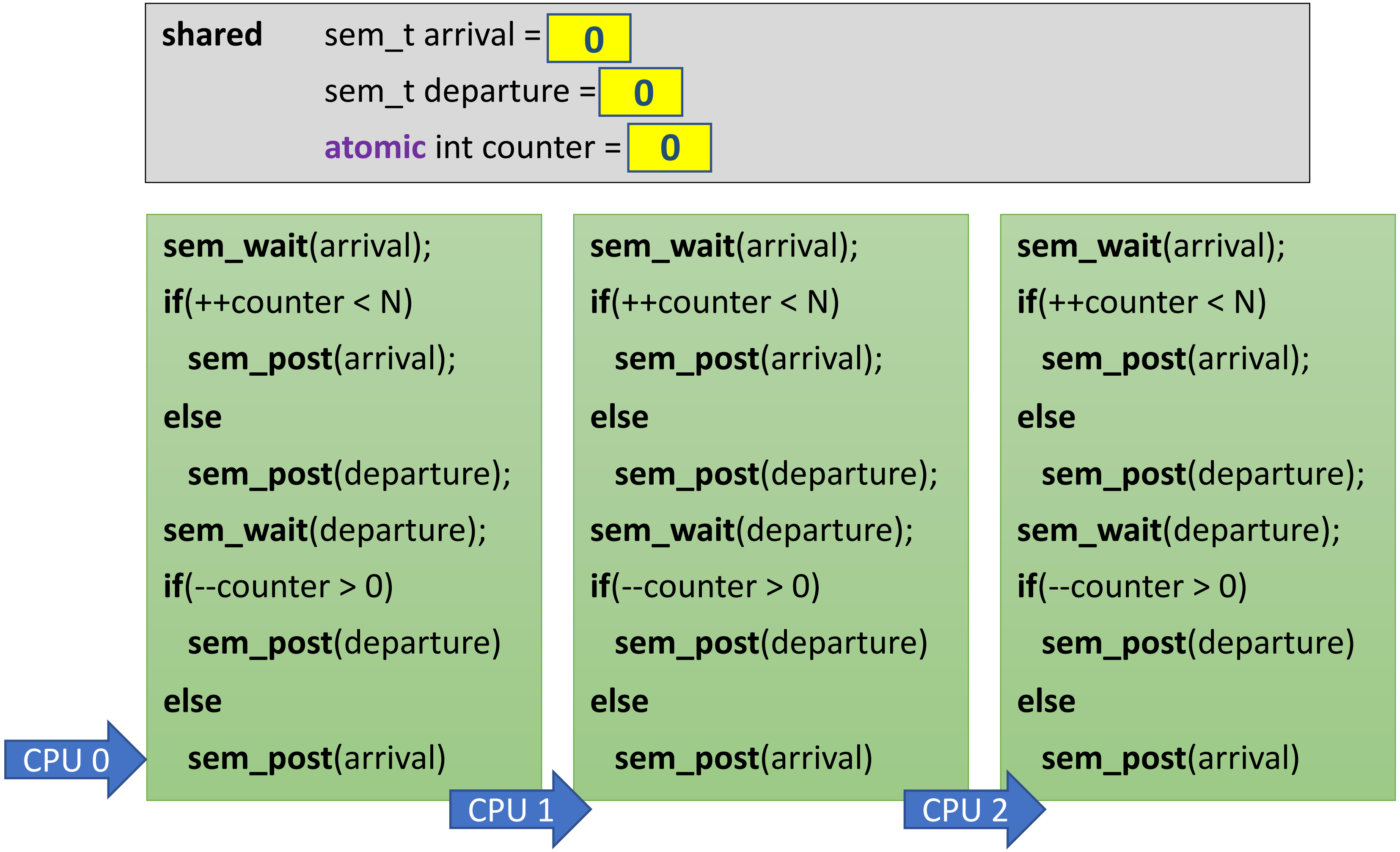
```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3



```
shared    sem_t arrival = 0
          sem_t departure = 0
          atomic int counter = 0
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

1

Semaphore Barrier Action Zone

N == 3

```
shared      sem_t arrival = 1
            sem_t departure = 0
            atomic int counter = 0
```

1

CPU 0

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 1

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 2

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Semaphore Barrier Action Zone

N == 3

```
shared    sem_t arrival = 1
          sem_t departure = 0
          atomic int counter = 0
```

1

CPU 0

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 1

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 2

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

Still correct if counter is not atomic?

Semaphore Barrier Action Zone

N == 3

```
shared    sem_t arrival = 1
          sem_t departure = 0
          atomic int counter = 0
```

1

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 0

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 1

```
sem_wait(arrival);
if(++counter < N)
  sem_post(arrival);
else
  sem_post(departure);
sem_wait(departure);
if(--counter > 0)
  sem_post(departure)
else
  sem_post(arrival)
```

CPU 2

Do we need two phases?

Still correct if counter is not atomic?

Barrier using Semaphores

Properties

- Pros:

- Cons:

Barrier using Semaphores

Properties

- Pros:
    - Very Simple
    - Space complexity O(1)
    - Symmetric
- Cons:

Barrier using Semaphores

Properties

- Pros:
  - Very Simple
  - Space complexity O(1)
  - Symmetric
- Cons:
  - Required a strong object
    - Requires some central manager
    - High contention on the semaphores
  - Propagation delay O(n)

Barriers based on counters

Counter Barrier Ingredients

Fetch-and-Increment register

- A shared register that supports a F&I operation:
- Input: register *r*
- Atomic operation:
  - *r* is incremented by 1
  - the old value of r is returned

```
function fetch-and-increment (r : register)
    orig_r := r;
    r:= r + 1;
    return (orig_r);
end-function
```

Await

- For brevity, we use the **await** macro
- Not an operation of an object
- This is also called: “spinning”

```
macro await (condition : boolean condition)
    repeat
        cond = eval(condition);
    until (cond)
end-macro
```

Simple Barrier Using an Atomic Counter

```
shared    counter: fetch and increment reg. – {0,..n}, initially = 0
          go: atomic bit, initial value is immaterial
local     local.go: a bit, initial value is immaterial
          local.counter: register
```

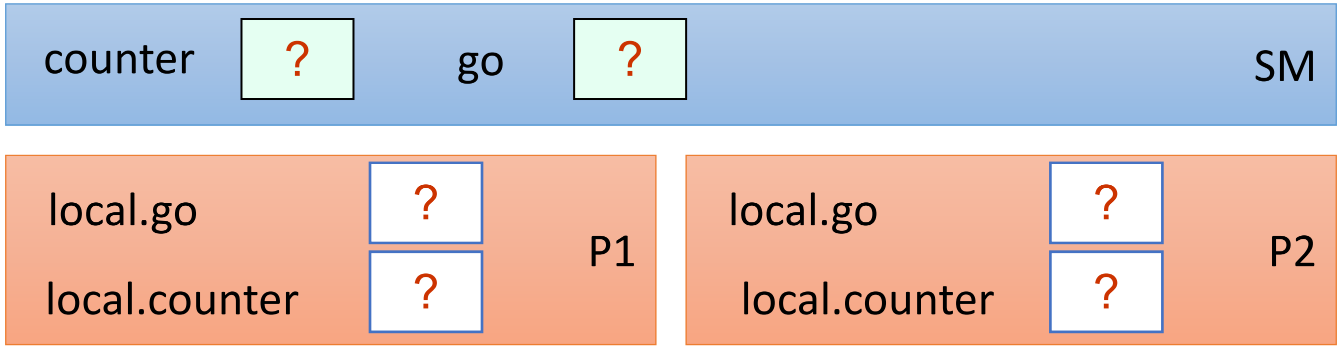
Simple Barrier Using an Atomic Counter

```
shared    counter: fetch and increment reg. – {0,..n}, initially = 0
          go: atomic bit, initial value is immaterial
local     local.go: a bit, initial value is immaterial
          local.counter: register
```

```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
    counter := 0
    go := 1 - go
else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

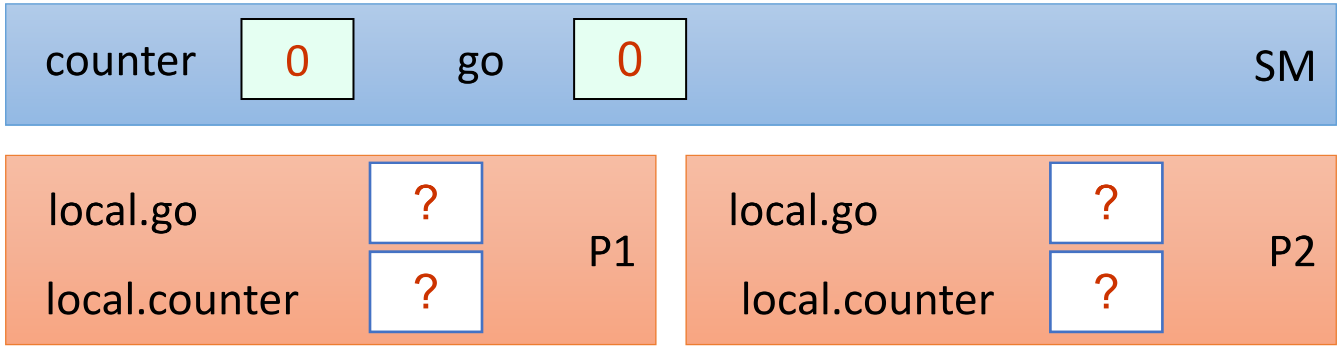
Run for n=2 Threads



```
local.go := go
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
    counter := 0
    go := 1 - go
**else await**(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

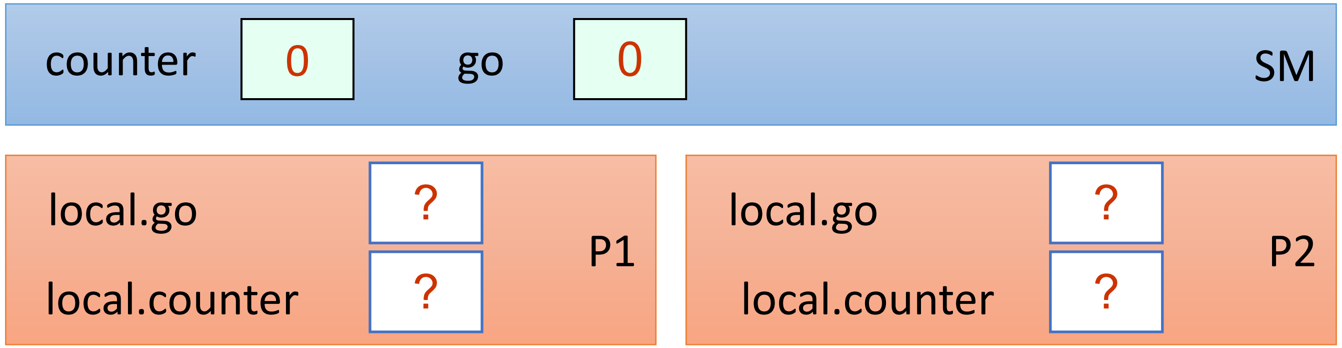
Run for n=2 Threads



```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
    counter := 0
    go := 1 - go
else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



counter 0 go 0 SM

local.go ? local.counter ? P1

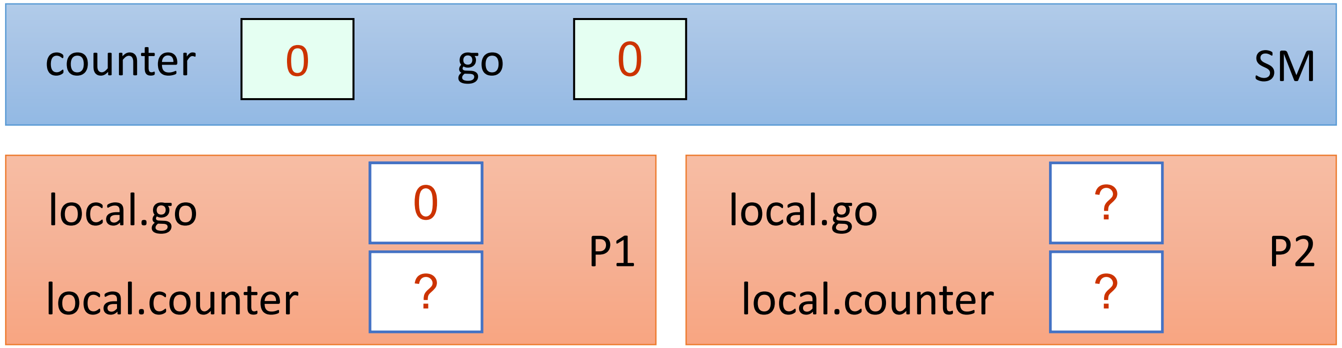
local.go ? local.counter ? P2

P1 →

```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 0, go 0

P1: local.go 0, local.counter ?

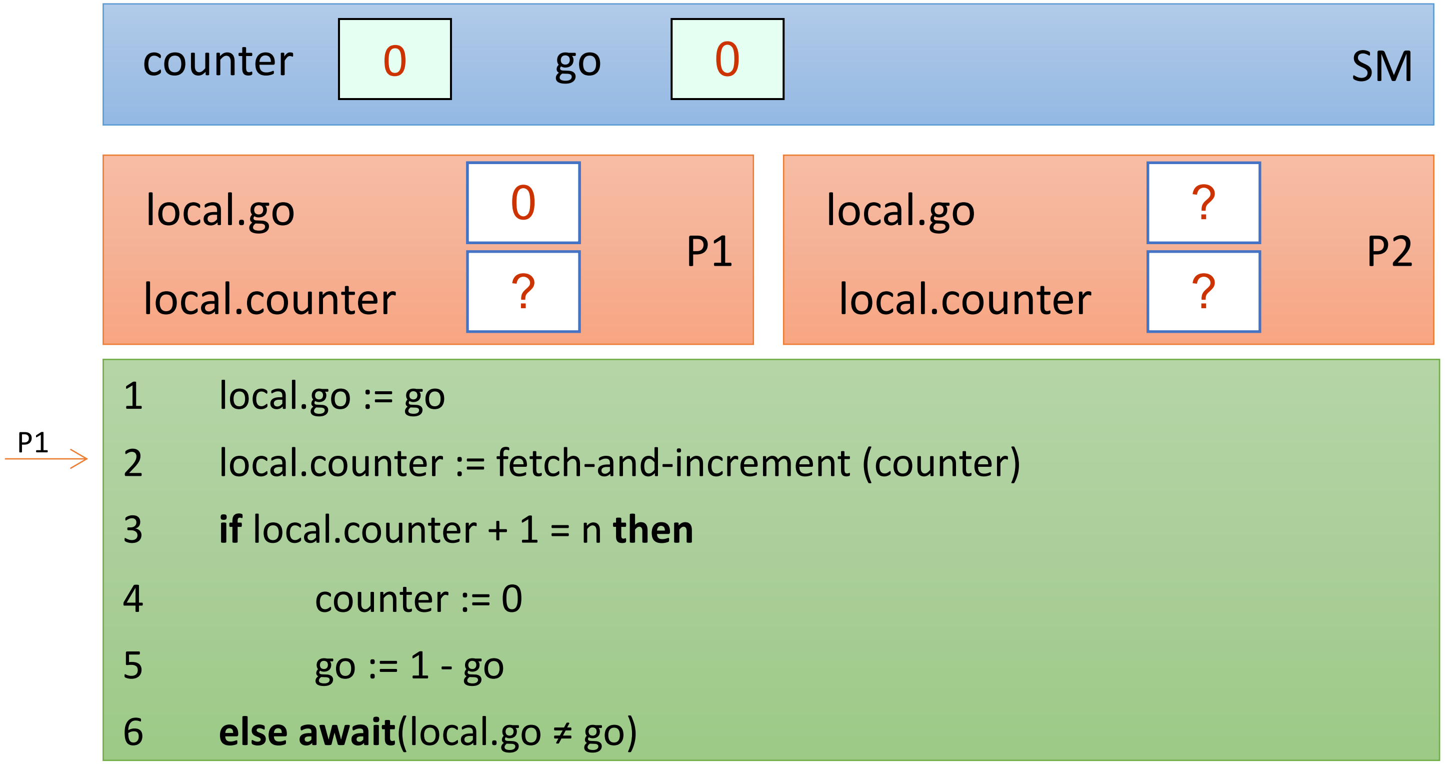
P2: local.go ?, local.counter ?

P1 →

```
local.go := go
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
        counter := 0
        go := 1 - go
**else await**(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 0, go 0

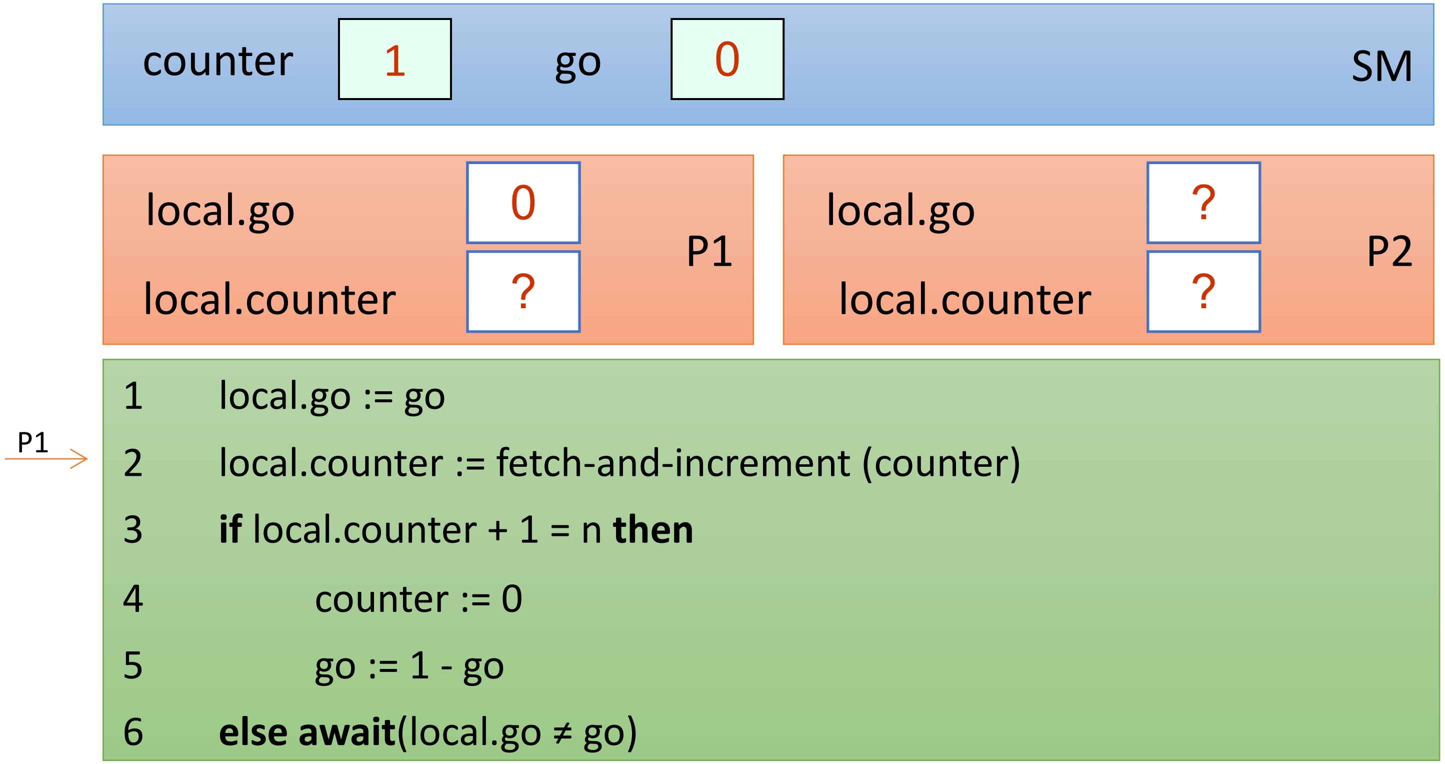
P1: local.go 0, local.counter ?

P2: local.go ?, local.counter ?

```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

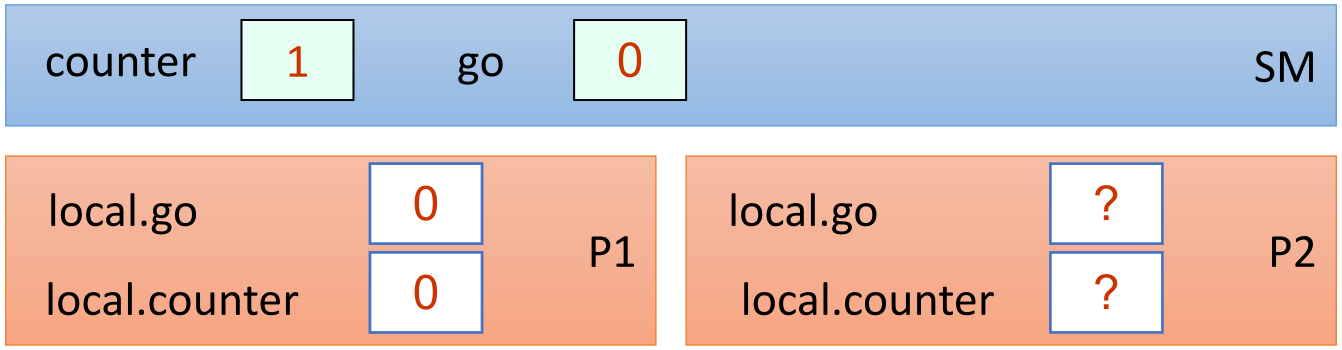
Run for n=2 Threads



```
1   local.go := go
2   local.counter := fetch-and-increment (counter)
3   **if** local.counter + 1 = n **then**
4           counter := 0
5           go := 1 - go
6   **else await**(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

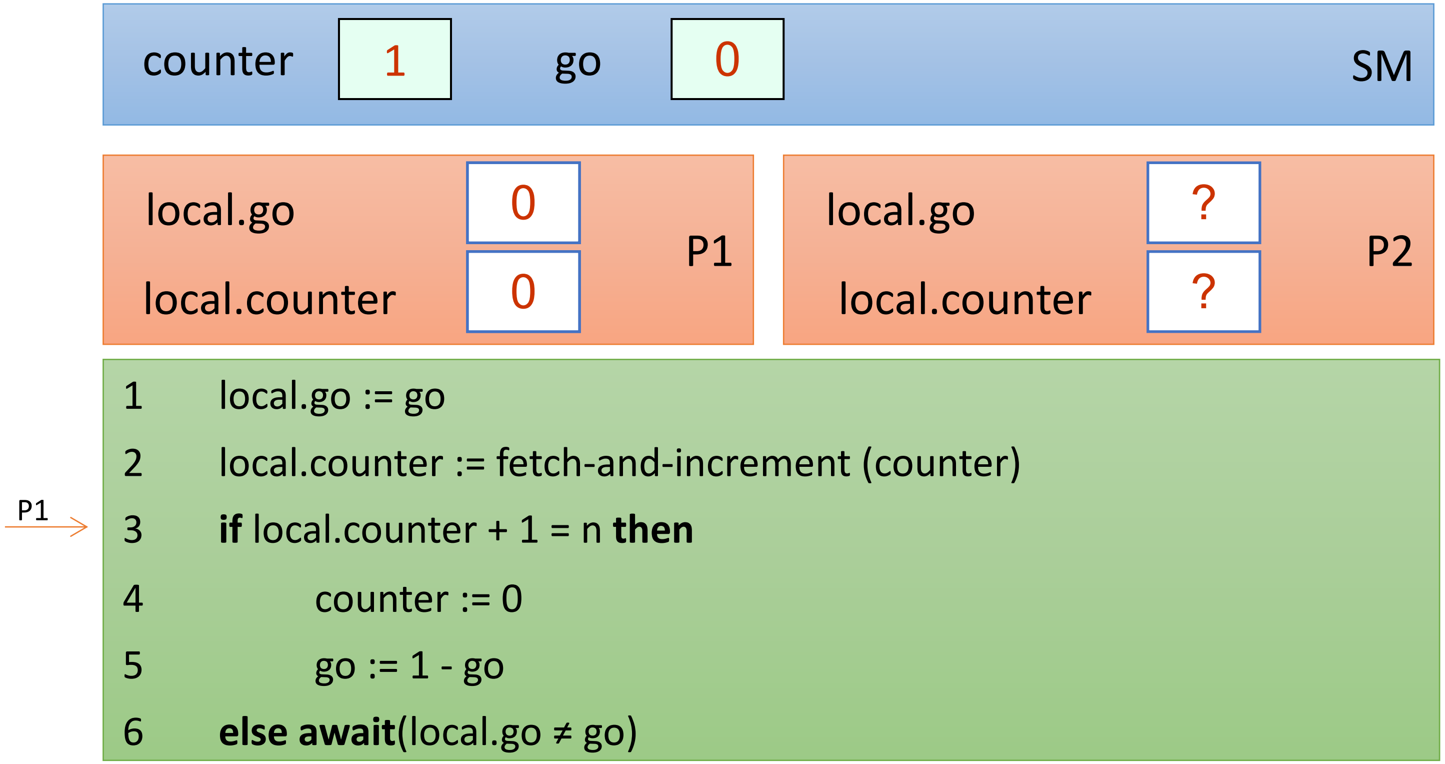
Run for n=2 Threads



```
      1   local.go := go
P1 →  2   local.counter := fetch-and-increment (counter)
      3   **if** local.counter + 1 = n **then**
      4         counter := 0
      5         go := 1 - go
      6   **else await**(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



counter 1 go 0 SM

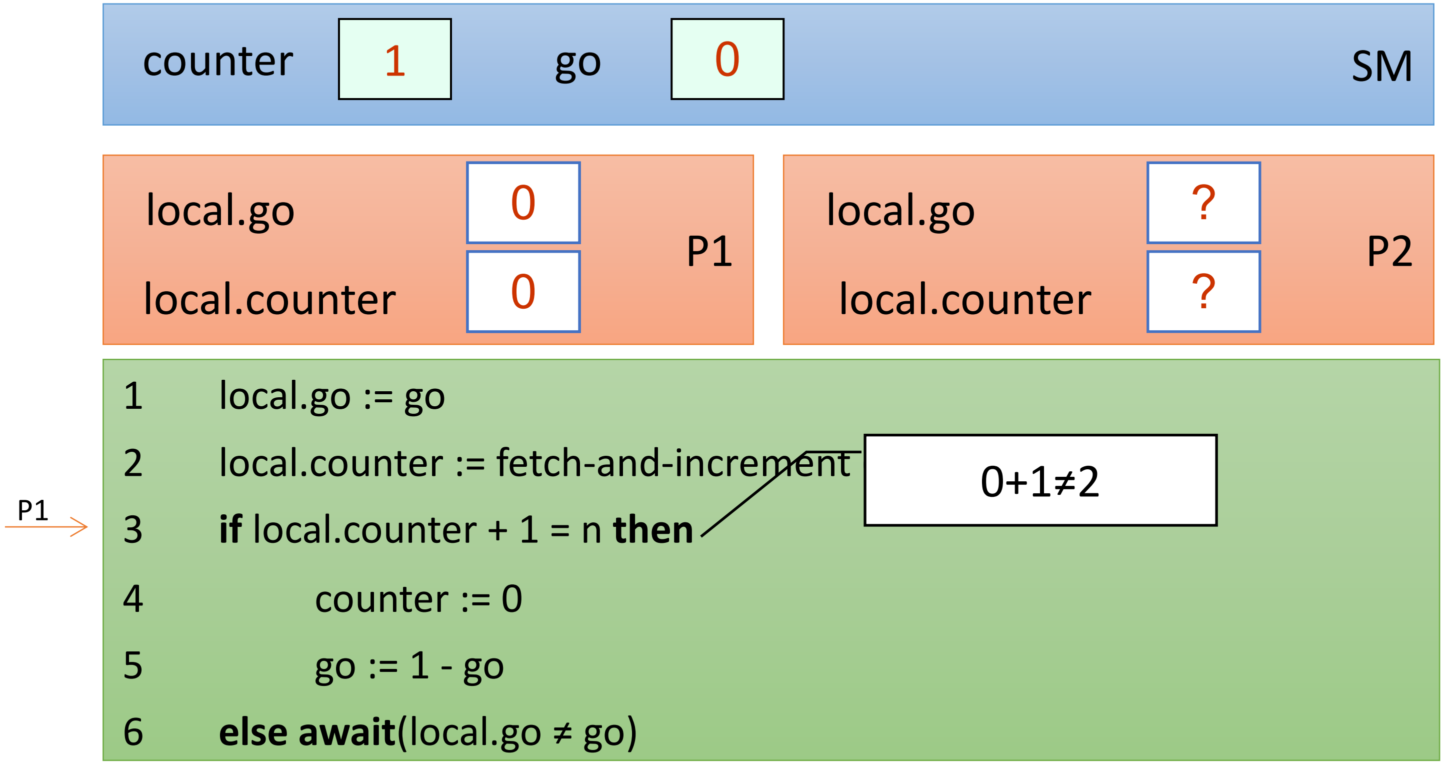
local.go 0
local.counter 0 P1

local.go ?
local.counter ? P2

```
1    local.go := go
2    local.counter := fetch-and-increment (counter)
3    if local.counter + 1 = n then
4        counter := 0
5        go := 1 - go
6    else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 1, go 0

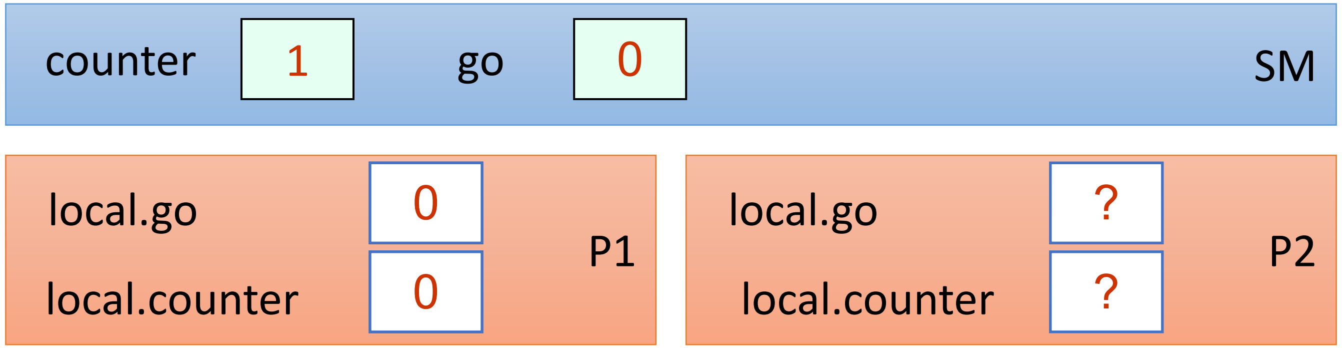
P1: local.go 0, local.counter 0

P2: local.go ?, local.counter ?

```
local.go := go
local.counter := fetch-and-increment
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 1, go 0

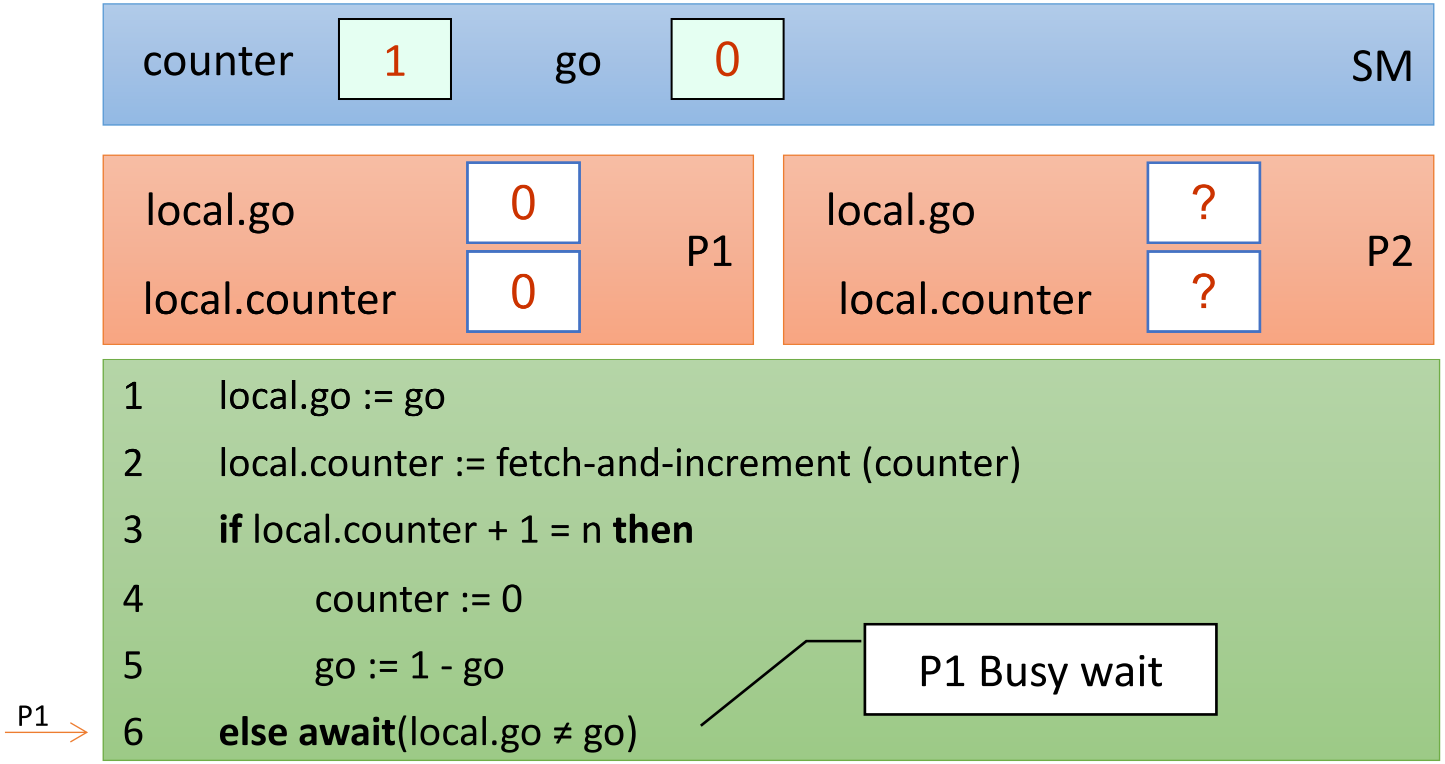
P1: local.go 0, local.counter 0

P2: local.go ?, local.counter ?

```
     1  local.go := go
     2  local.counter := fetch-and-increment (counter)
     3  if local.counter + 1 = n then
     4      counter := 0
     5      go := 1 - go
P1 → 6  else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads

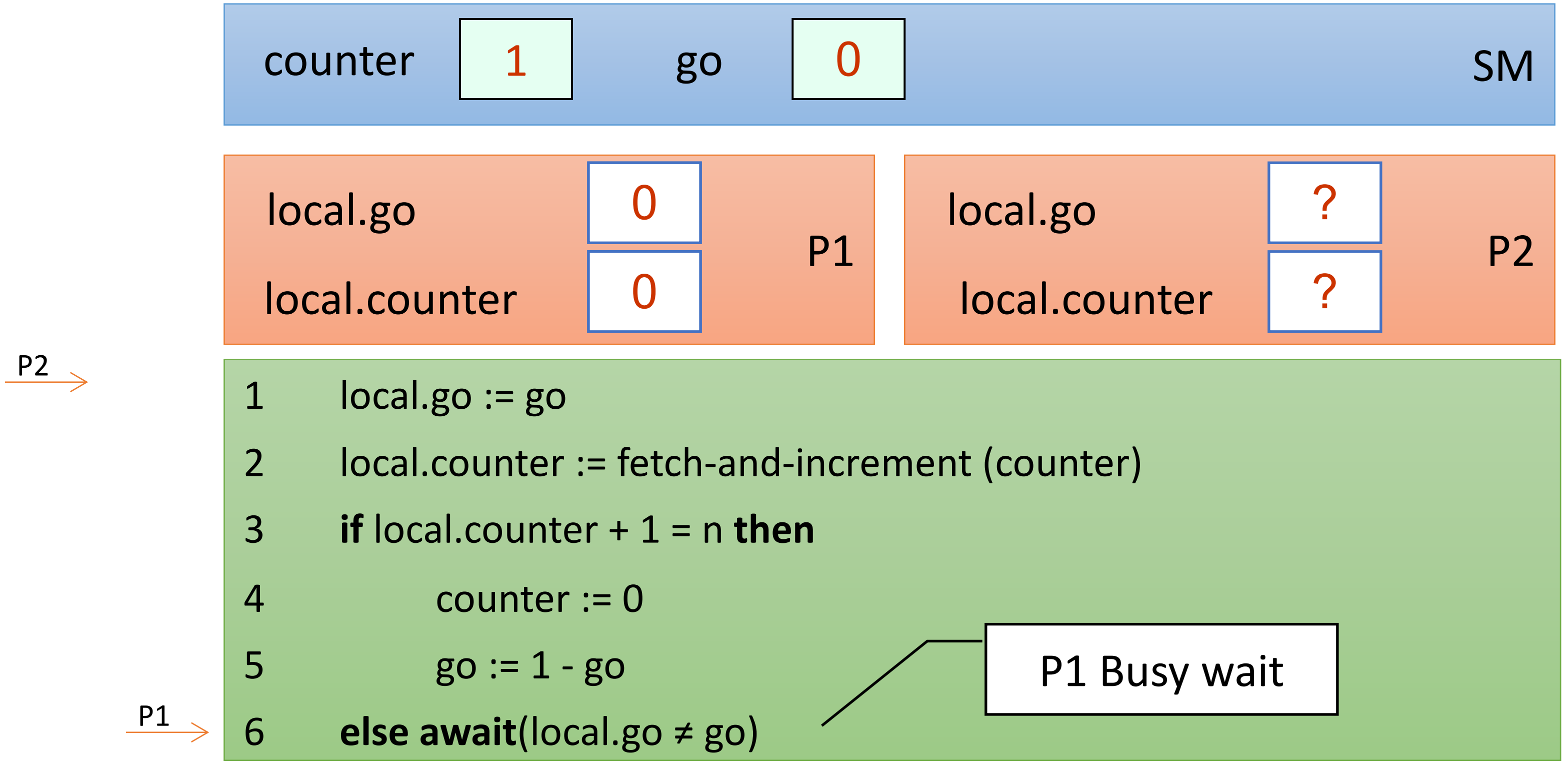


```
local.go := go
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
    counter := 0
    go := 1 - go
**else await**(local.go ≠ go)
```

P1 Busy wait

Simple Barrier Using an Atomic Counter

Run for n=2 Threads

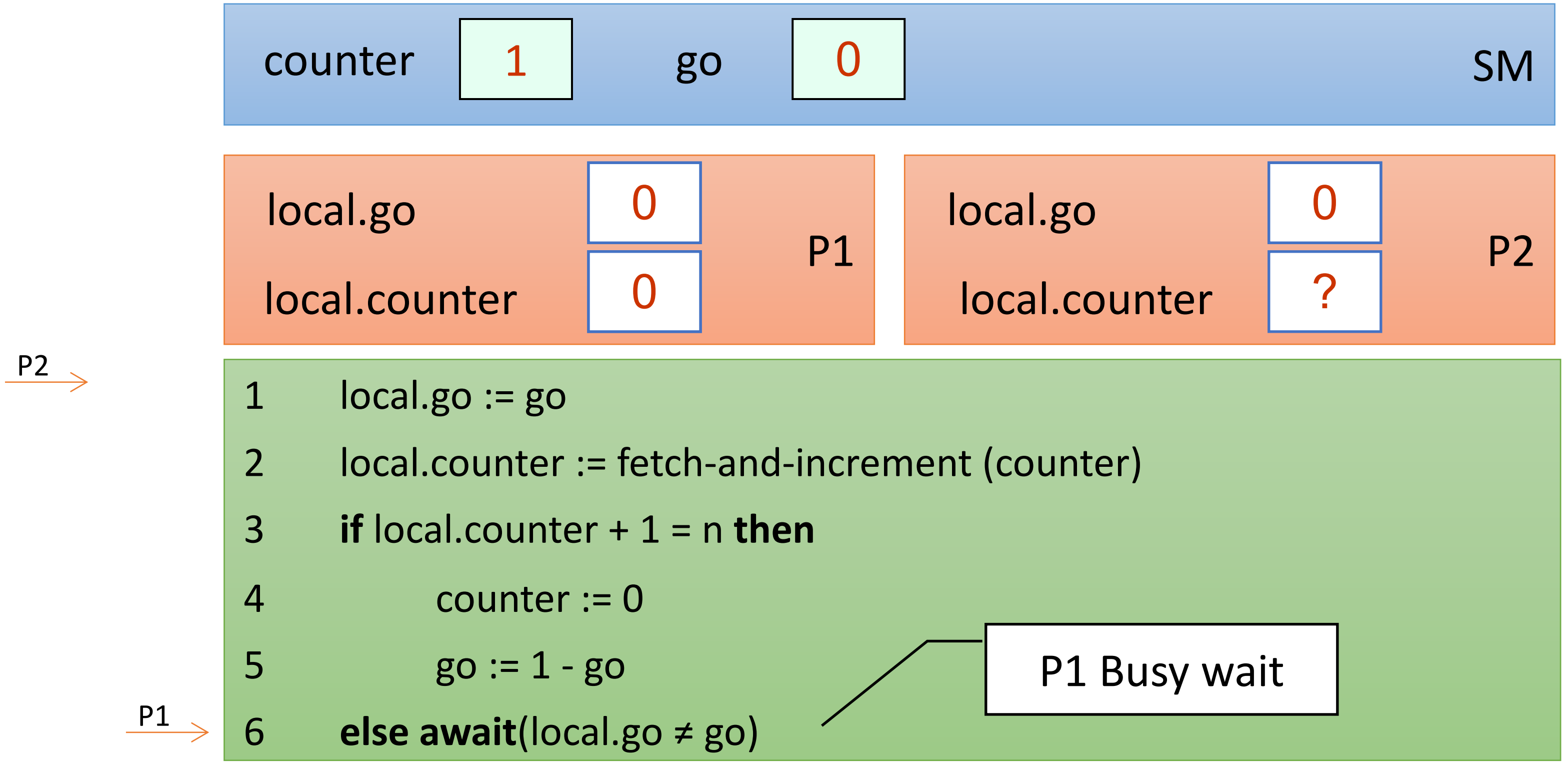


```
local.go := go
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
        counter := 0
        go := 1 - go
**else await**(local.go ≠ go)
```

P1 Busy wait

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



counter 1 go 0 SM

local.go 0 local.counter 0 P1

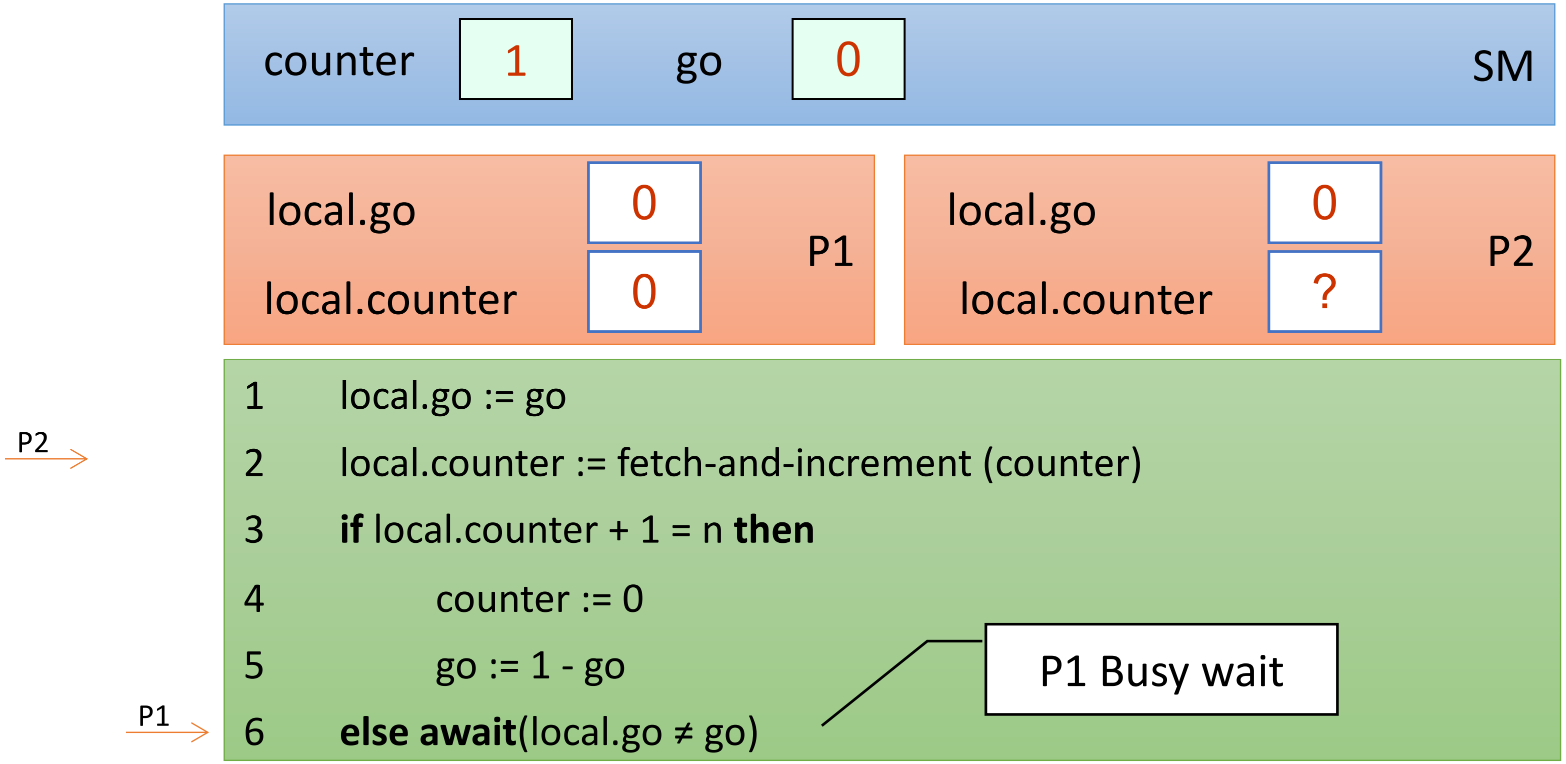
local.go 0 local.counter ? P2

```
P2 →  1   local.go := go
      2   local.counter := fetch-and-increment (counter)
      3   if local.counter + 1 = n then
      4           counter := 0
      5           go := 1 - go
P1 →  6   else await(local.go ≠ go)
```

P1 Busy wait

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 1, go 0

P1: local.go 0, local.counter 0

P2: local.go 0, local.counter ?

```
1   local.go := go
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       go := 1 - go
6   else await(local.go ≠ go)
```

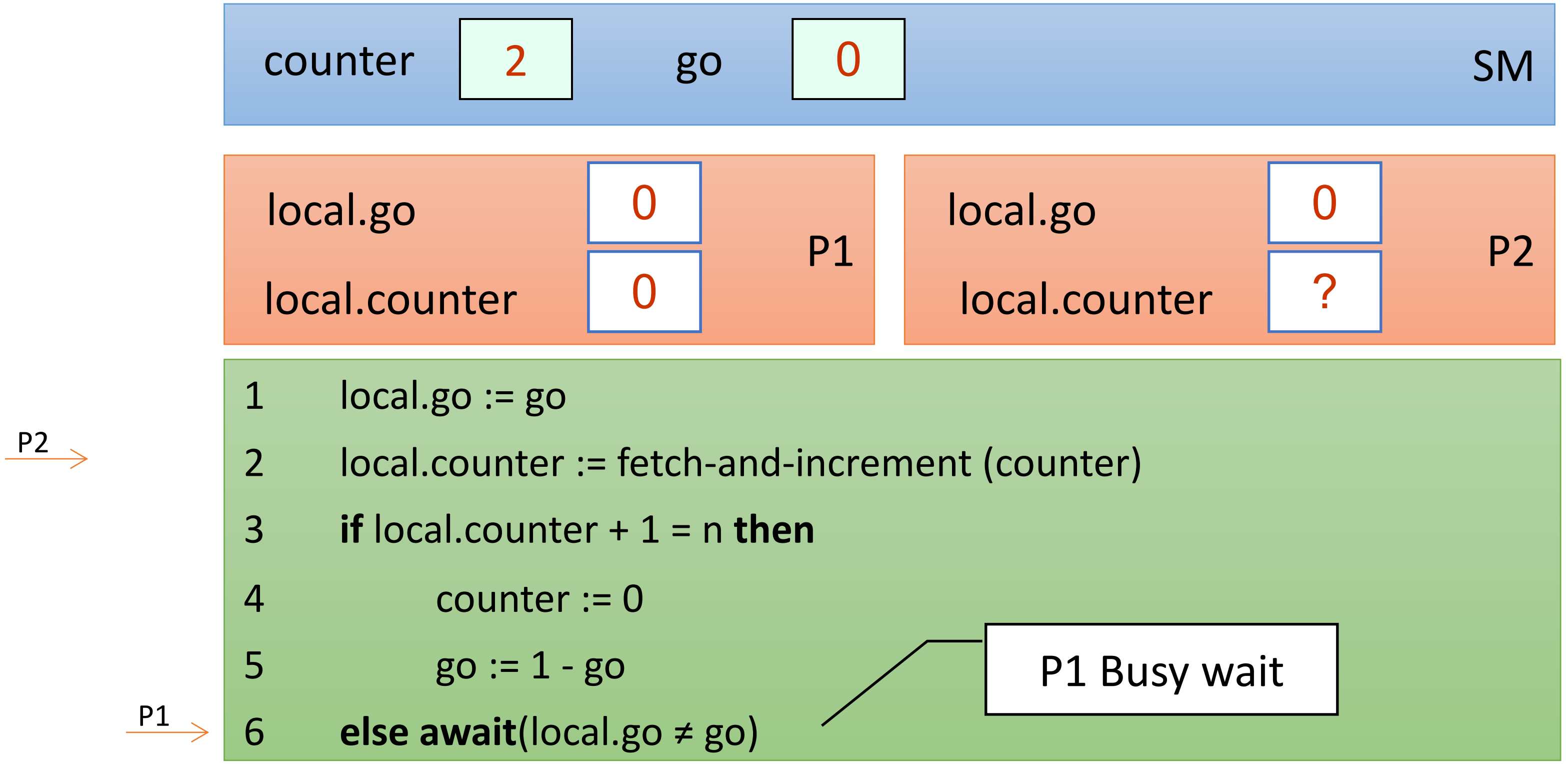
P2 → line 2

P1 → line 6

P1 Busy wait

Simple Barrier Using an Atomic Counter

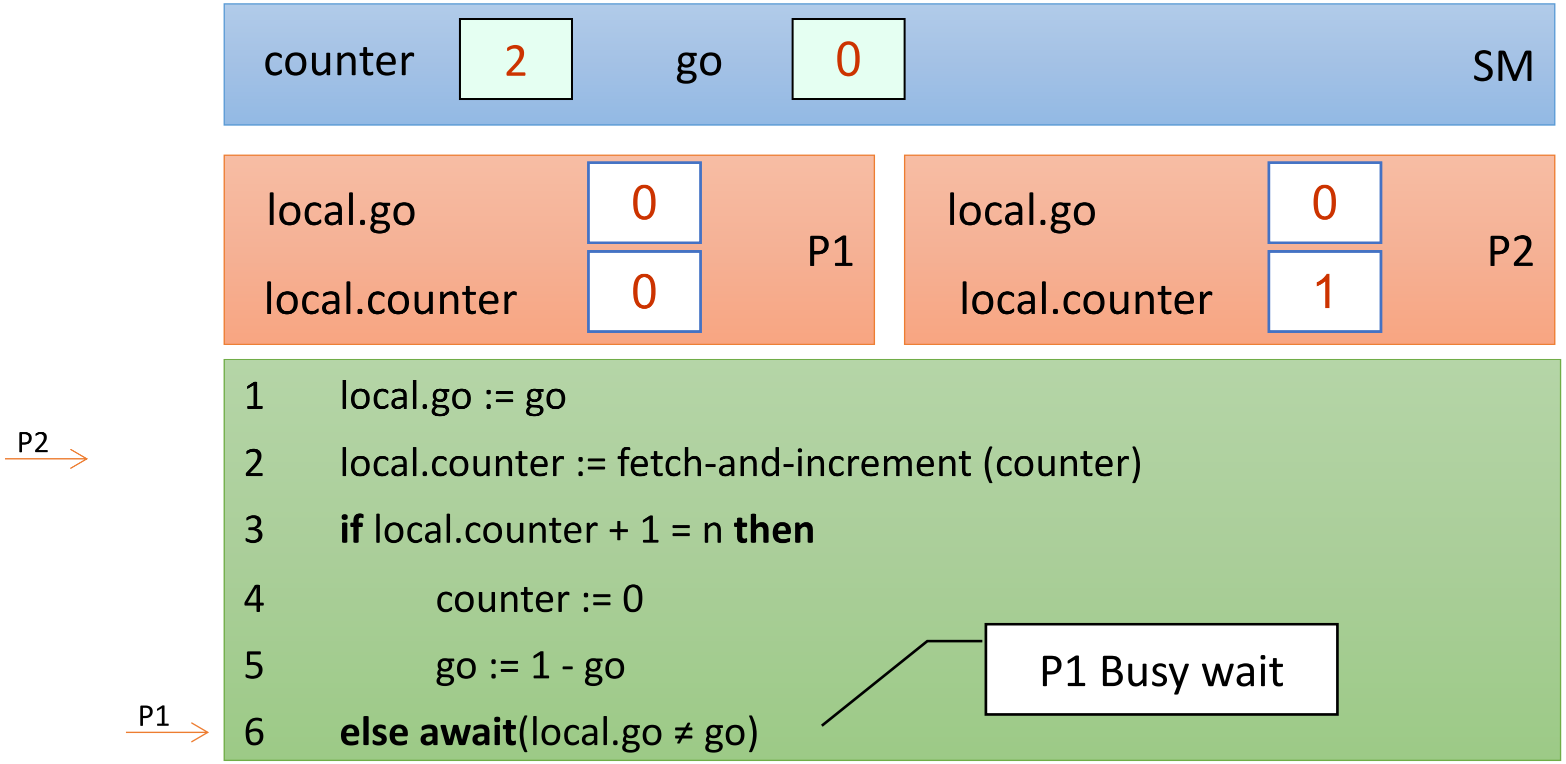
Run for n=2 Threads



```
1    local.go := go
2    local.counter := fetch-and-increment (counter)
3    if local.counter + 1 = n then
4        counter := 0
5        go := 1 - go
6    else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

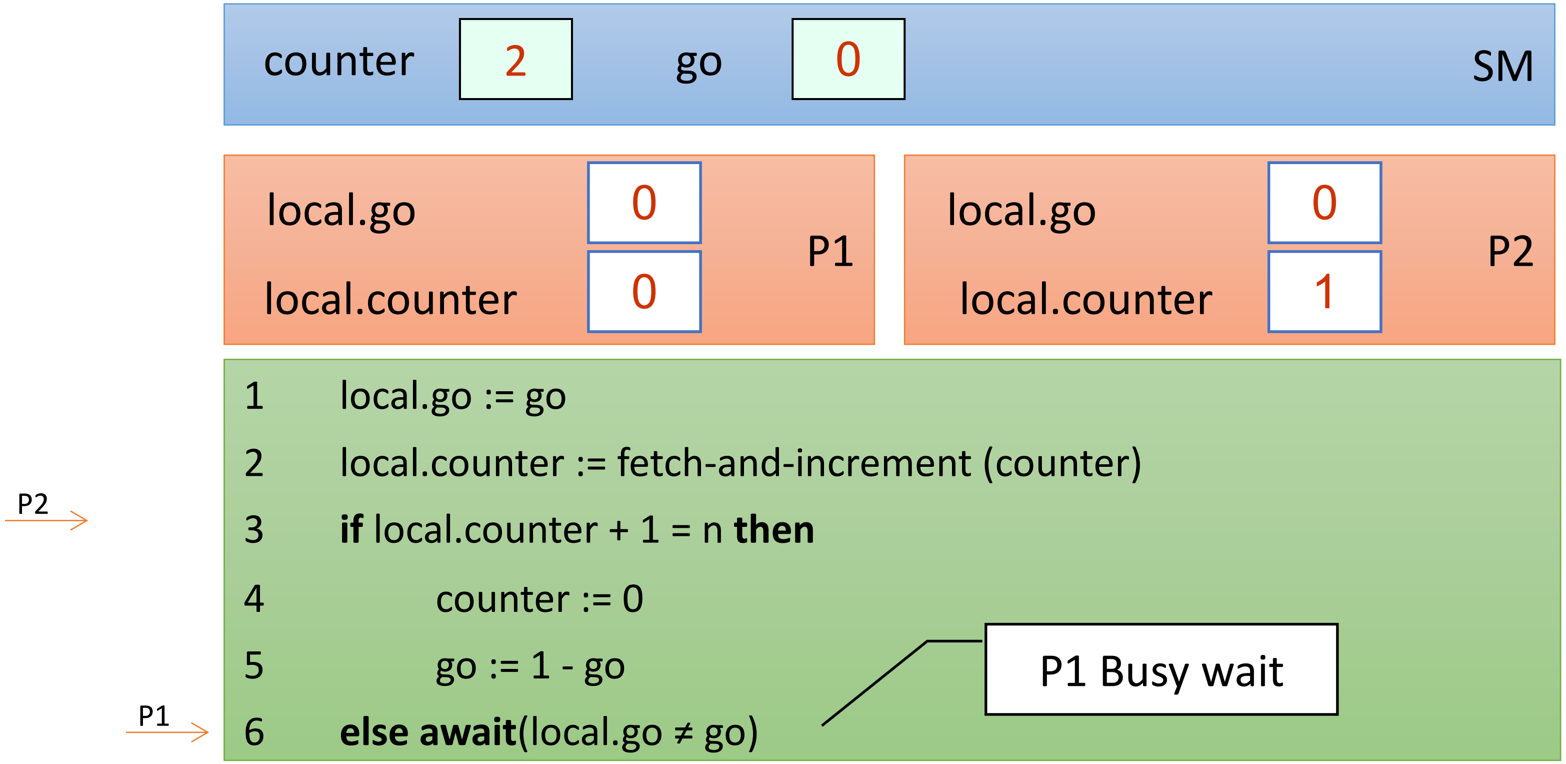
Run for n=2 Threads



```
1    local.go := go
2    local.counter := fetch-and-increment (counter)
3    if local.counter + 1 = n then
4         counter := 0
5         go := 1 - go
6    else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

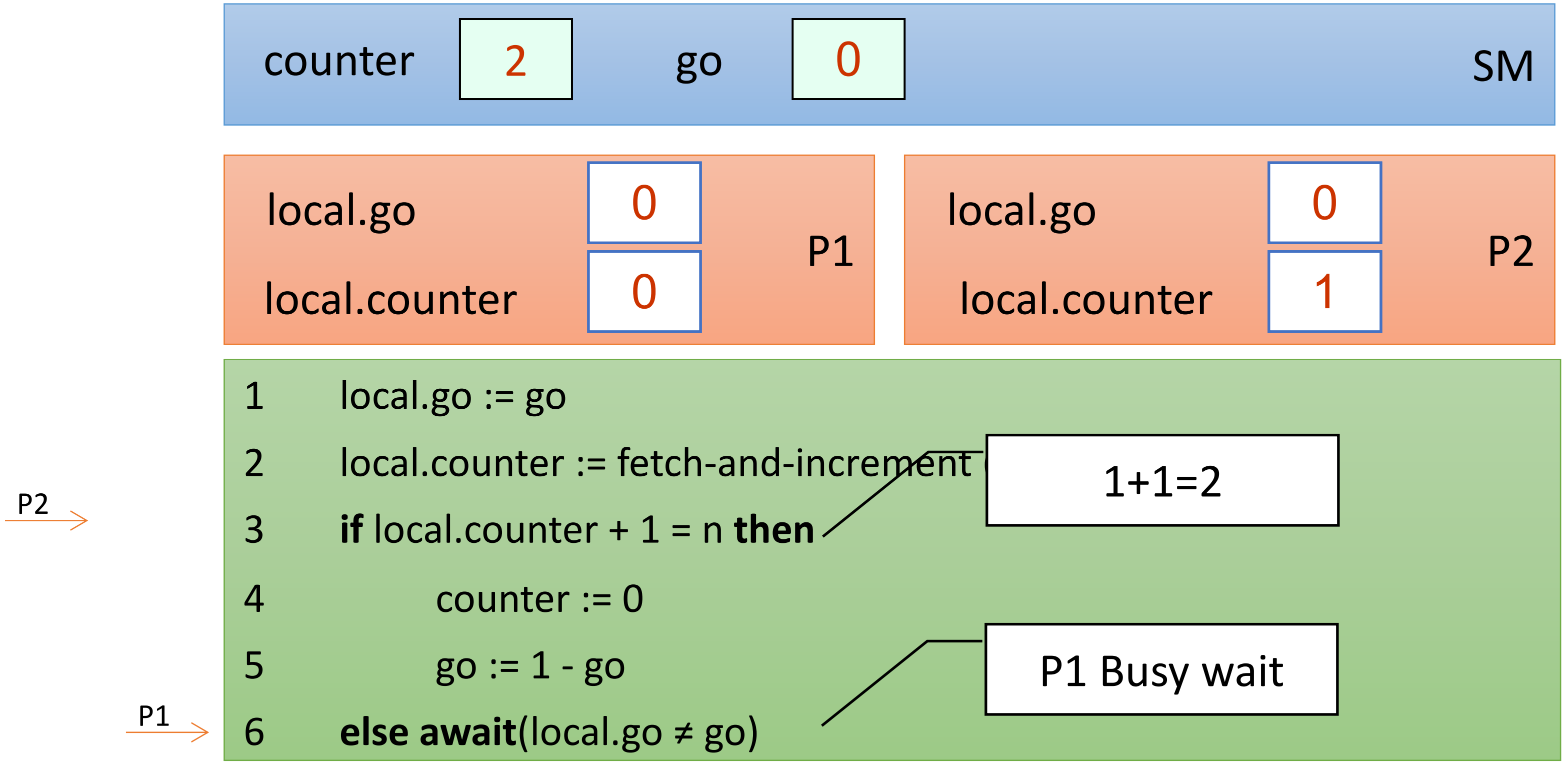
Run for n=2 Threads



```
1   local.go := go
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       go := 1 - go
6   else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



counter 2 go 0 SM

P1: local.go 0, local.counter 0

P2: local.go 0, local.counter 1

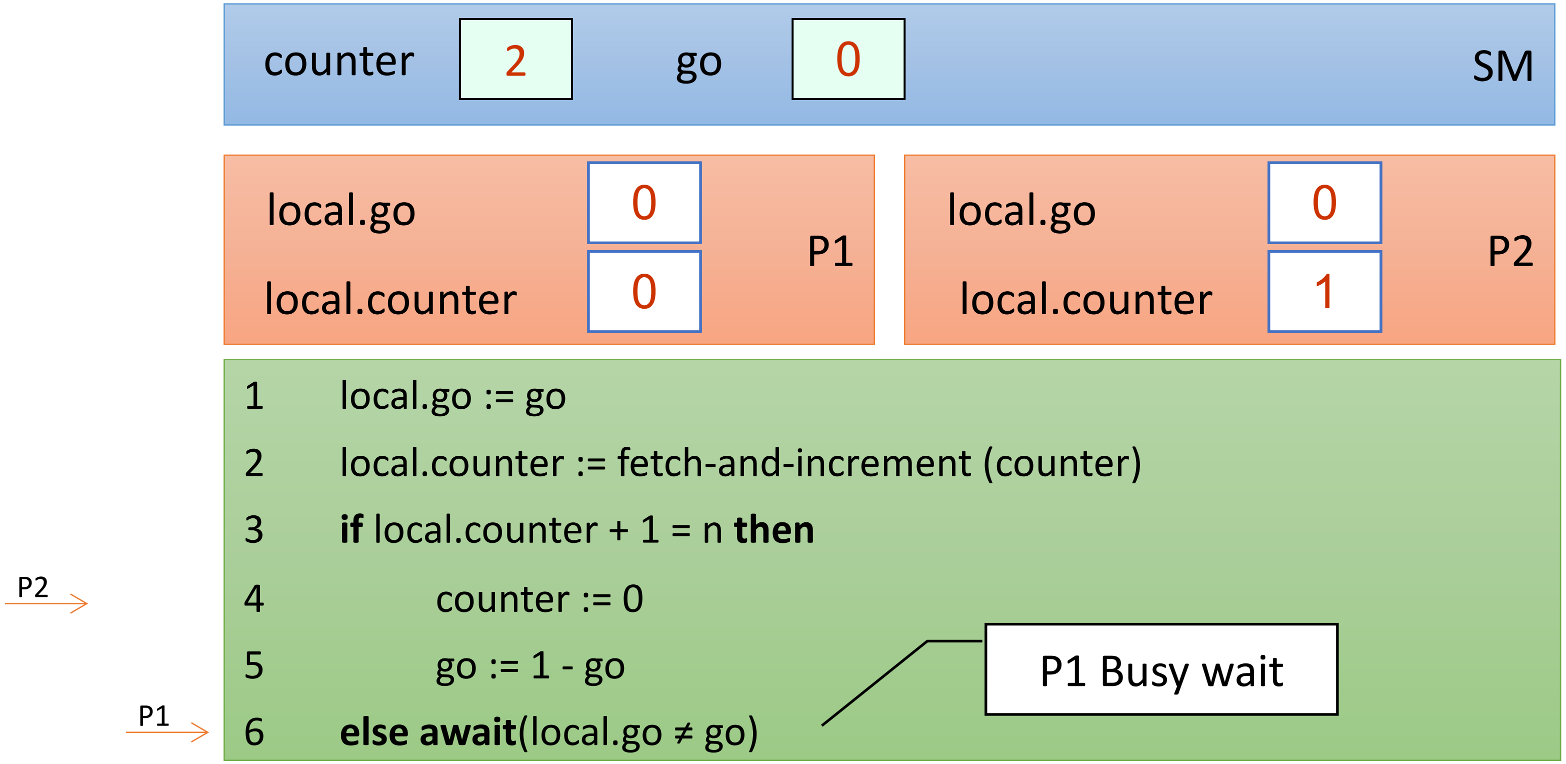
```
local.go := go
local.counter := fetch-and-increment(counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

1+1=2

P1 Busy wait

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 2 go 0

P1: local.go 0 local.counter 0

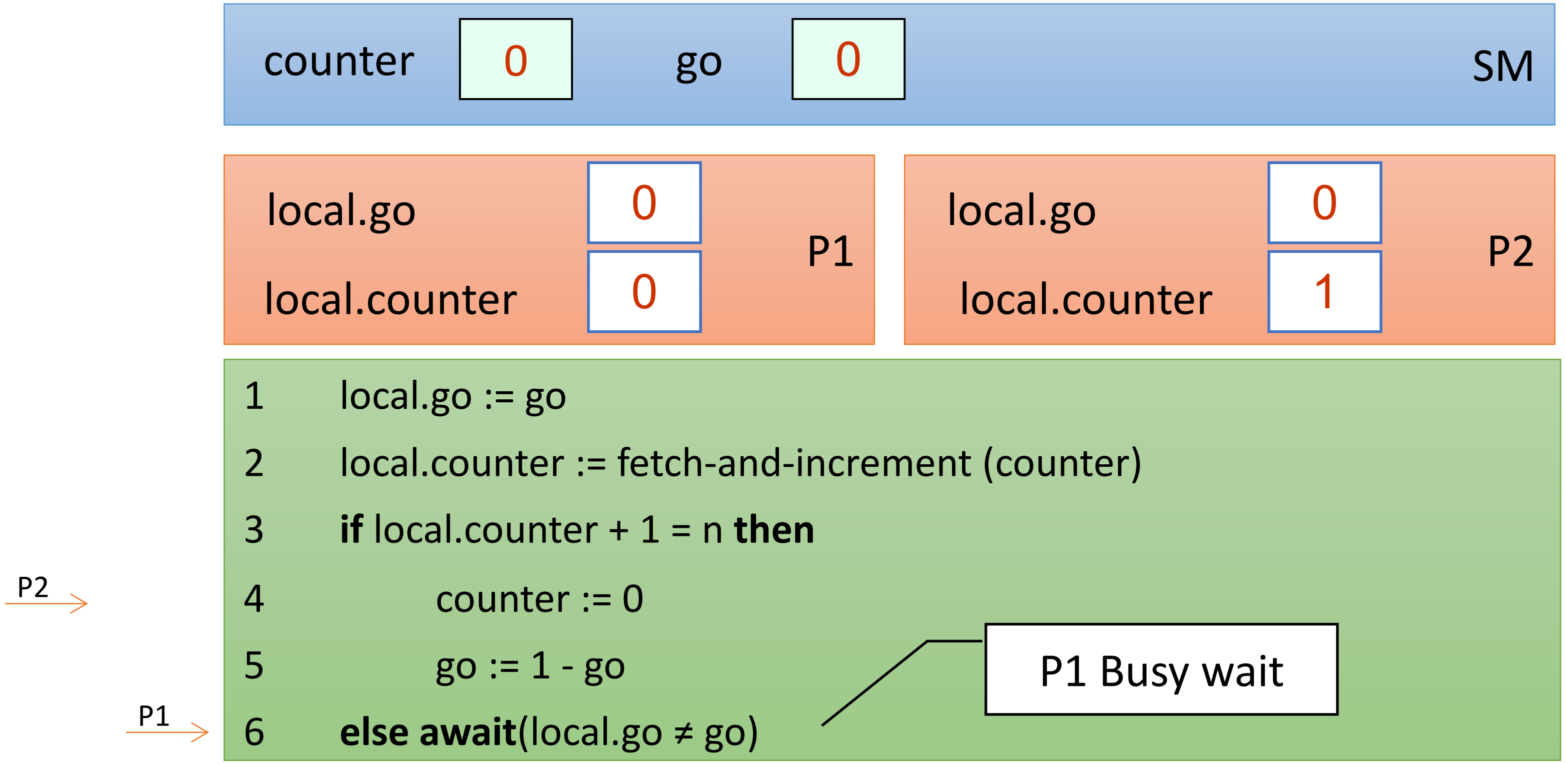
P2: local.go 0 local.counter 1

```
1   local.go := go
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       go := 1 - go
6   else await(local.go ≠ go)
```

P1 Busy wait

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 0, go 0

P1: local.go 0, local.counter 0

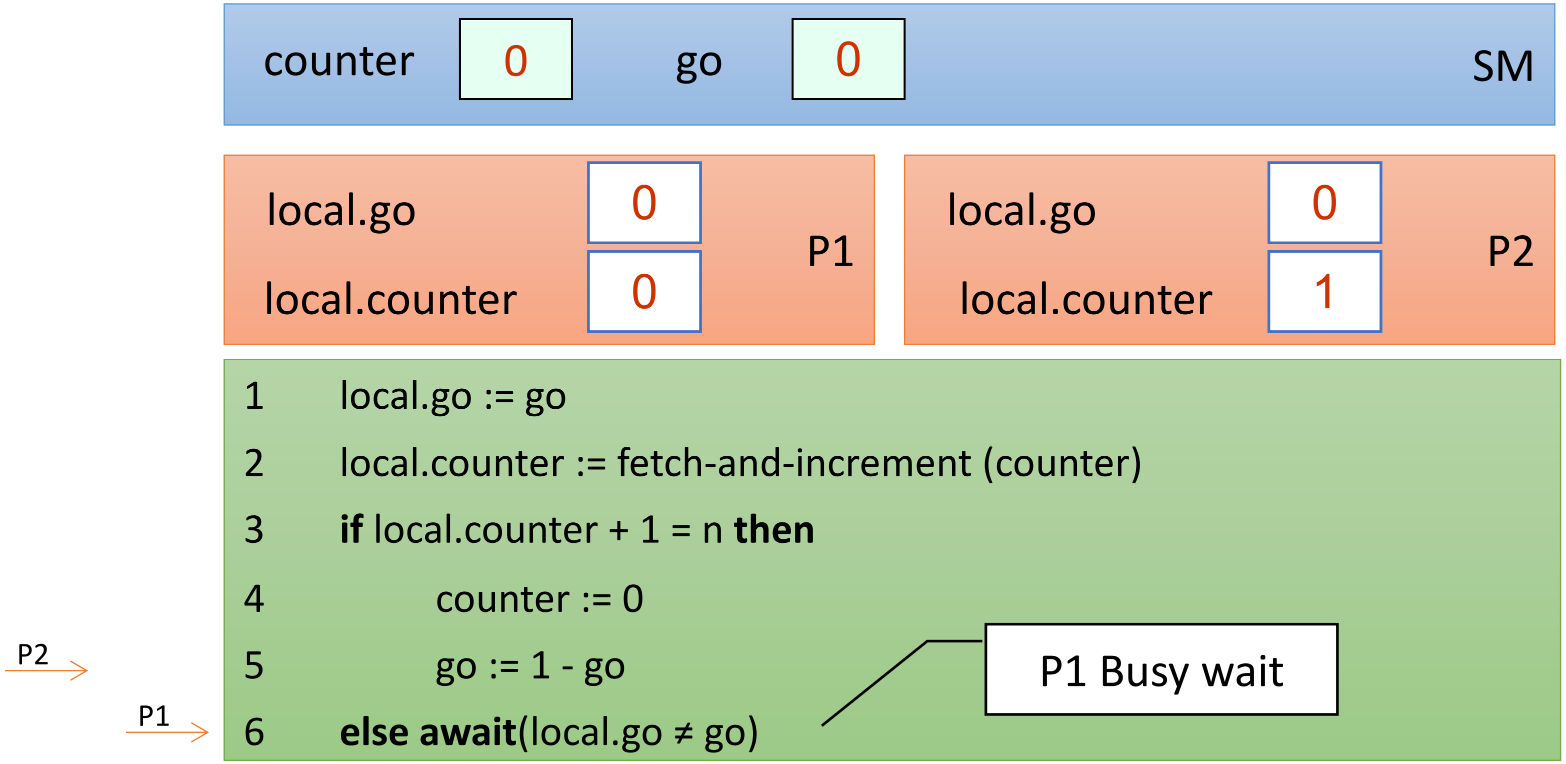
P2: local.go 0, local.counter 1

```
1   local.go := go
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       go := 1 - go
6   else await(local.go ≠ go)
```

P1 Busy wait

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



SM: counter 0, go 0

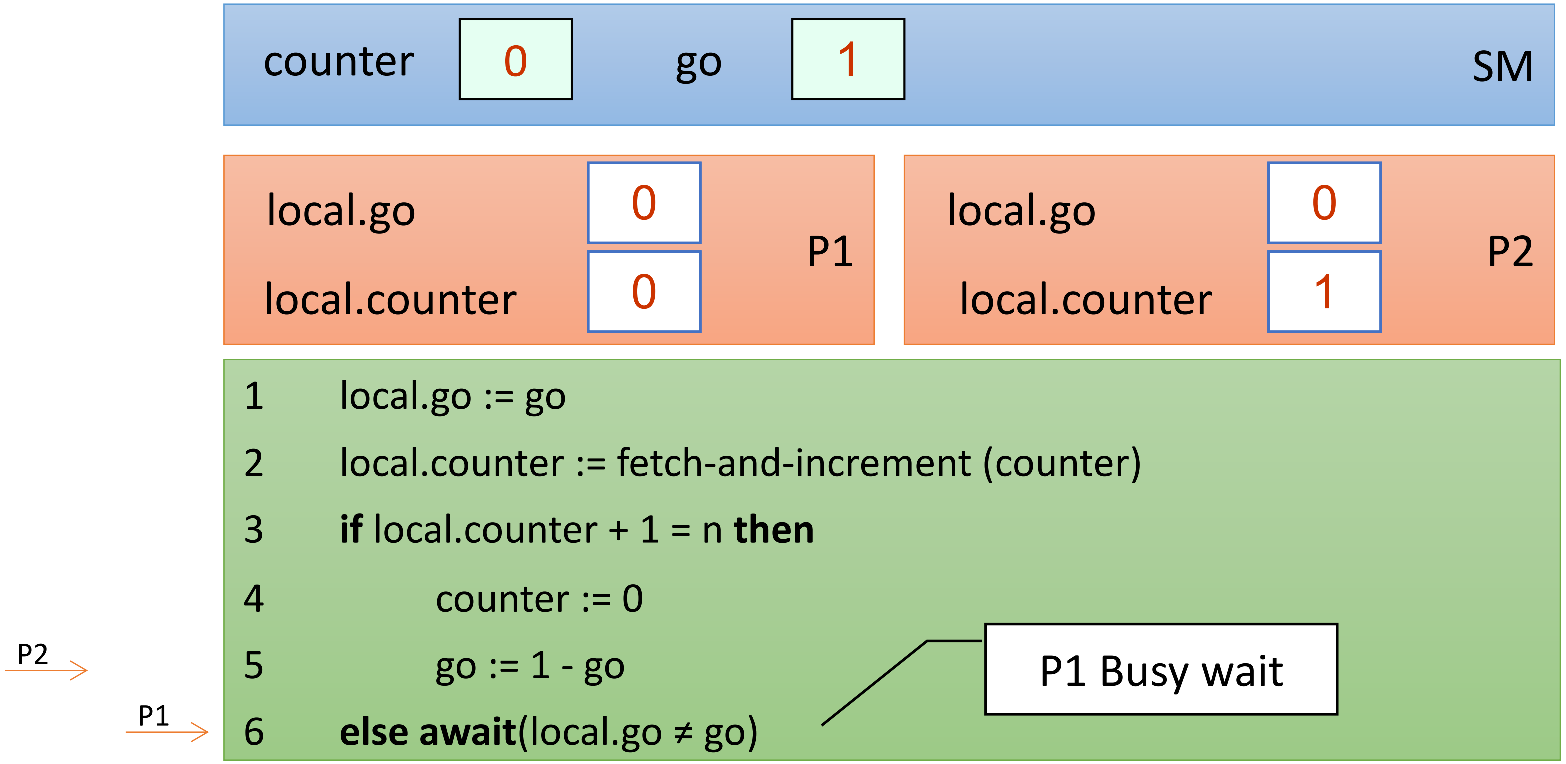
P1: local.go 0, local.counter 0

P2: local.go 0, local.counter 1

```
1   local.go := go
2   local.counter := fetch-and-increment (counter)
3   **if** local.counter + 1 = n **then**
4       counter := 0
5       go := 1 - go
6   **else await**(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



```
1    local.go := go
2    local.counter := fetch-and-increment (counter)
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5        go := 1 - go
6    **else await**(local.go ≠ go)
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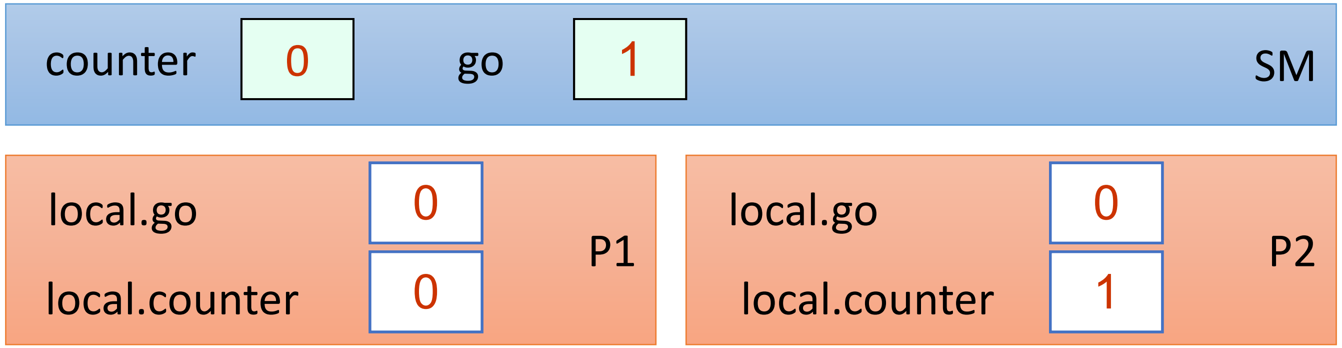
P1 Busy wait

Simple Barrier Using an Atomic Counter

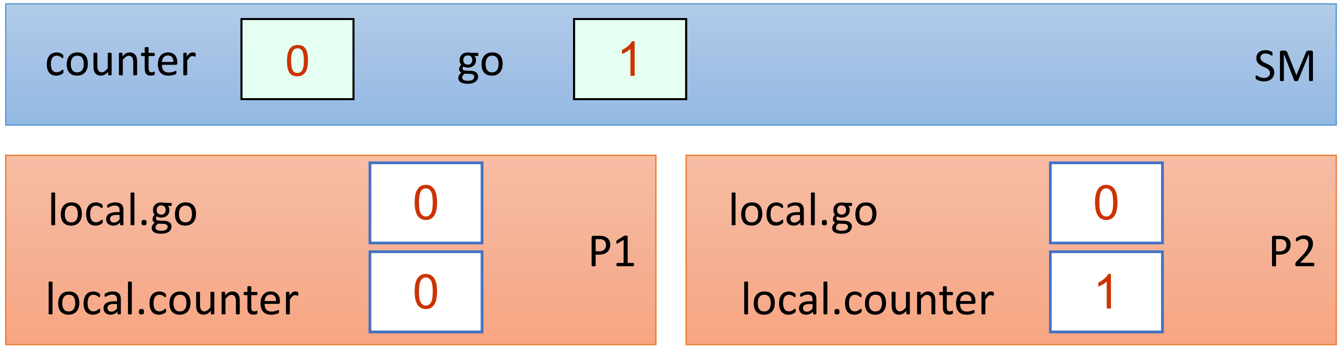
Run for n=2 Threads



```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



counter 0 go 1 SM

local.go 0

local.counter 0

P1

local.go 0

local.counter 1

P2

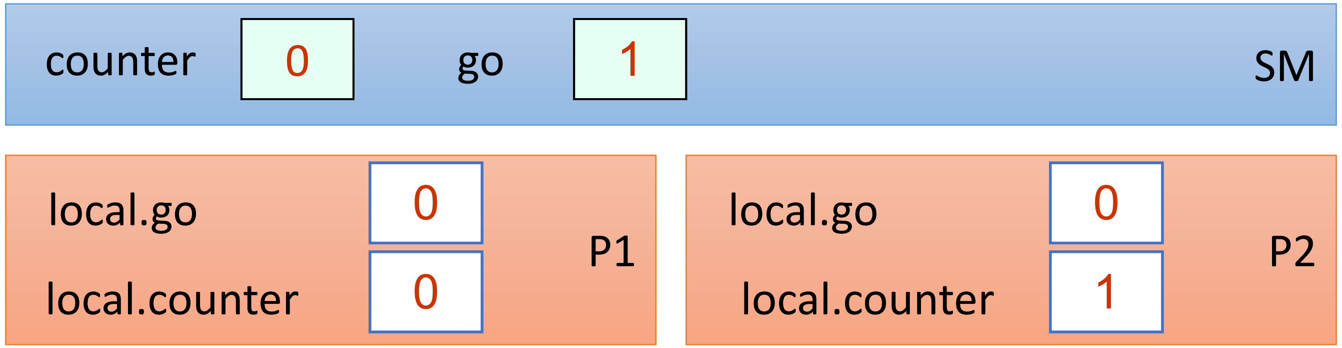
```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

Pros/Cons?

P2 → P1 →

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
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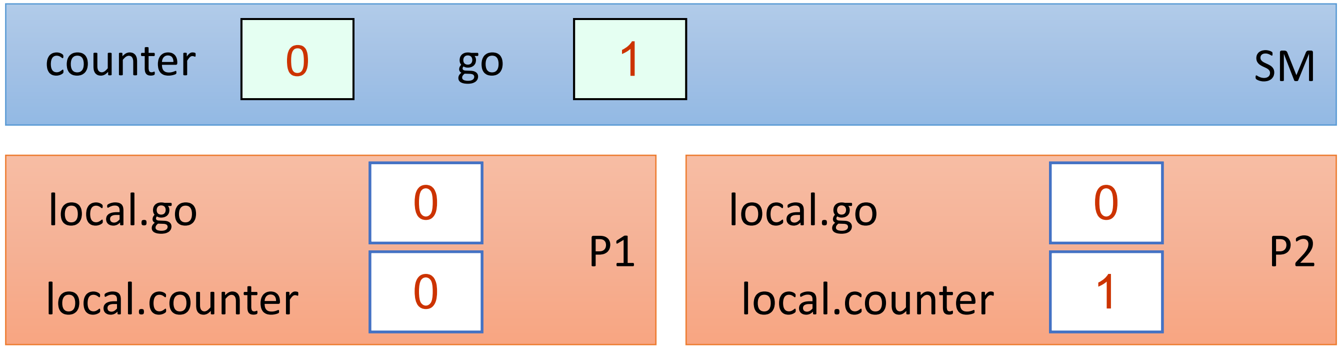
```

Pros/Cons?

P2 → P1 →

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



counter 0 go 1 SM

local.go 0 local.counter 0 P1

local.go 0 local.counter 1 P2

```
local.go := go
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
        counter := 0
        go := 1 - go
**else await**(local.go ≠ go)
```

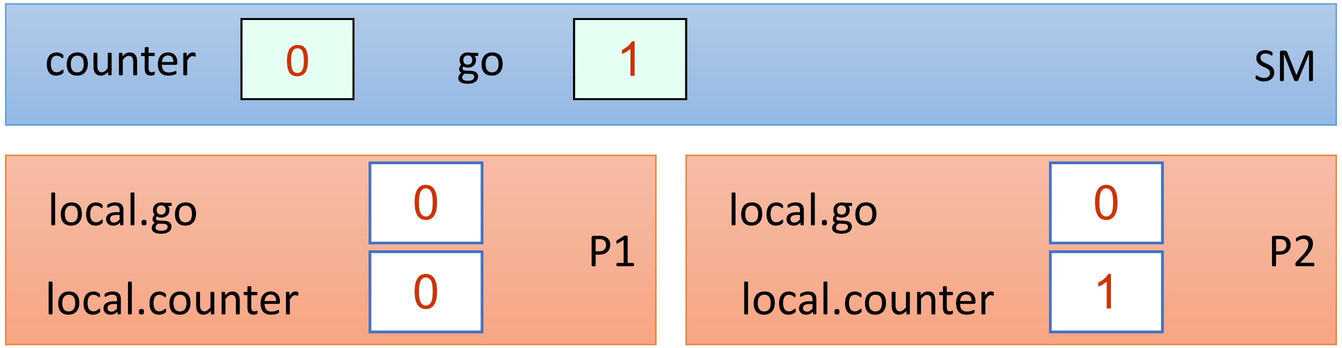
P2 → P1 →

Pros/Cons?

- There is high memory contention on *go* bit

Simple Barrier Using an Atomic Counter

Run for n=2 Threads



```
local.go := go
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        go := 1 - go
else await(local.go ≠ go)
```

P2 → P1 →

Pros/Cons?

- There is high memory contention on *go* bit
- Reducing the contention:
  - Replace the *go* bit with *n* bits: *go*[1],…,*go*[*n*]
  - Process p_i may spin only on the bit *go*[*i*]

A Local Spinning Counter Barrier

Program of a Thread i

```
shared    counter: fetch and increment reg. – {0,..n}, initially = 0
          go[1..n]: array of atomic bits, initial values are immaterial
local     local.go: a bit, initial value is immaterial
          local.counter: register
```

A Local Spinning Counter Barrier

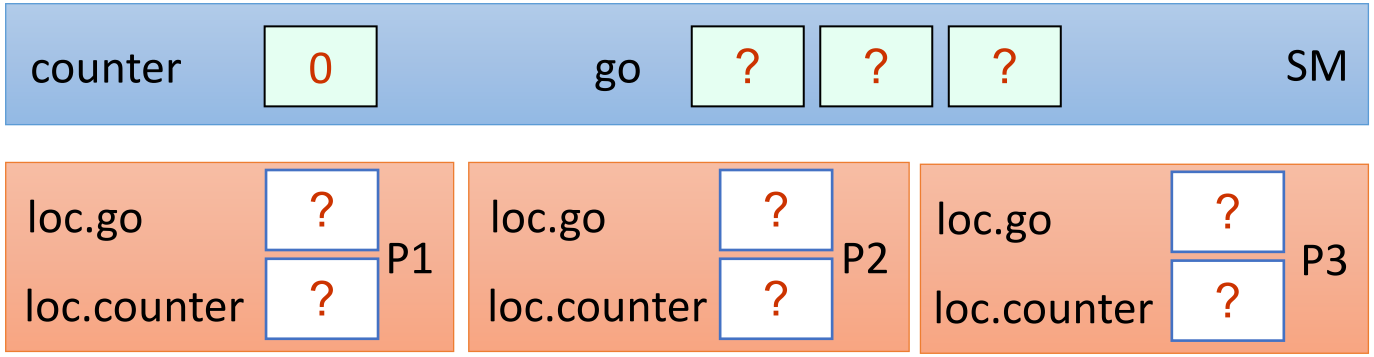
Program of a Thread i

```
shared    counter: fetch and increment reg. – {0,..n}, initially = 0
          go[1..n]: array of atomic bits, initial values are immaterial
local     local.go: a bit, initial value is immaterial
          local.counter: register
```

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
    counter := 0
    for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 0

go ? ? ?

SM

loc.go ?

loc.counter ?

P1

loc.go ?

loc.counter ?

P2

loc.go ?

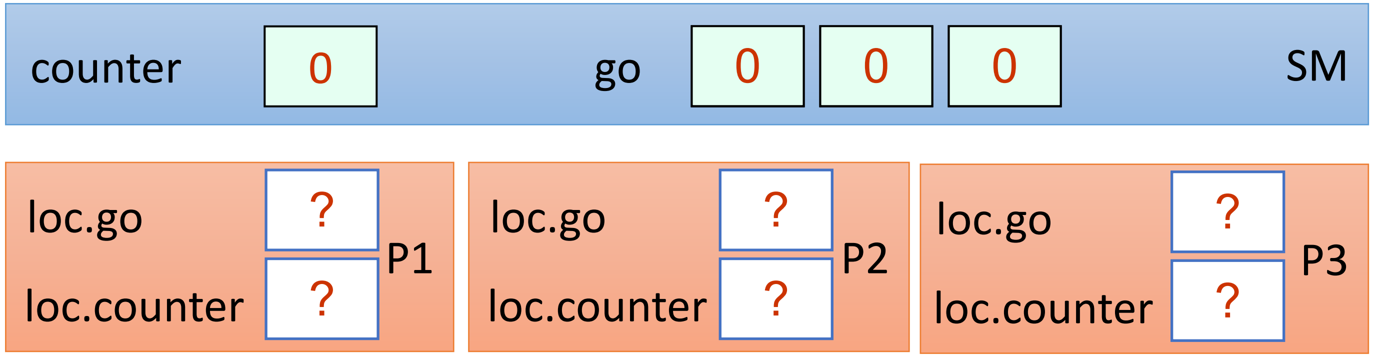
loc.counter ?

P3

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
        counter := 0
        **for** j=1 **to** n **{** go[j] := 1 – go[j] **}**
**else await**(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

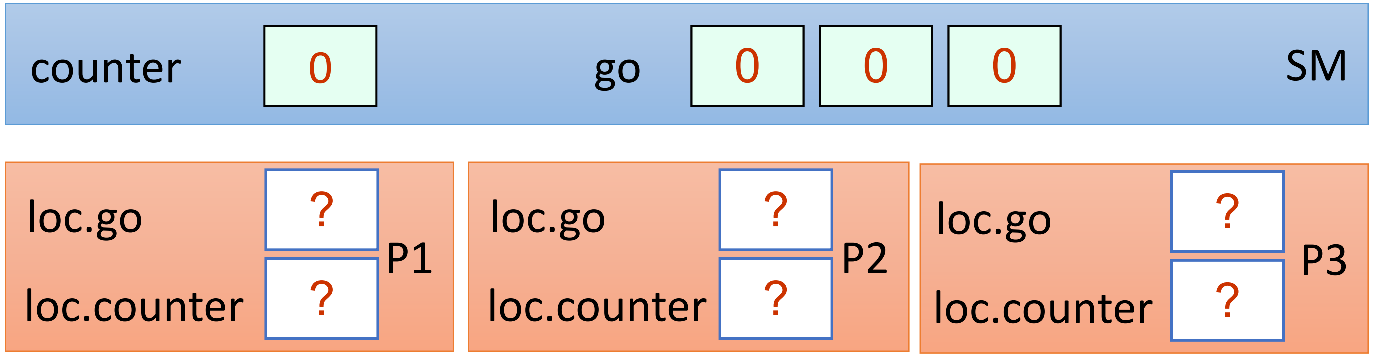
Example Run for n=3 Threads



```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
    counter := 0
    for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads


counter 0

go 0 0 0

SM

loc.go ? loc.counter ? P1

loc.go ? loc.counter ? P2

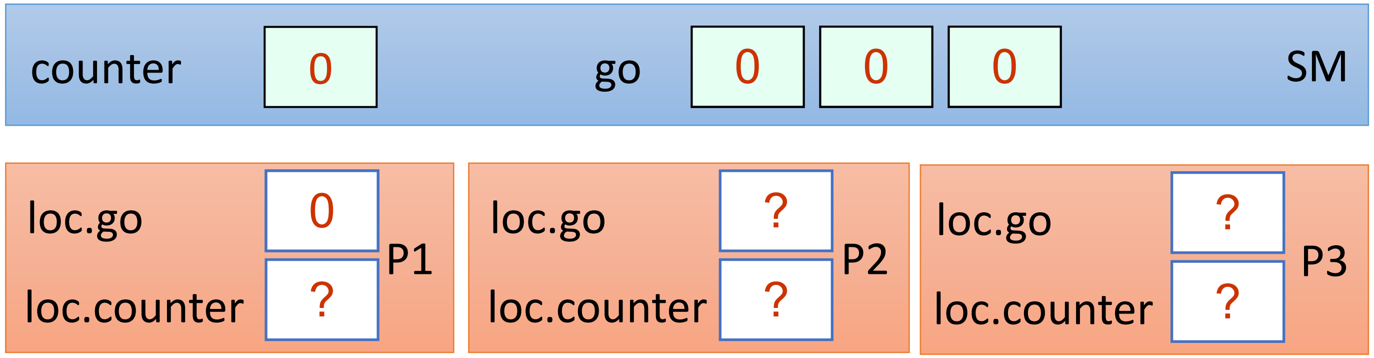
loc.go ? loc.counter ? P3

P1 →

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
    counter := 0
    **for** j=1 **to** n { go[j] := 1 – go[j] }
**else await**(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads

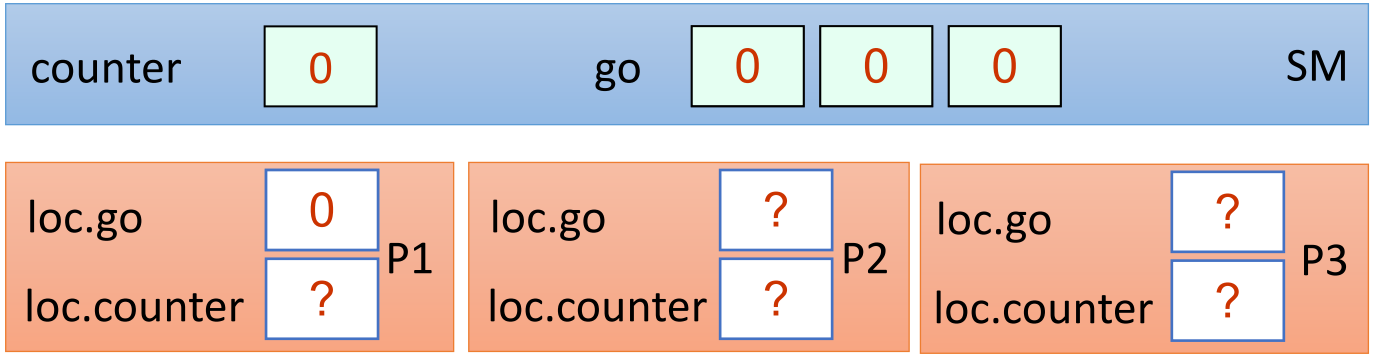


P1 →

```
1    local.go := go[i]
2    local.counter := fetch-and-increment (counter)
3    **if** local.counter + 1 = n **then**
4        counter := 0
5        **for** j=1 **to** n **{** go[j] := 1 – go[j] **}**
6    **else await**(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 0 go 0 0 0 SM

P1: loc.go 0, loc.counter ?

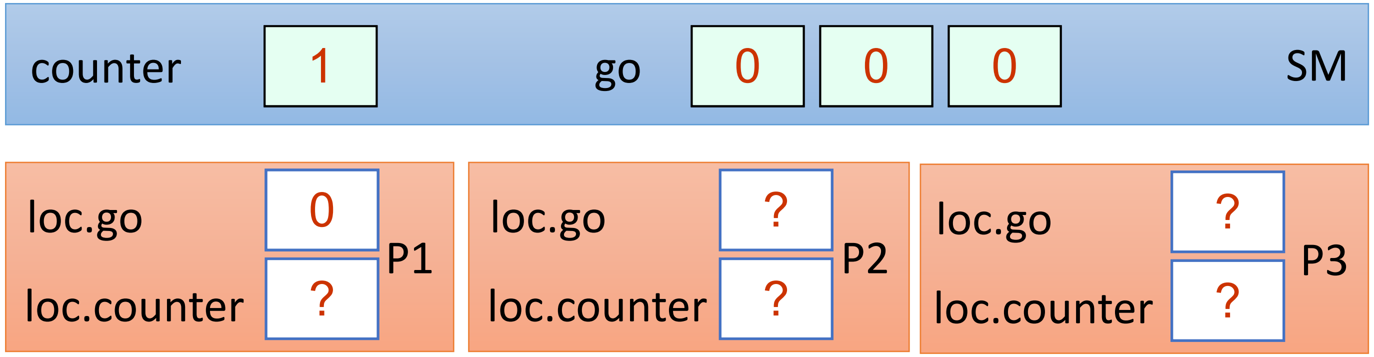
P2: loc.go ?, loc.counter ?

P3: loc.go ?, loc.counter ?

```
     1   local.go := go[i]
P1 → 2   local.counter := fetch-and-increment (counter)
     3   if local.counter + 1 = n then
     4       counter := 0
     5       for j=1 to n { go[j] := 1 – go[j] }
     6   else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 1 go 0 0 0 SM

loc.go 0 loc.counter ? P1

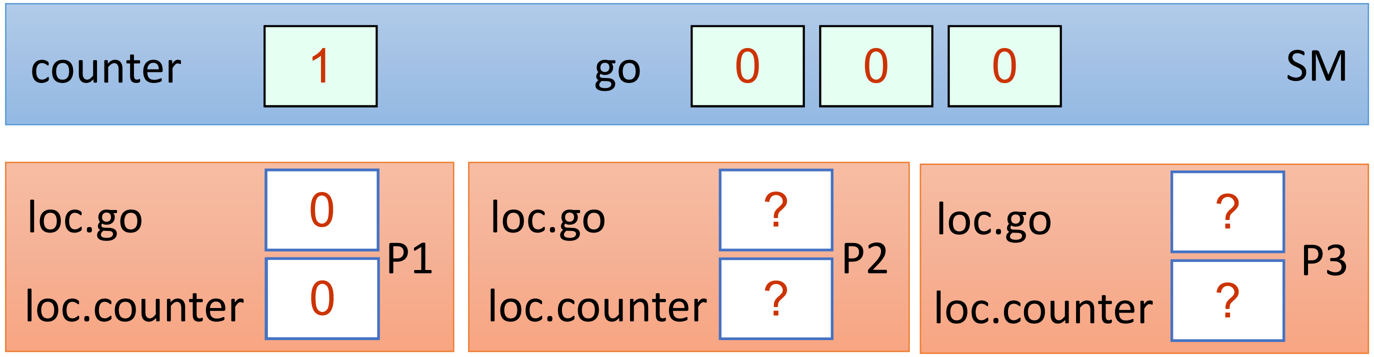
loc.go ? loc.counter ? P2

loc.go ? loc.counter ? P3

```
      1   local.go := go[i]
P1 →  2   local.counter := fetch-and-increment (counter)
      3   if local.counter + 1 = n then
      4       counter := 0
      5       for j=1 to n { go[j] := 1 – go[j] }
      6   else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



SM: counter 1 go 0 0 0

P1: loc.go 0 loc.counter 0

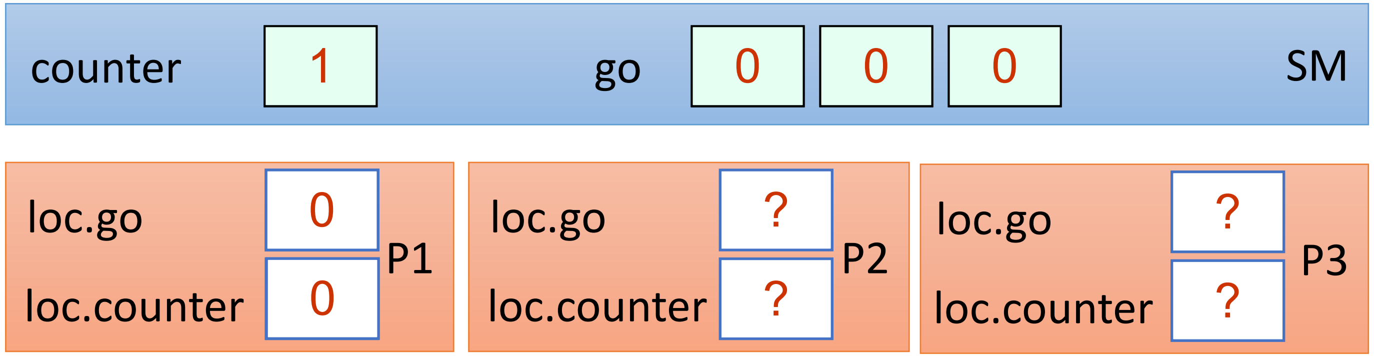
P2: loc.go ? loc.counter ?

P3: loc.go ? loc.counter ?

```
     1   local.go := go[i]
P1 → 2   local.counter := fetch-and-increment (counter)
     3   **if** local.counter + 1 = n **then**
     4       counter := 0
     5       **for** j=1 **to** n **{** go[j] := 1 – go[j] **}**
     6   **else await**(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

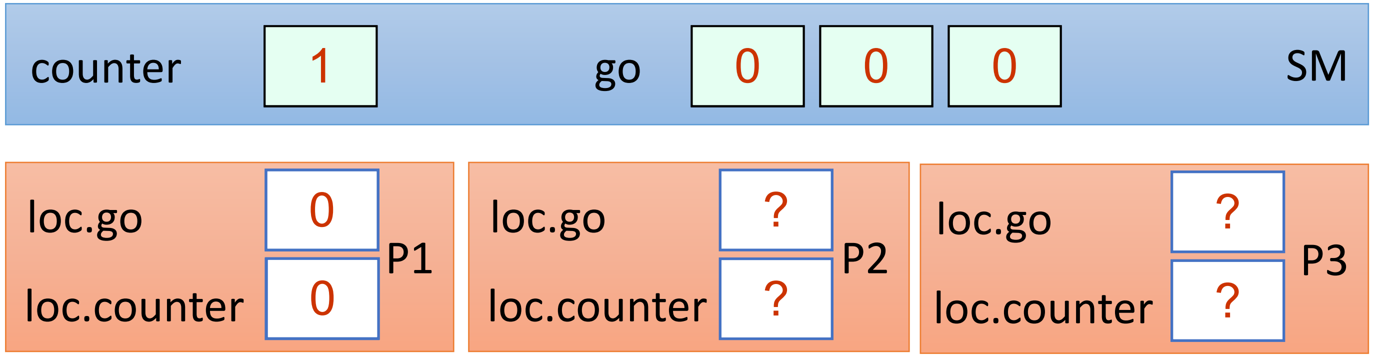
Example Run for n=3 Threads



```
      1   local.go := go[i]
      2   local.counter := fetch-and-increment (counter)
P1 →  3   if local.counter + 1 = n then
      4       counter := 0
      5       for j=1 to n { go[j] := 1 – go[j] }
      6   else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



SM				
counter	1			
go	0	0	0	

	P1	P2	P3
loc.go	0	?	?
loc.counter	0	?	?

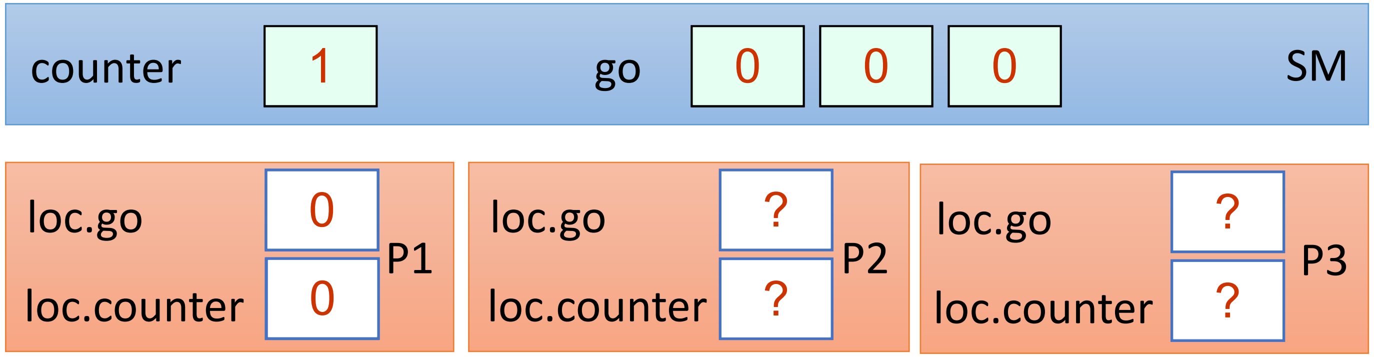
```
local.go := go[i]
local.counter := fetch-and-increment (
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

P1

0+1≠3

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 1 go 0 0 0 SM

loc.go 0
loc.counter 0 P1

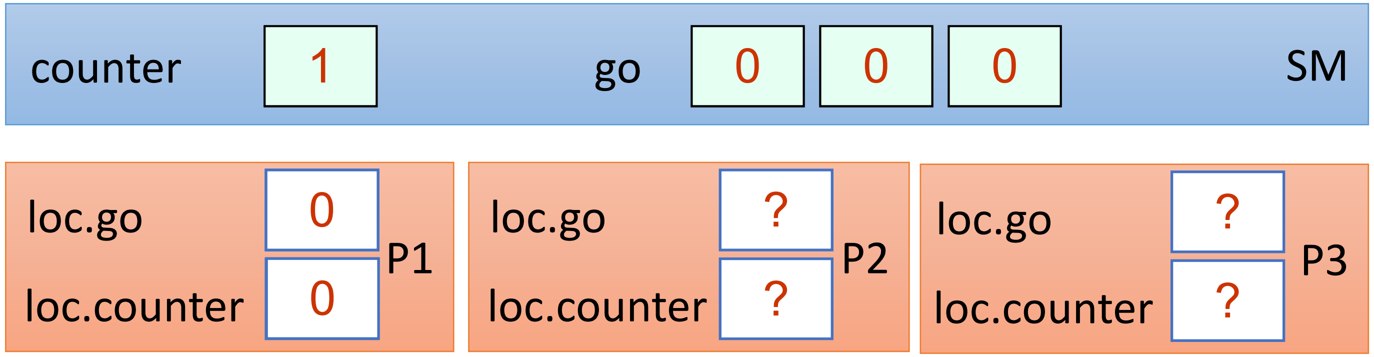
loc.go ?
loc.counter ? P2

loc.go ?
loc.counter ? P3

```
   1  local.go := go[i]
   2  local.counter := fetch-and-increment (counter)
   3  if local.counter + 1 = n then
   4      counter := 0
   5      for j=1 to n { go[j] := 1 – go[j] }
P1 → 6  else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 1

go 0 0 0

SM

P1: loc.go 0, loc.counter 0

P2: loc.go ?, loc.counter ?

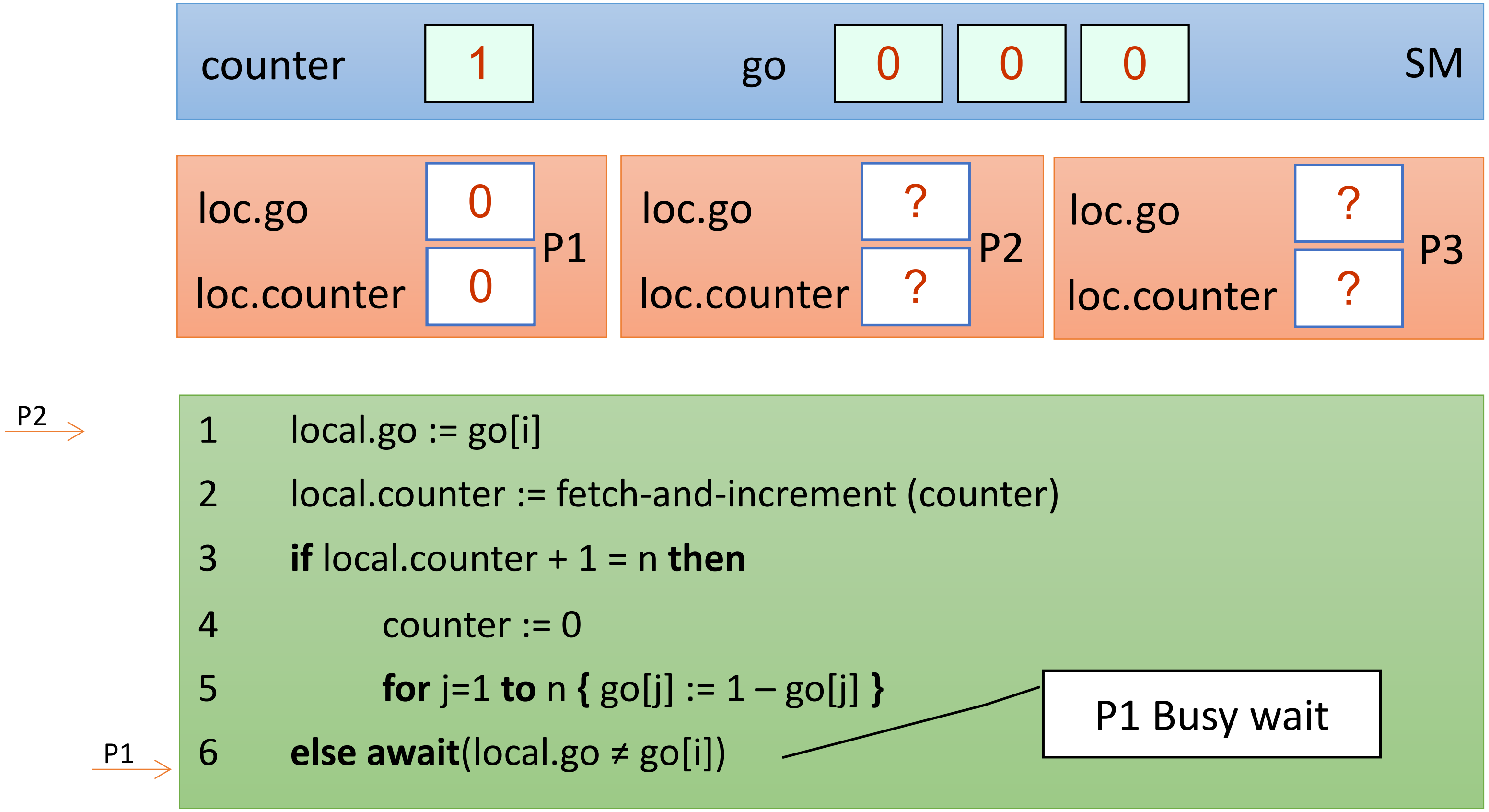
P3: loc.go ?, loc.counter ?

```
     local.go := go[i]
     local.counter := fetch-and-increment (counter)
     **if** local.counter + 1 = n **then**
          counter := 0
          **for** j=1 **to** n **{** go[j] := 1 – go[j] **}**
P1 → **else await**(local.go ≠ go[i])
```

P1 Busy wait

A Local Spinning Counter Barrier

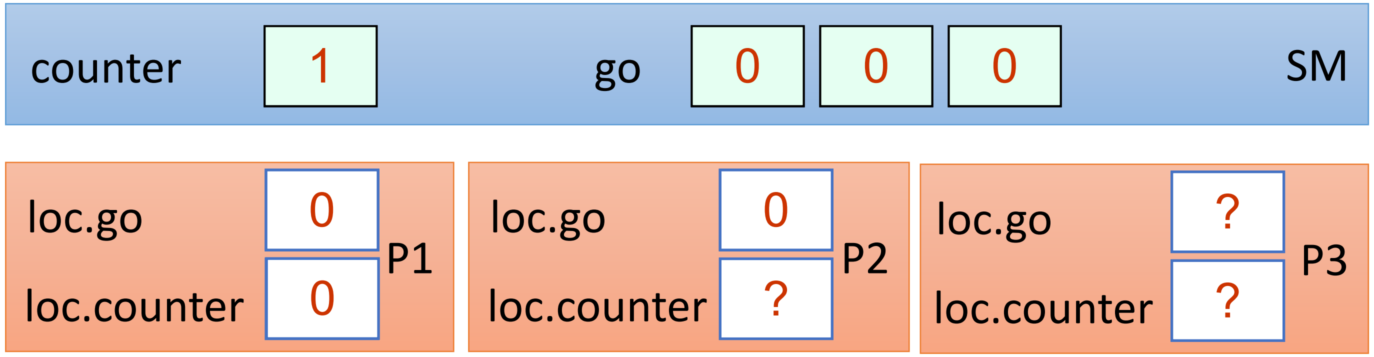
Example Run for n=3 Threads



```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 1 go 0 0 0 SM

loc.go 0 loc.counter 0 P1

loc.go 0 loc.counter ? P2

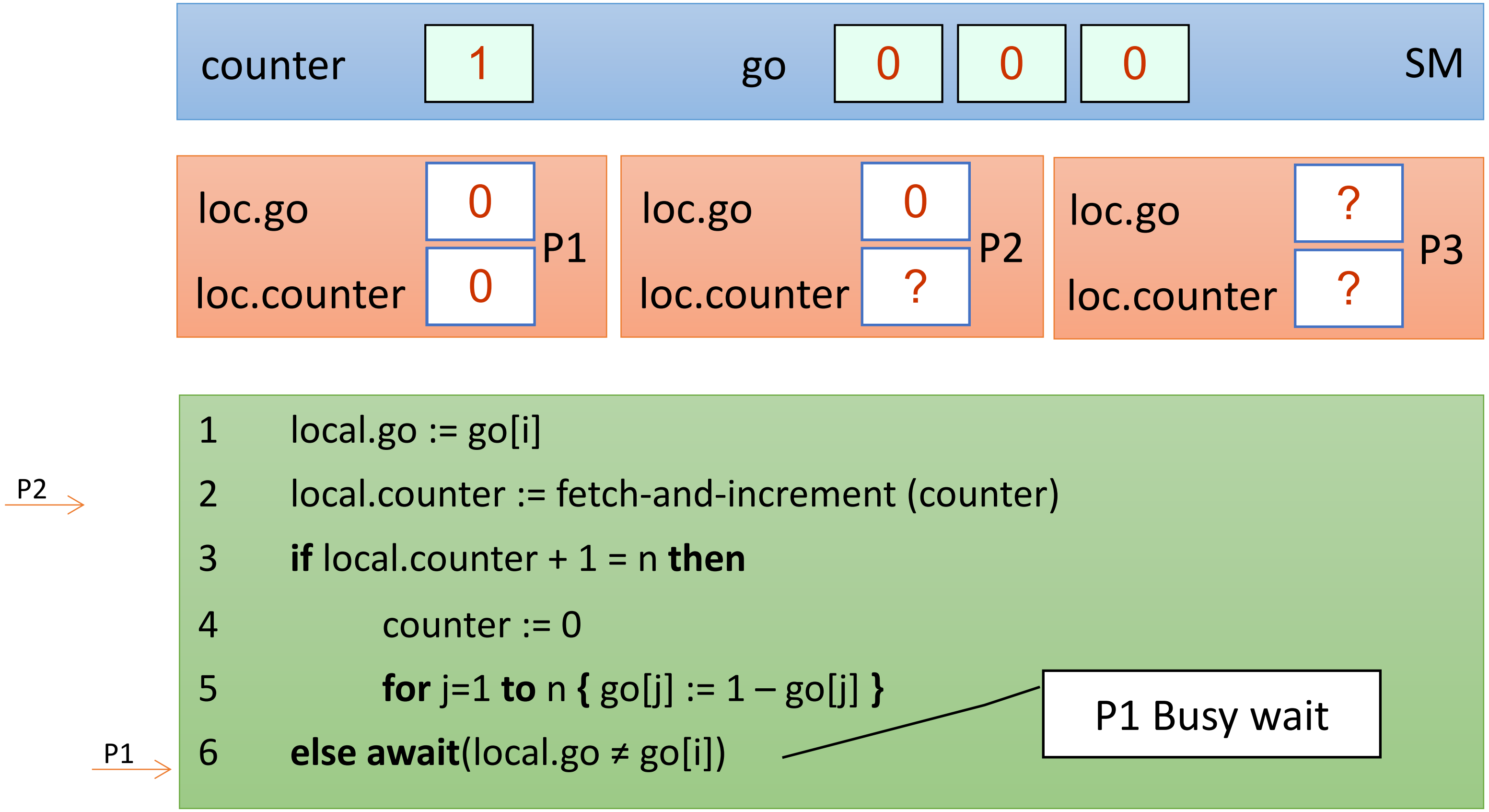
loc.go ? loc.counter ? P3

```
P2 →   1   local.go := go[i]
       2   local.counter := fetch-and-increment (counter)
       3   if local.counter + 1 = n then
       4       counter := 0
       5       for j=1 to n { go[j] := 1 – go[j] }
P1 →   6   else await(local.go ≠ go[i])
```

P1 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 1 go 0 0 0 SM

P1: loc.go 0, loc.counter 0

P2: loc.go 0, loc.counter ?

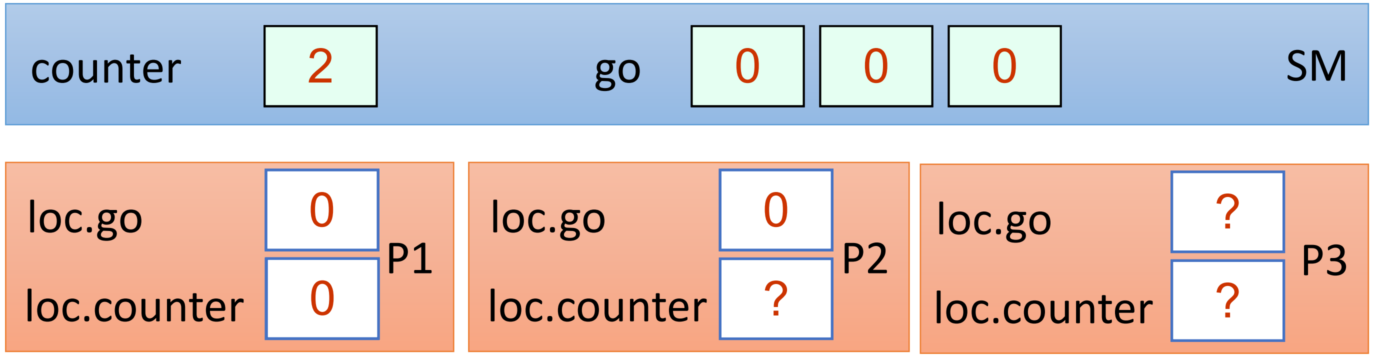
P3: loc.go ?, loc.counter ?

```
   1   local.go := go[i]
P2 →  2   local.counter := fetch-and-increment (counter)
   3   if local.counter + 1 = n then
   4       counter := 0
   5       for j=1 to n { go[j] := 1 – go[j] }
P1 →  6   else await(local.go ≠ go[i])
```

P1 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 2

go 0 0 0

SM

loc.go 0

loc.counter 0

P1

loc.go 0

loc.counter ?

P2

loc.go ?

loc.counter ?

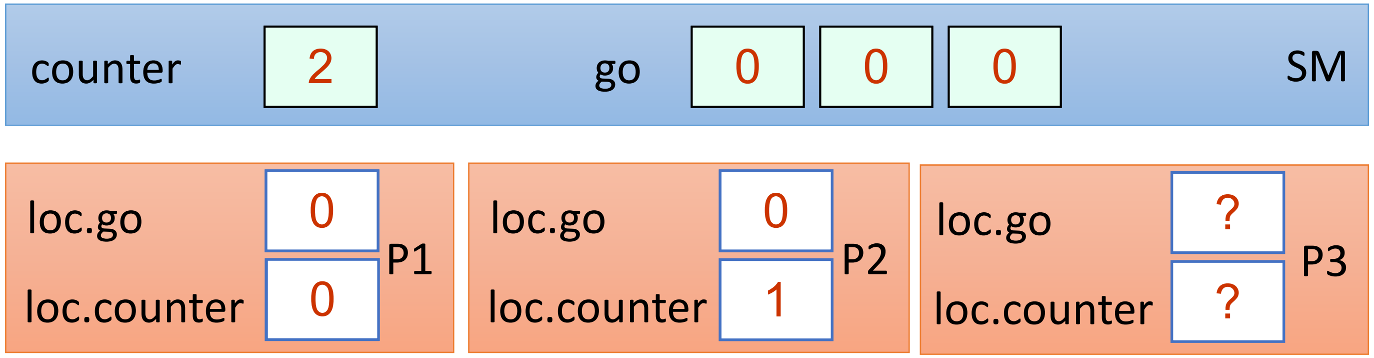
P3

```
      1   local.go := go[i]
P2 →  2   local.counter := fetch-and-increment (counter)
      3   if local.counter + 1 = n then
      4           counter := 0
      5           for j=1 to n { go[j] := 1 – go[j] }
P1 →  6   else await(local.go ≠ go[i])
```

P1 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads

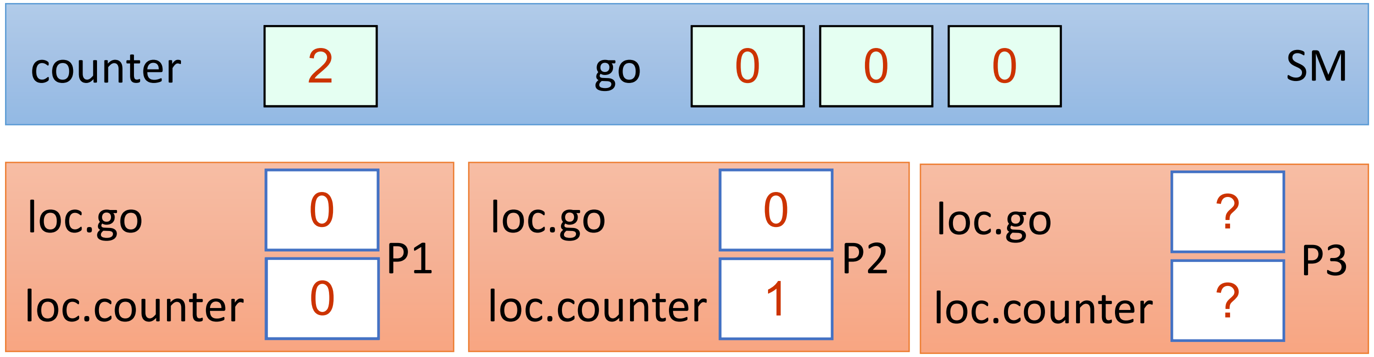


```
      local.go := go[i]
P2 →  local.counter := fetch-and-increment (counter)
      if local.counter + 1 = n then
            counter := 0
            for j=1 to n { go[j] := 1 – go[j] }
P1 →  else await(local.go ≠ go[i])
```

P1 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



SM				
counter	2	go	0	0	0

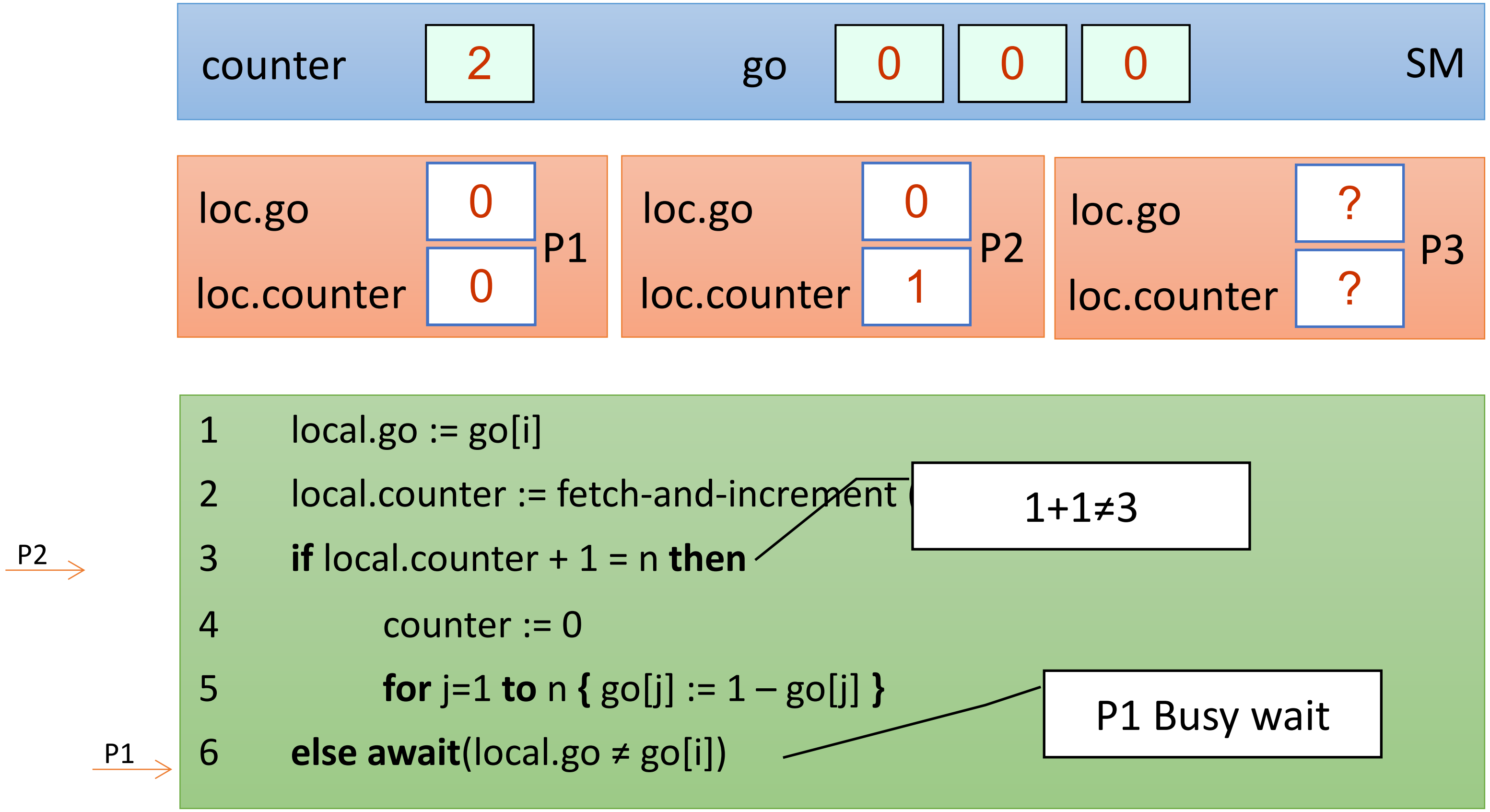
	P1	P2	P3
loc.go	0	0	?
loc.counter	0	1	?

```
      local.go := go[i]
      local.counter := fetch-and-increment (counter)
P2 →  if local.counter + 1 = n then
          counter := 0
          for j=1 to n { go[j] := 1 – go[j] }
P1 →  else await(local.go ≠ go[i])
```

P1 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



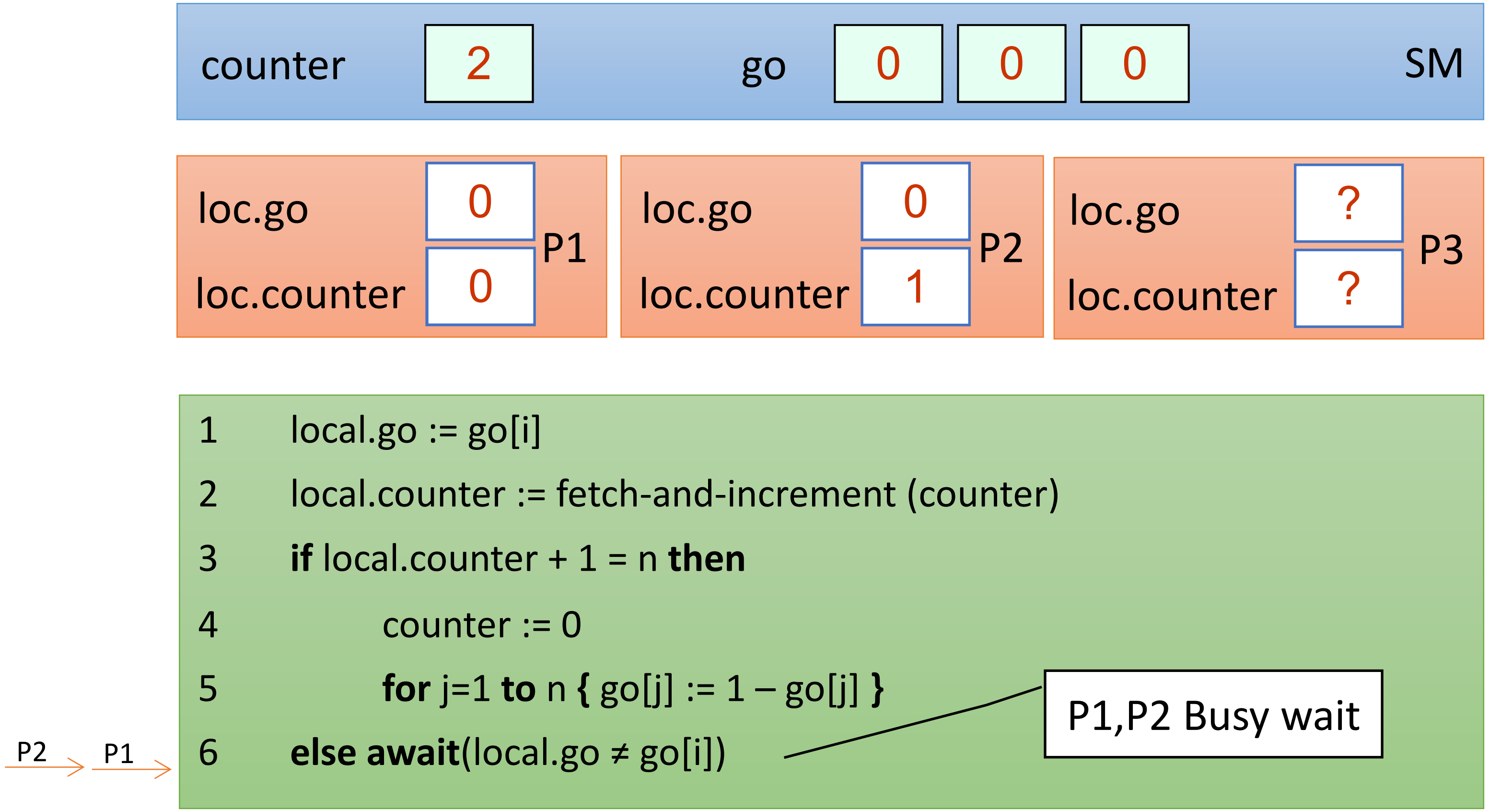
```
local.go := go[i]
local.counter := fetch-and-increment (
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

1+1≠3

P1 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



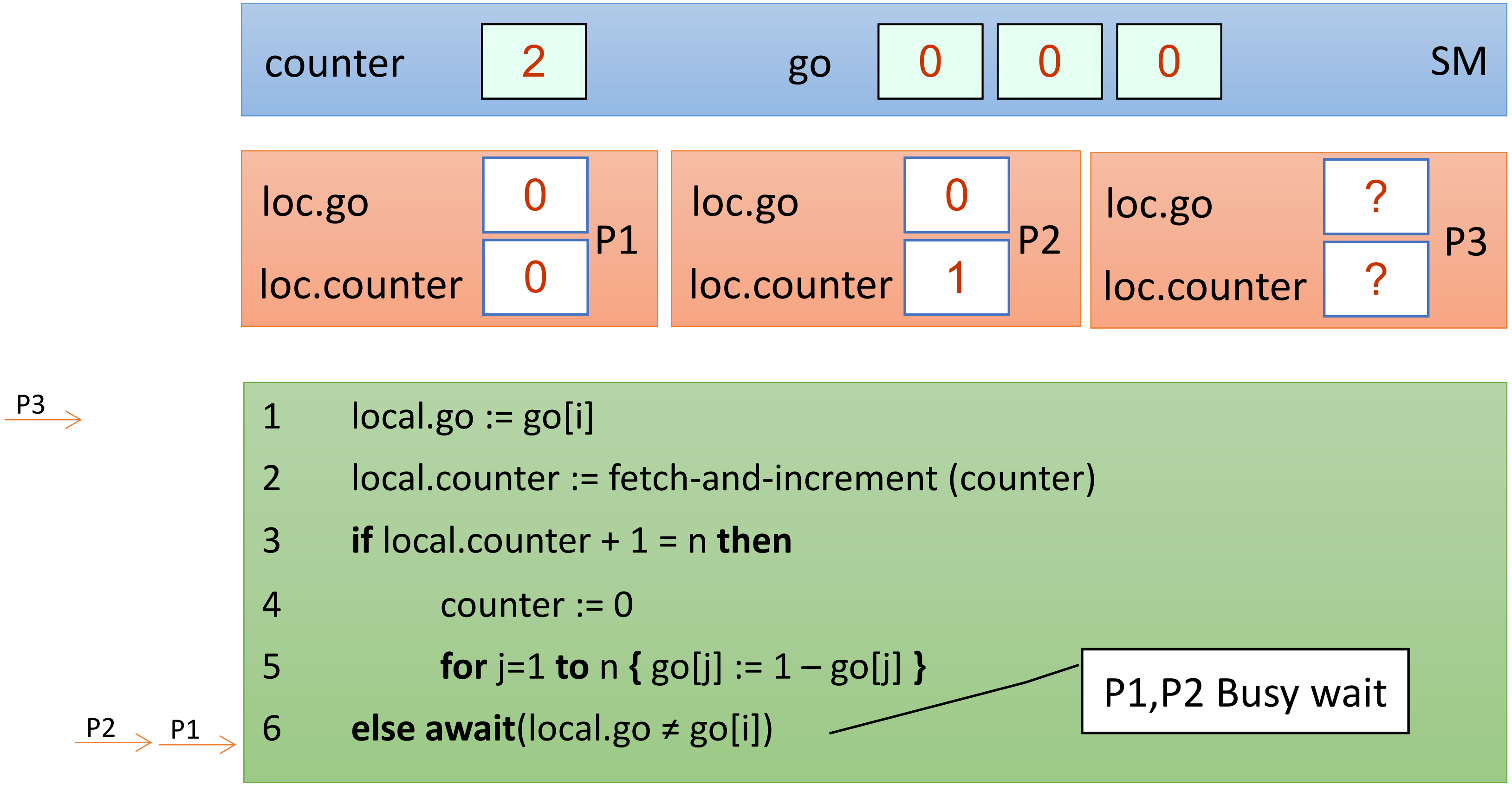
SM				
counter	2			
go	0	0	0	

	P1	P2	P3
loc.go	0	0	?
loc.counter	0	1	?

```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads

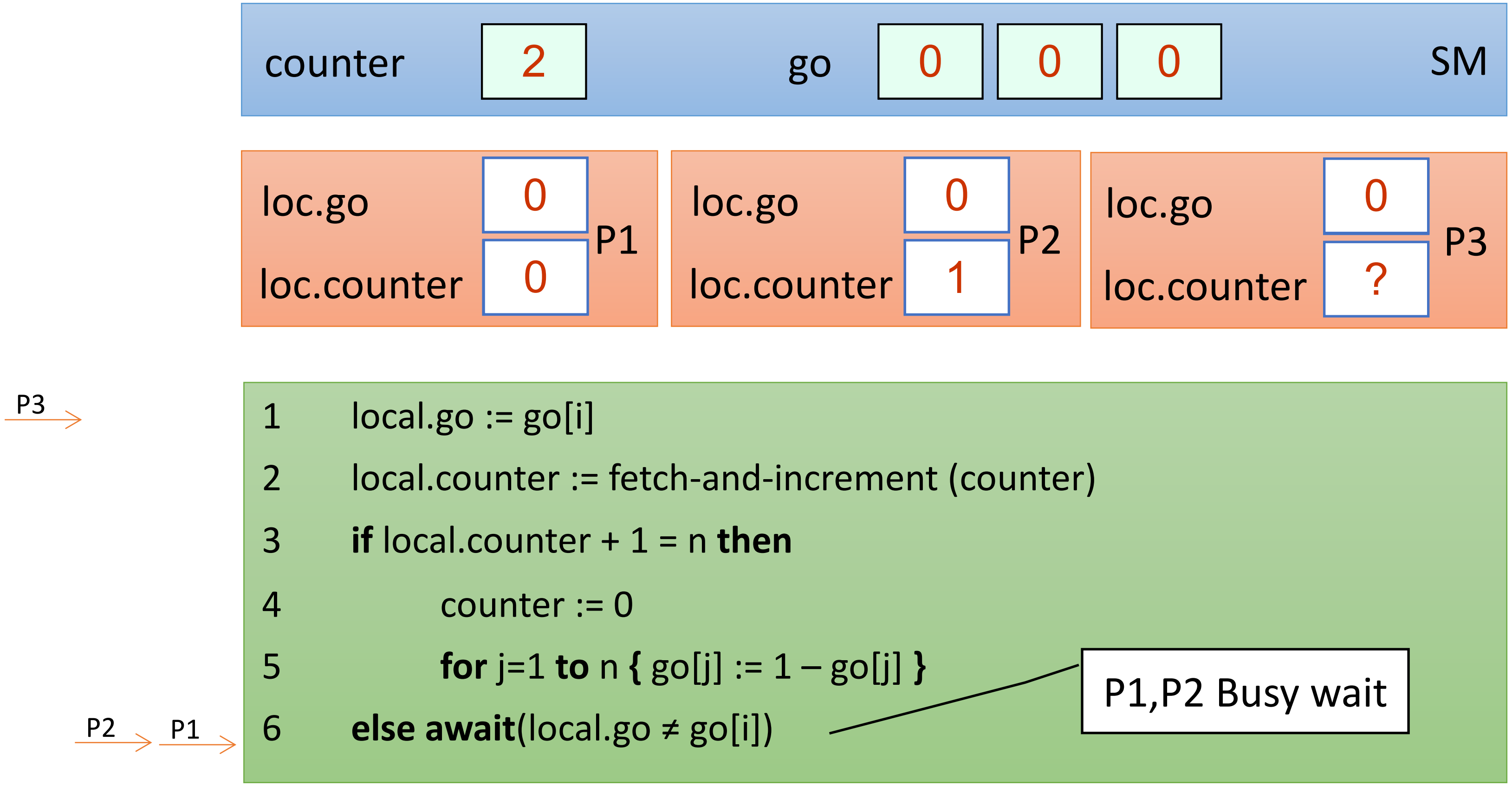


```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

P1,P2 Busy wait

A Local Spinning Counter Barrier

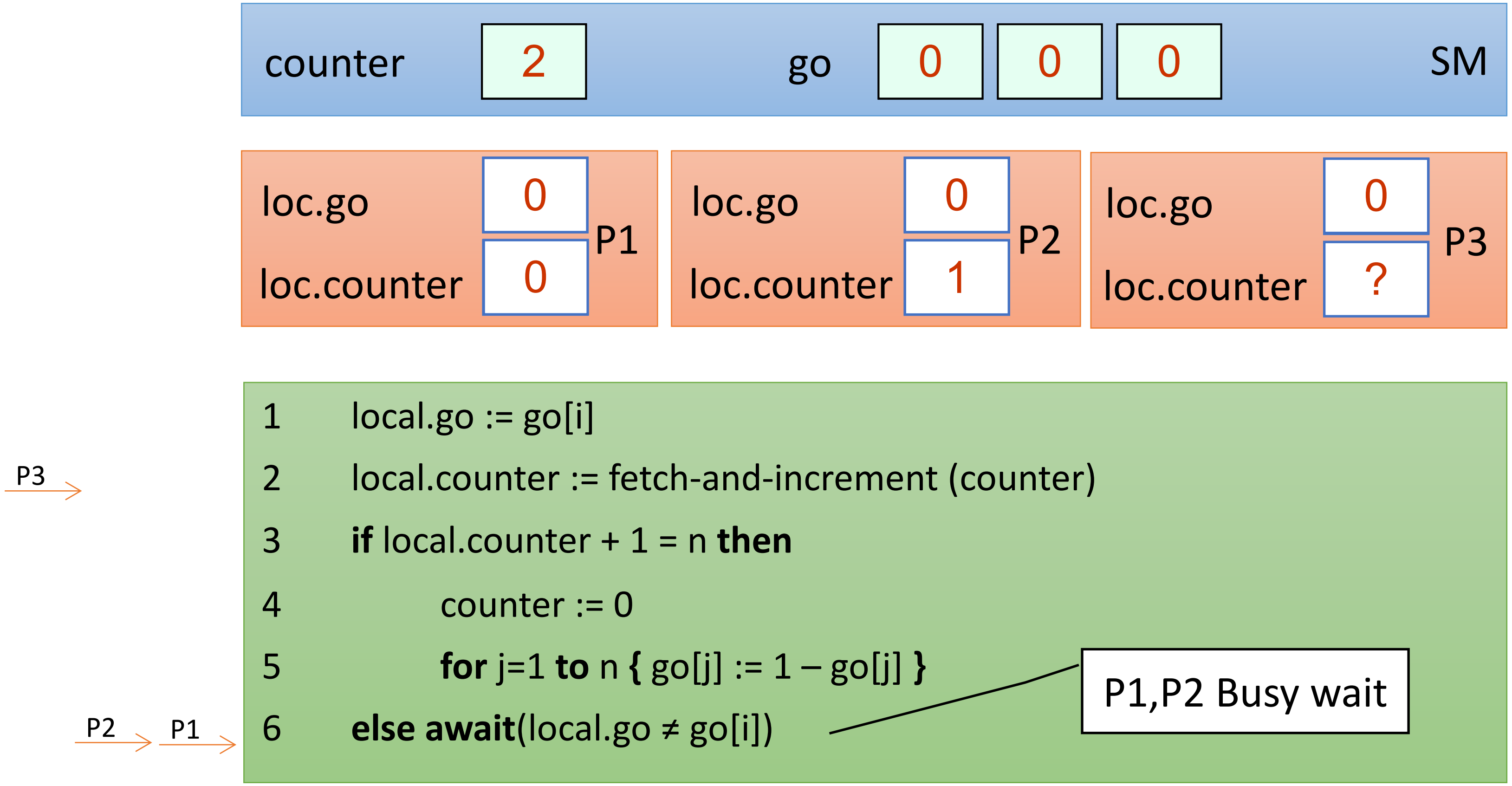
Example Run for n=3 Threads



```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4           counter := 0
5           for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 2 go 0 0 0 SM

loc.go 0 loc.counter 0 P1

loc.go 0 loc.counter 1 P2

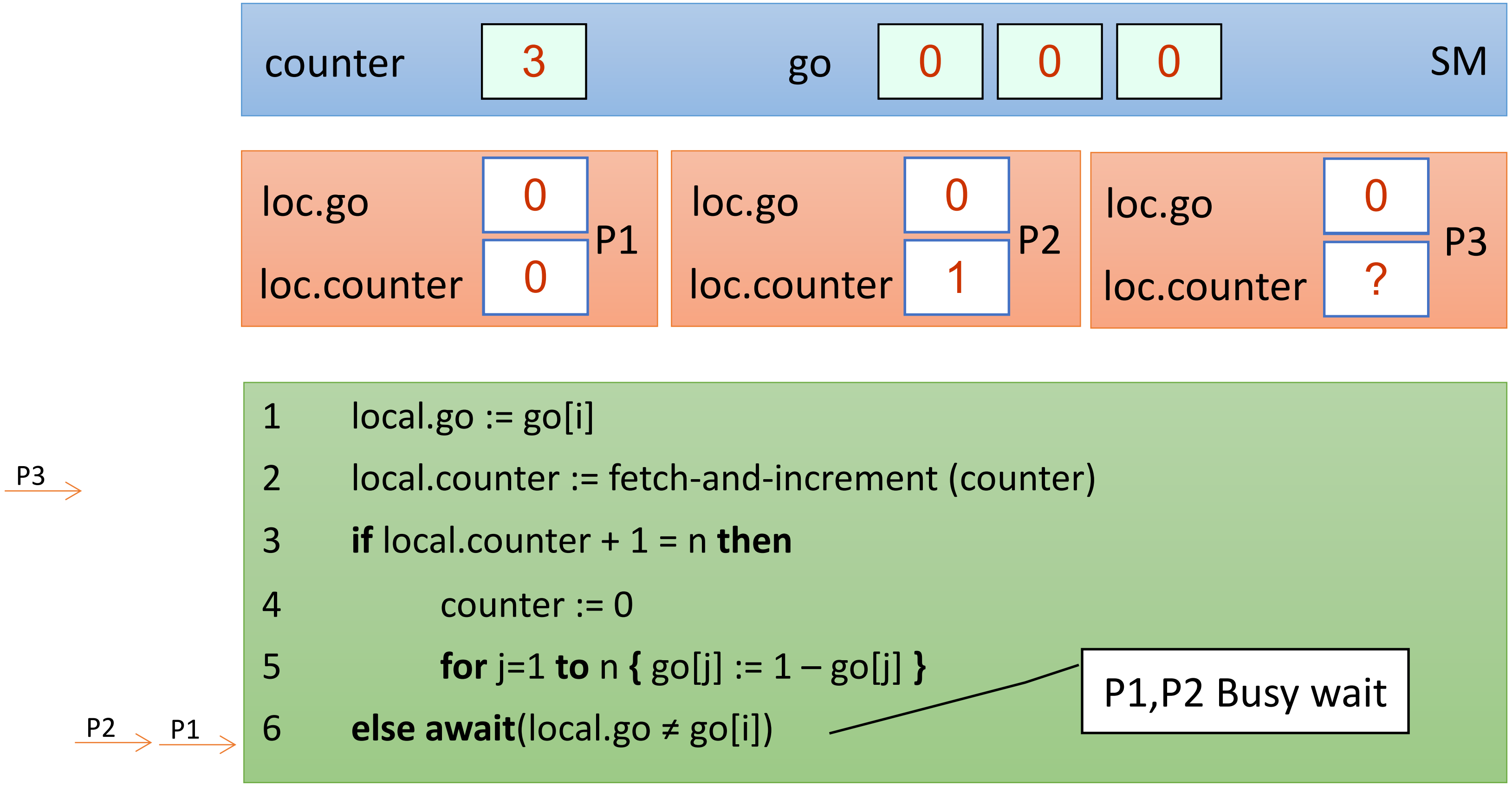
loc.go 0 loc.counter ? P3

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
    counter := 0
    for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 3 go 0 0 0 SM

P1: loc.go 0, loc.counter 0

P2: loc.go 0, loc.counter 1

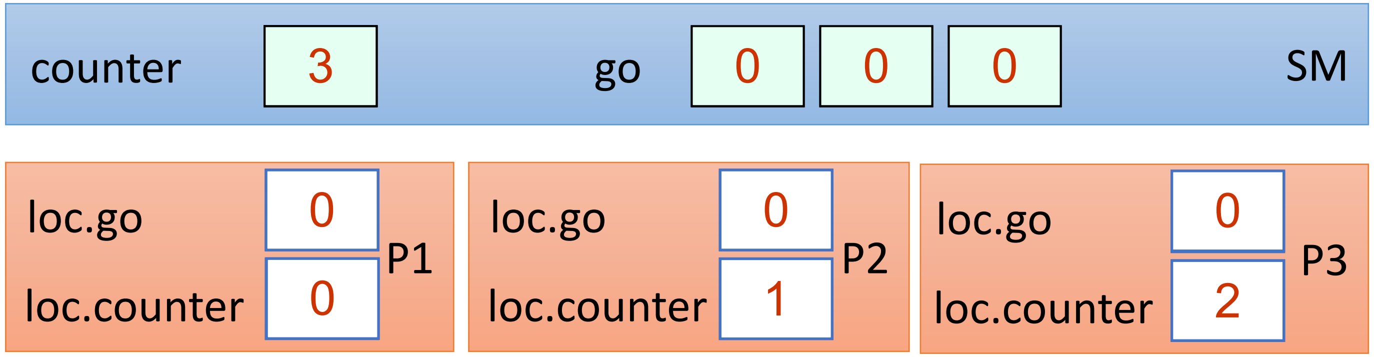
P3: loc.go 0, loc.counter ?

```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)    ← P3
3   if local.counter + 1 = n then
4       counter := 0
5       for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])                      ← P2, P1
```

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 3 go 0 0 0 SM

loc.go 0 loc.counter 0 P1

loc.go 0 loc.counter 1 P2

loc.go 0 loc.counter 2 P3

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

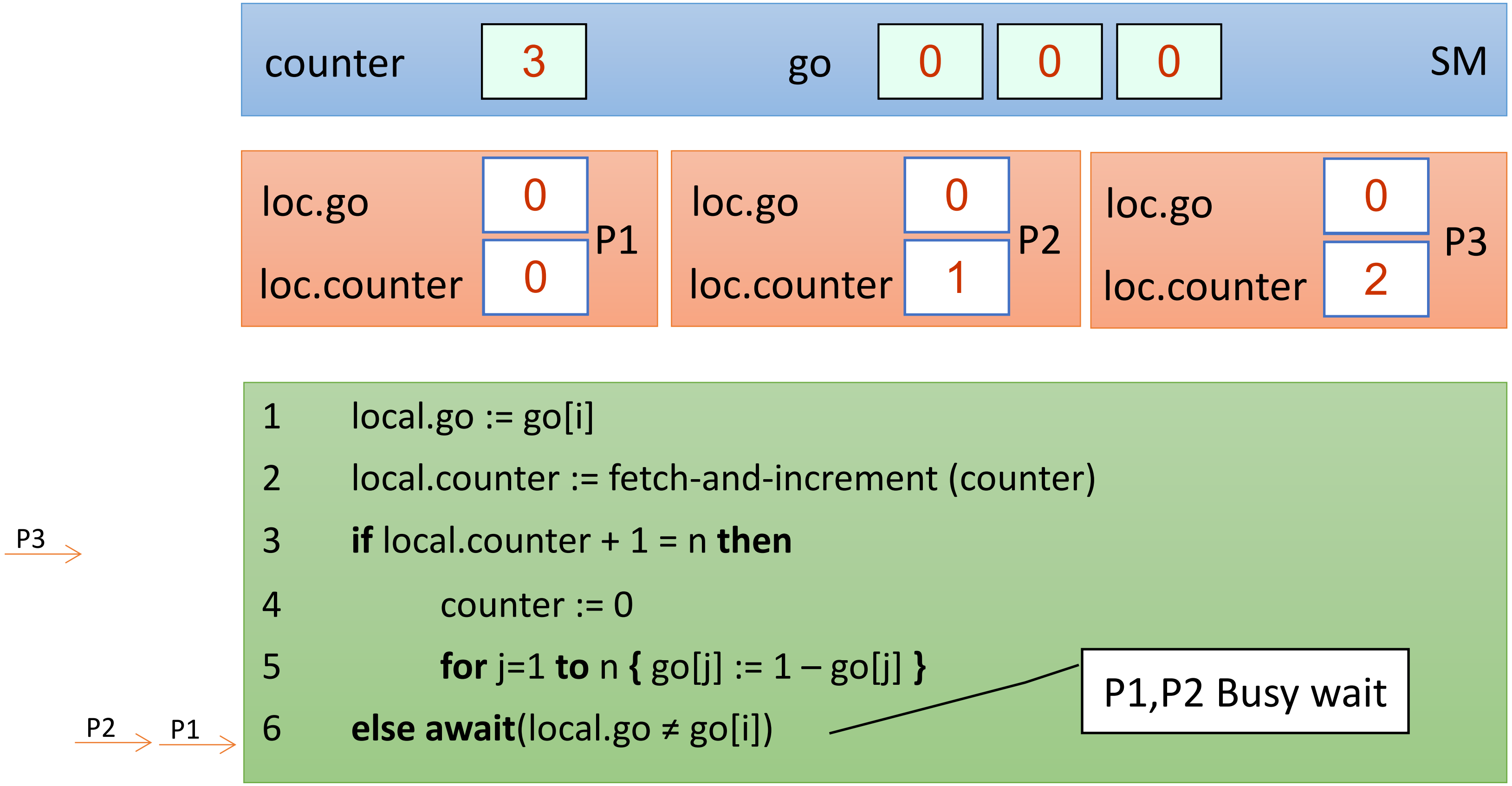
P3 →

P2 → P1 →

P1,P2 Busy wait

A Local Spinning Counter Barrier

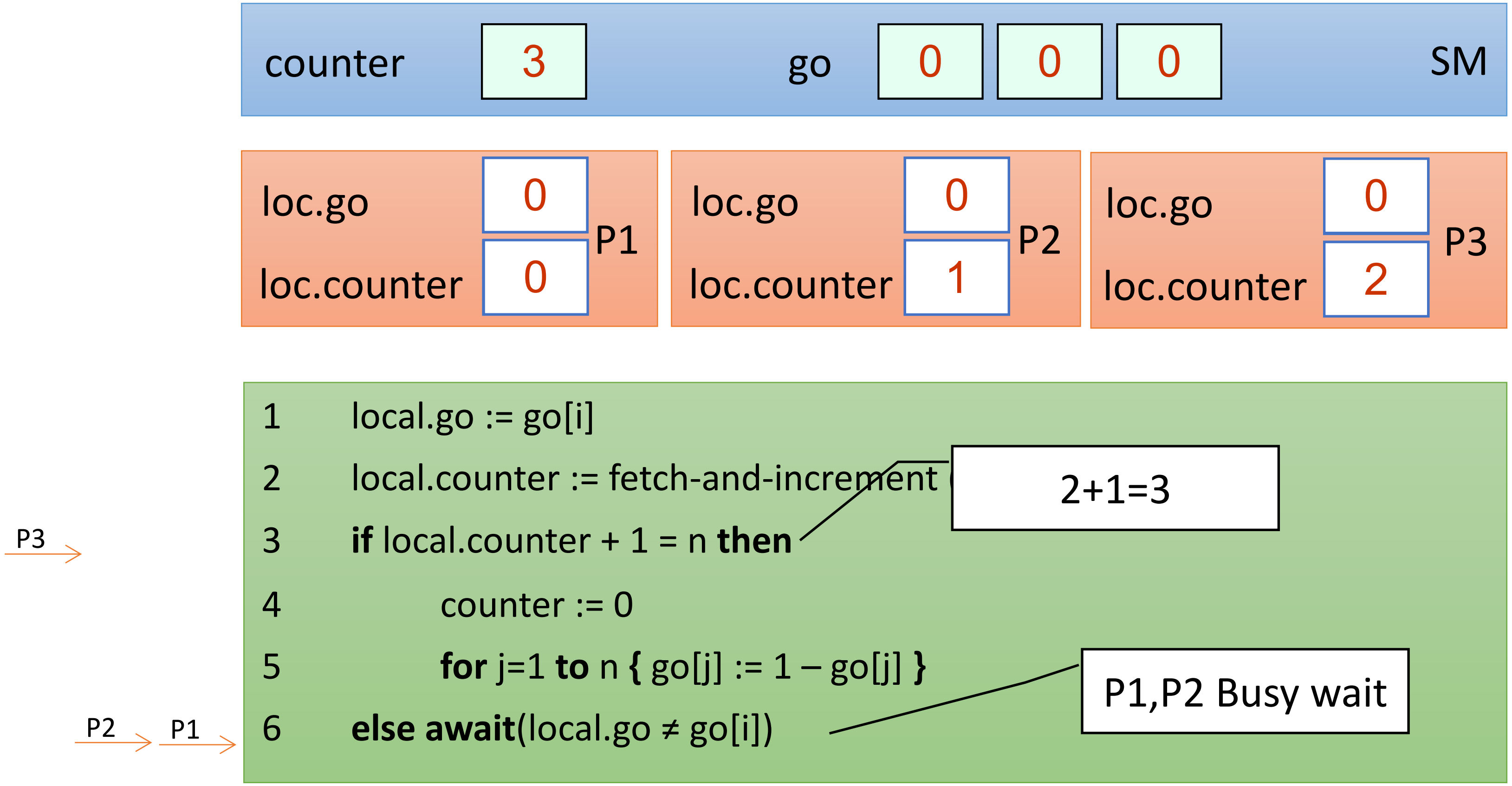
Example Run for n=3 Threads



```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



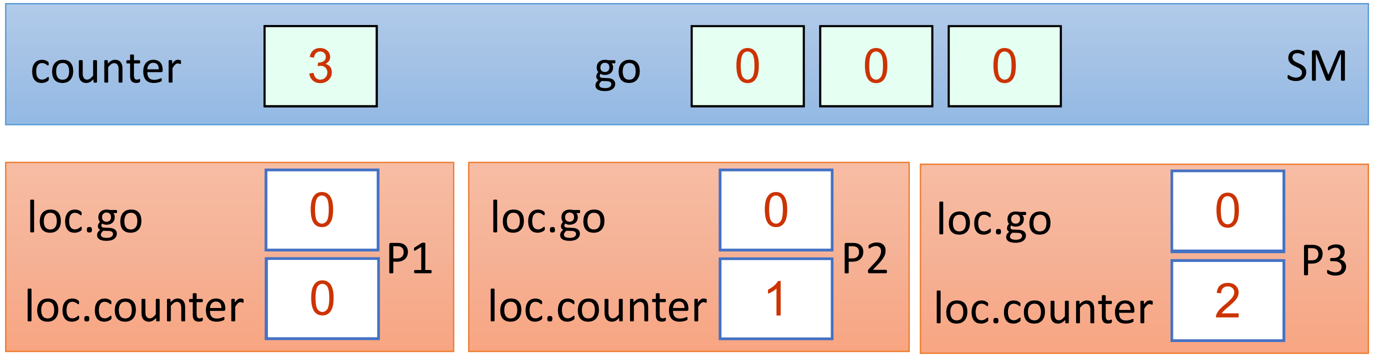
```
local.go := go[i]
local.counter := fetch-and-increment (
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

2+1=3

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 3 go 0 0 0 SM

loc.go 0 loc.counter 0 P1

loc.go 0 loc.counter 1 P2

loc.go 0 loc.counter 2 P3

```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4           counter := 0
5           for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

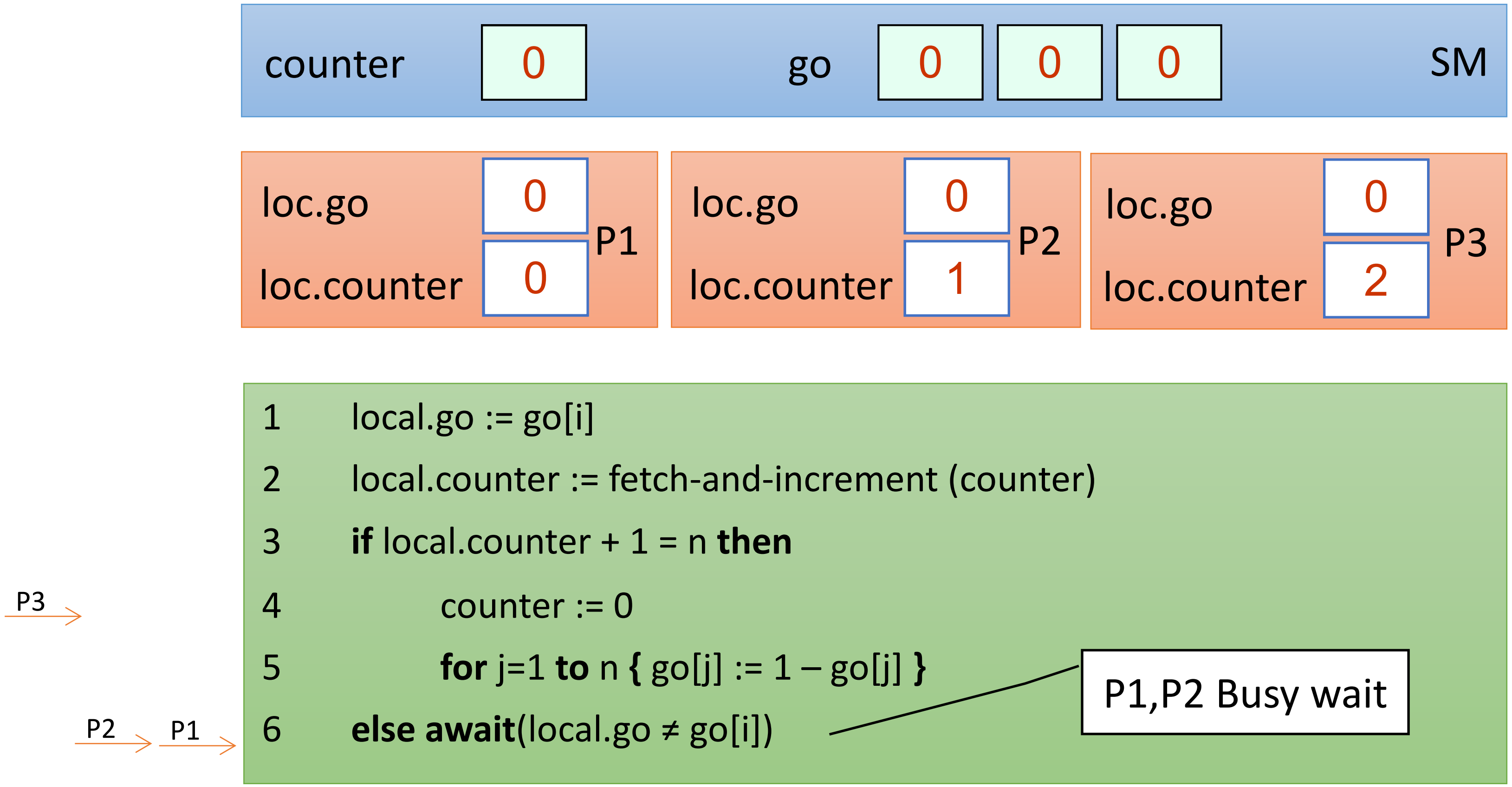
P3 → 4

P2 → P1 → 6

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



SM				
counter	0	go	0	0	0

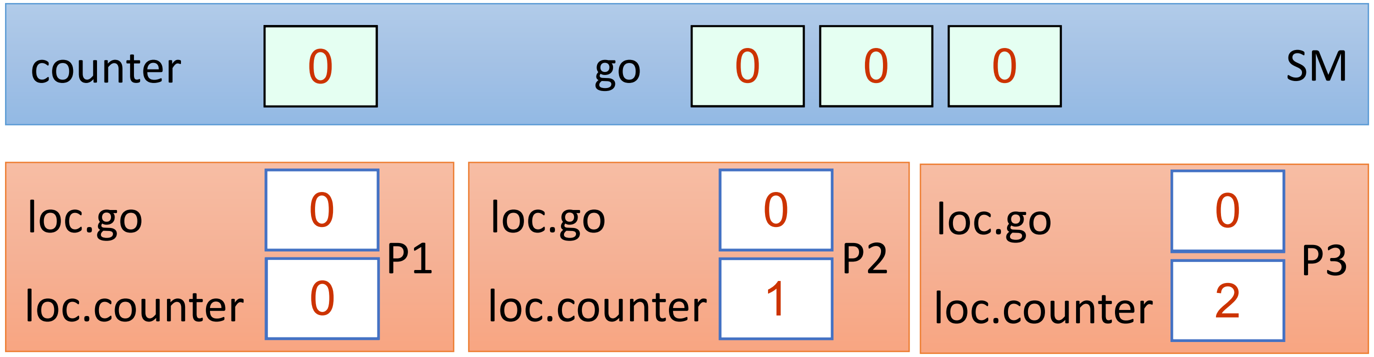
	P1	P2	P3
loc.go	0	0	0
loc.counter	0	1	2

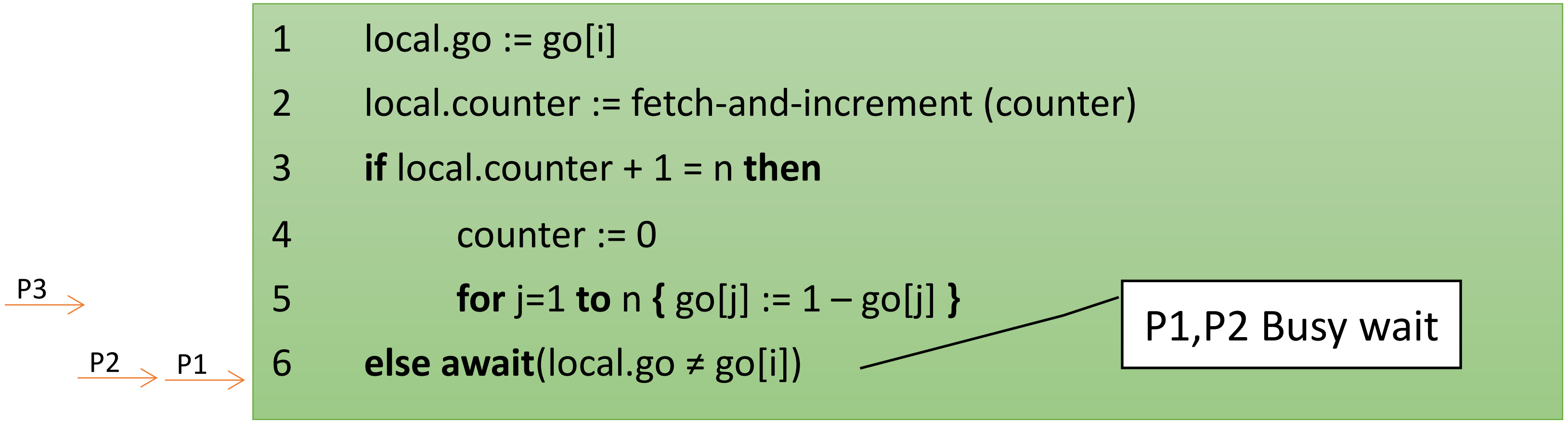
```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads

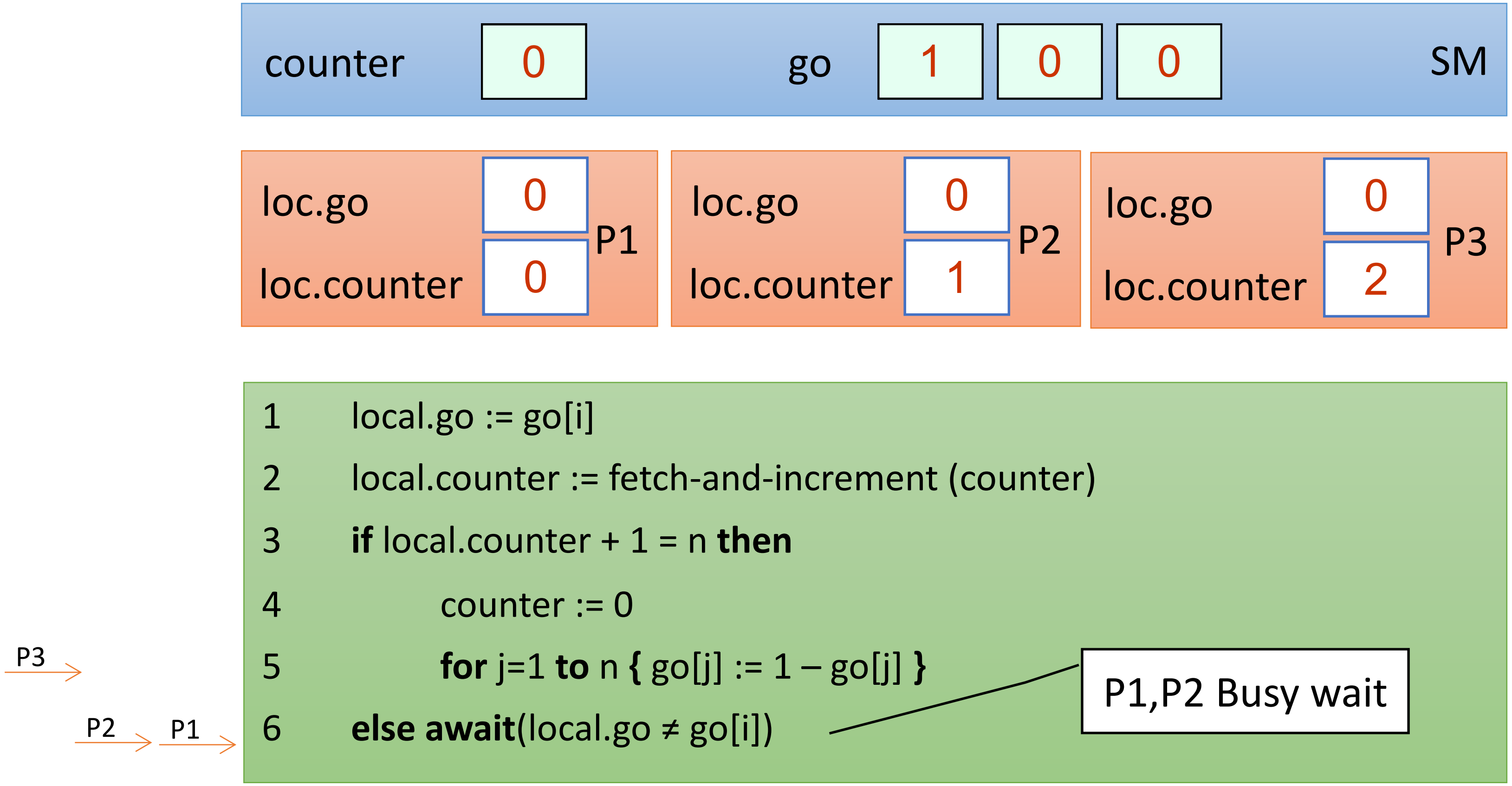




```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

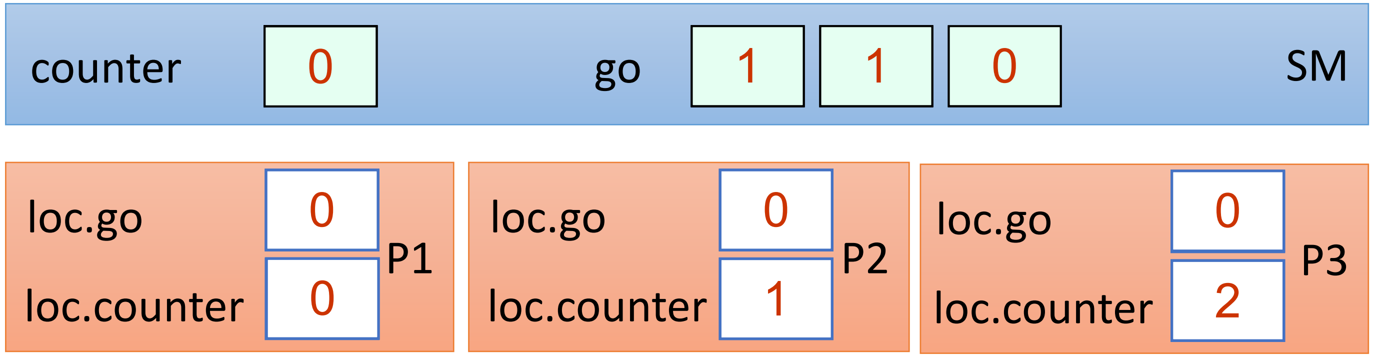
Example Run for n=3 Threads



```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   **if** local.counter + 1 = n **then**
4       counter := 0
5       **for** j=1 **to** n **{** go[j] := 1 – go[j] **}**
6   **else await**(local.go ≠ go[i])
```

A Local Spinning Counter Barrier

Example Run for n=3 Threads



SM: counter 0; go 1 1 0

P1: loc.go 0; loc.counter 0

P2: loc.go 0; loc.counter 1

P3: loc.go 0; loc.counter 2

```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4       counter := 0
5       for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

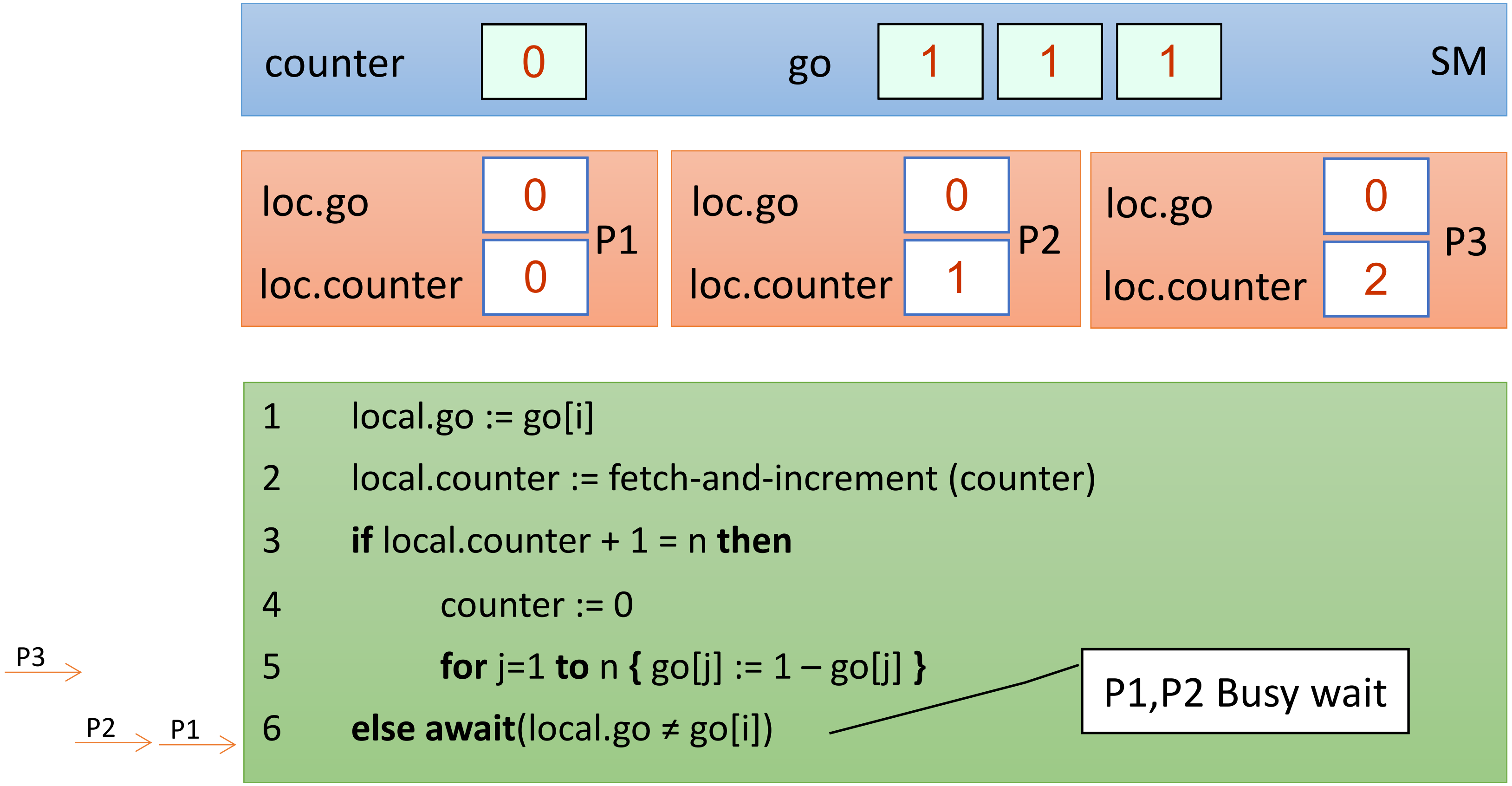
P3 → line 5

P2 → P1 → line 6

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



SM				
counter	0	go	1	1	1

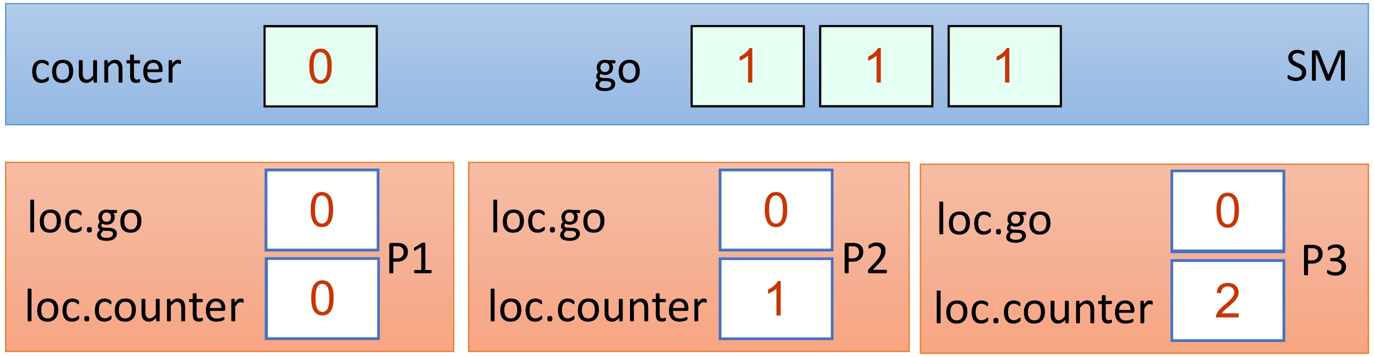
P1		P2		P3	
loc.go	0	loc.go	0	loc.go	0
loc.counter	0	loc.counter	1	loc.counter	2

```
1   local.go := go[i]
2   local.counter := fetch-and-increment (counter)
3   if local.counter + 1 = n then
4           counter := 0
5           for j=1 to n { go[j] := 1 – go[j] }
6   else await(local.go ≠ go[i])
```

P1,P2 Busy wait

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 0 go 1 1 1 SM

loc.go 0
loc.counter 0
P1

loc.go 0
loc.counter 1
P2

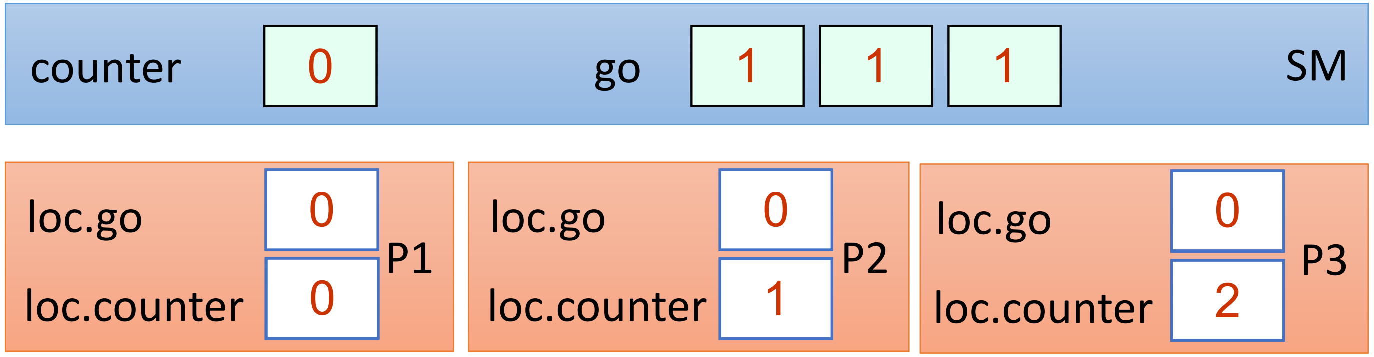
loc.go 0
loc.counter 2
P3

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
if local.counter + 1 = n then
        counter := 0
        for j=1 to n { go[j] := 1 – go[j] }
else await(local.go ≠ go[i])
```

P3 → P2 → P1

A Local Spinning Counter Barrier

Example Run for n=3 Threads



counter 0 go 1 1 1 SM

loc.go 0 loc.counter 0 P1

loc.go 0 loc.counter 1 P2

loc.go 0 loc.counter 2 P3

```
local.go := go[i]
local.counter := fetch-and-increment (counter)
**if** local.counter + 1 = n **then**
    counter := 0
    **for** j=1 **to** n **{** go[j] := 1 – go[j] **}**
**else await**(local.go ≠ go[i])
```

P3 → P2 → P1

Pros/Cons?

Does this actually reduce contention?

Comparison of counter-based Barriers

Simple Barrier

- Pros:

- Cons:

Simple Barrier with go array

- Pros:

- Cons:

Comparison of counter-based Barriers

Simple Barrier

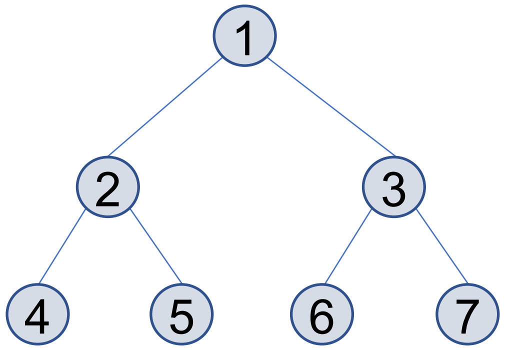
- Pros:
  - Very Simple
  - Shared memory: O(log n) ***bits***
  - Takes O(1) until last waiting p is awaken
- Cons:
  - High contention on the go bit
  - Contention on the counter register (*)

Simple Barrier with go array

- Pros:
  - Low contention on the go array
  - In some models:
    - spinning is done on local memory
    - remote mem. ref.: O(1)
- Cons:
  - Shared memory: O(n)
  - Still contention on the counter register (*)
  - Takes O(n) until last waiting p is awaken

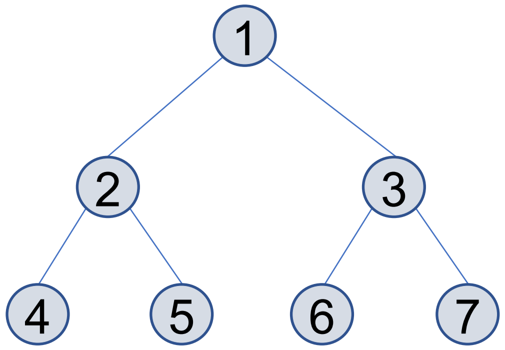
Tree Barriers

A Tree-based Barrier



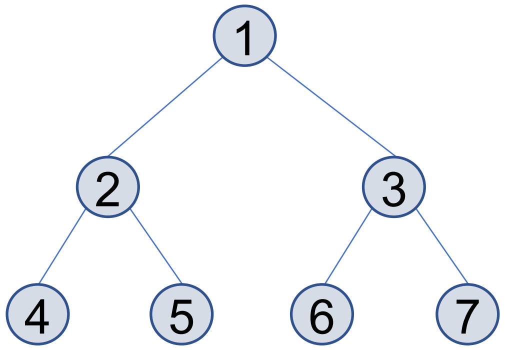
A Tree-based Barrier

- Threads are organized in a binary tree
- Each node is owned by a predetermined thread



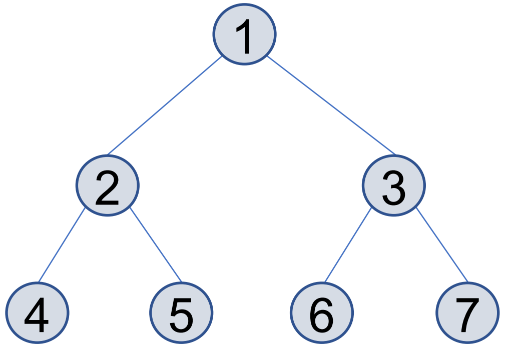
A Tree-based Barrier

- Threads are organized in a binary tree
- Each node is owned by a predetermined thread
- Each thread waits until its 2 children arrive
    - combines results
    - passes them on to its parent

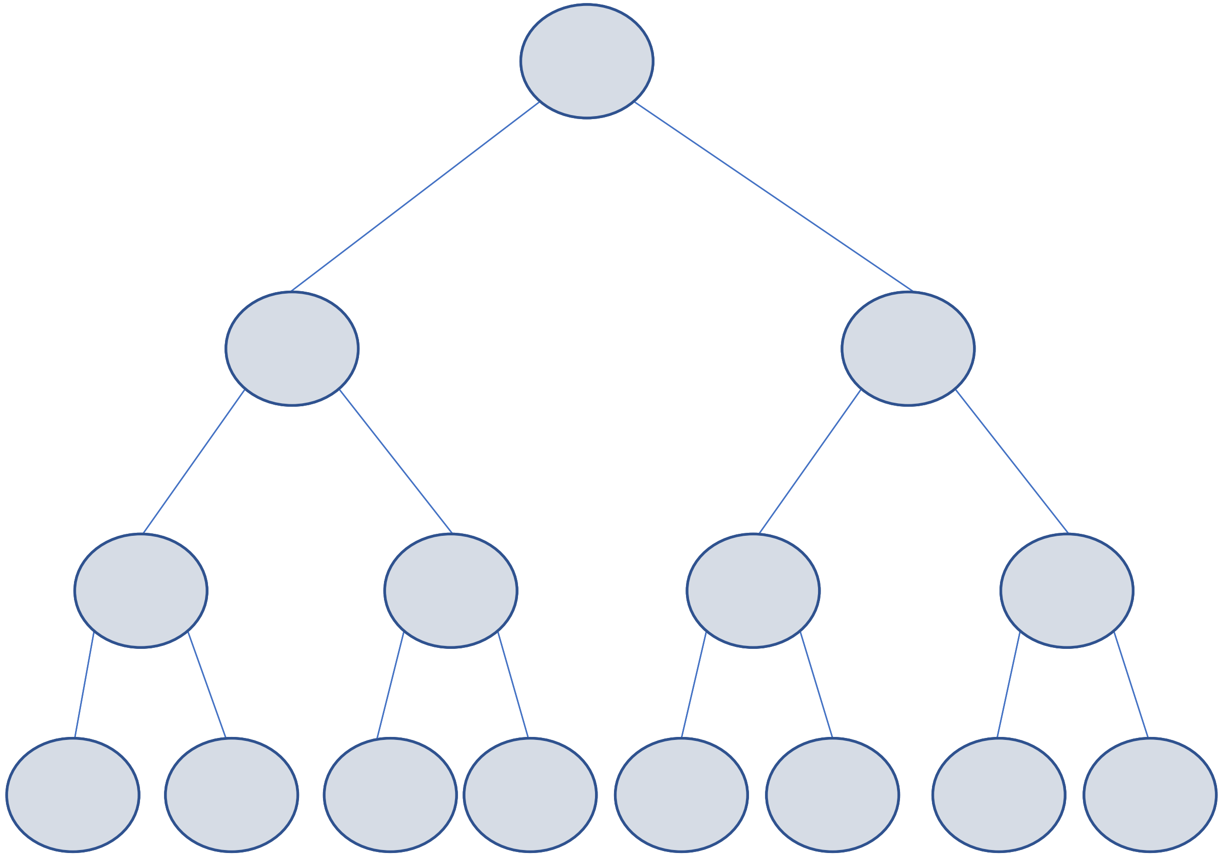


A Tree-based Barrier

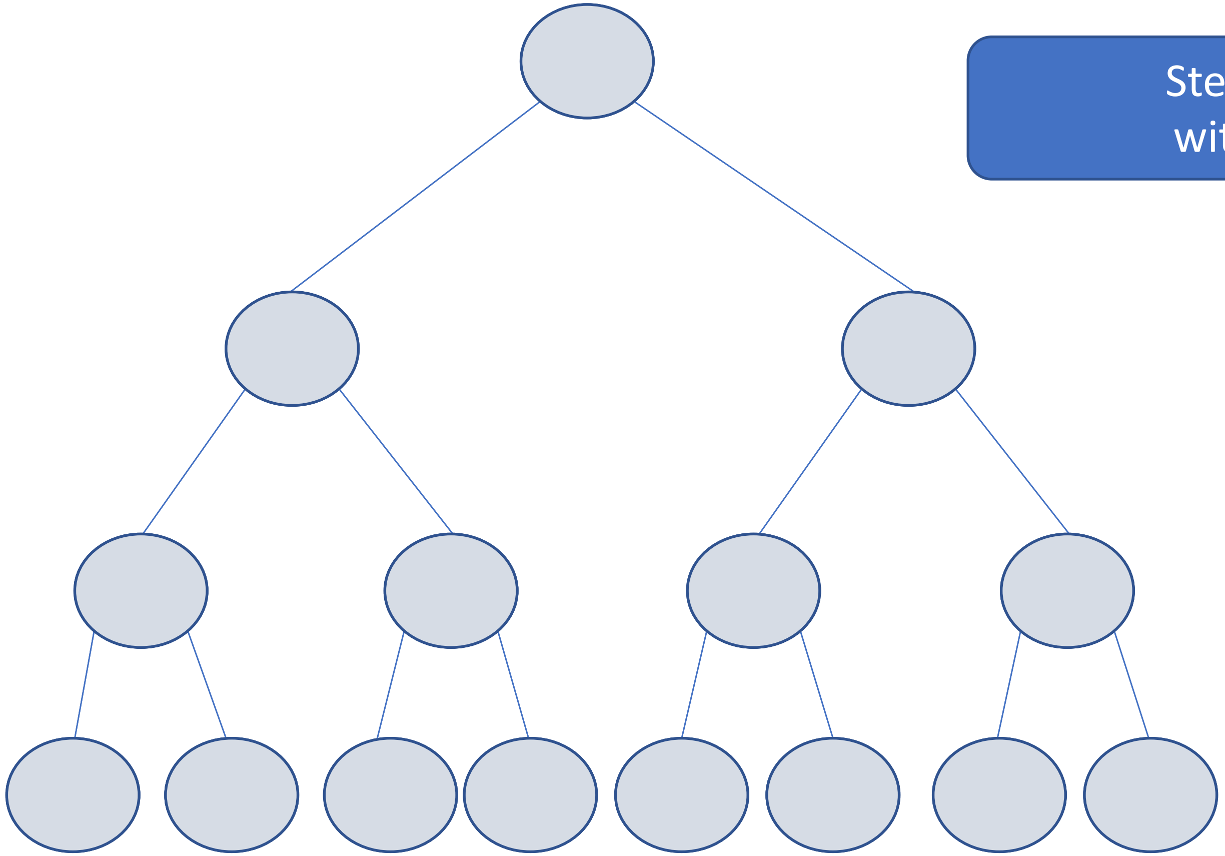
- Threads are organized in a binary tree
- Each node is owned by a predetermined thread
- Each thread waits until its 2 children arrive
  - combines results
  - passes them on to its parent
- Root learns that its 2 children have arrived→tells children they can go
- The signal propagates down the tree until all the threads get the message



A Tree-based Barrier: indexing

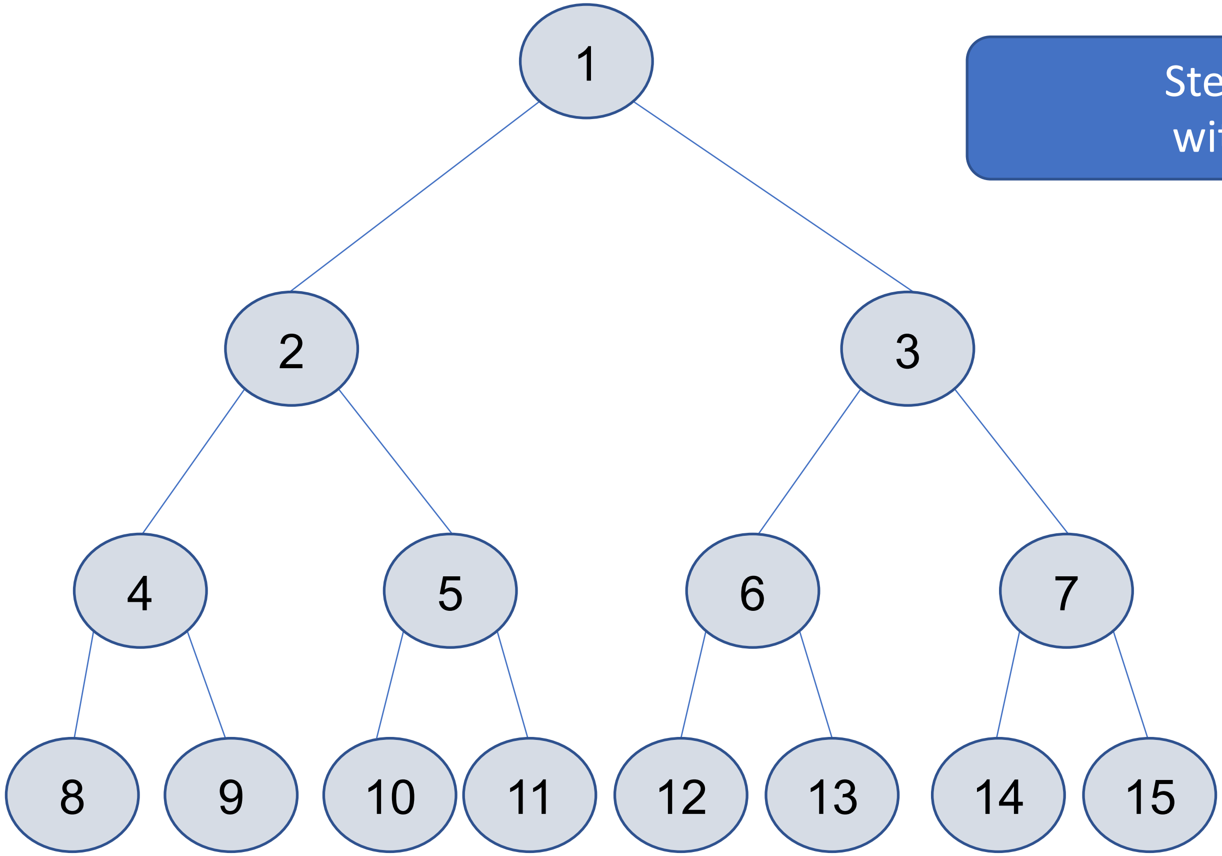


A Tree-based Barrier: indexing



Step 1: label numerically with depth-first traveral

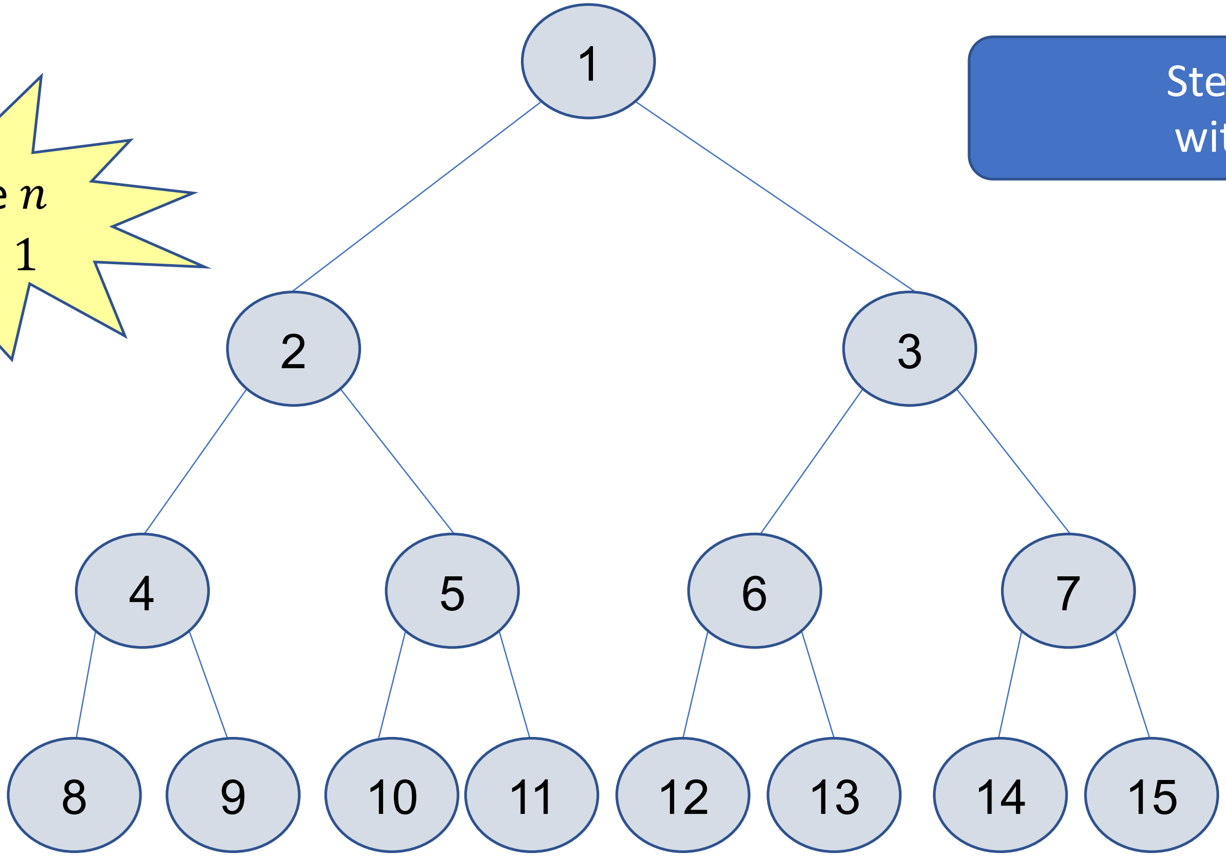
A Tree-based Barrier: indexing



Step 1: label numerically
with depth-first traveral

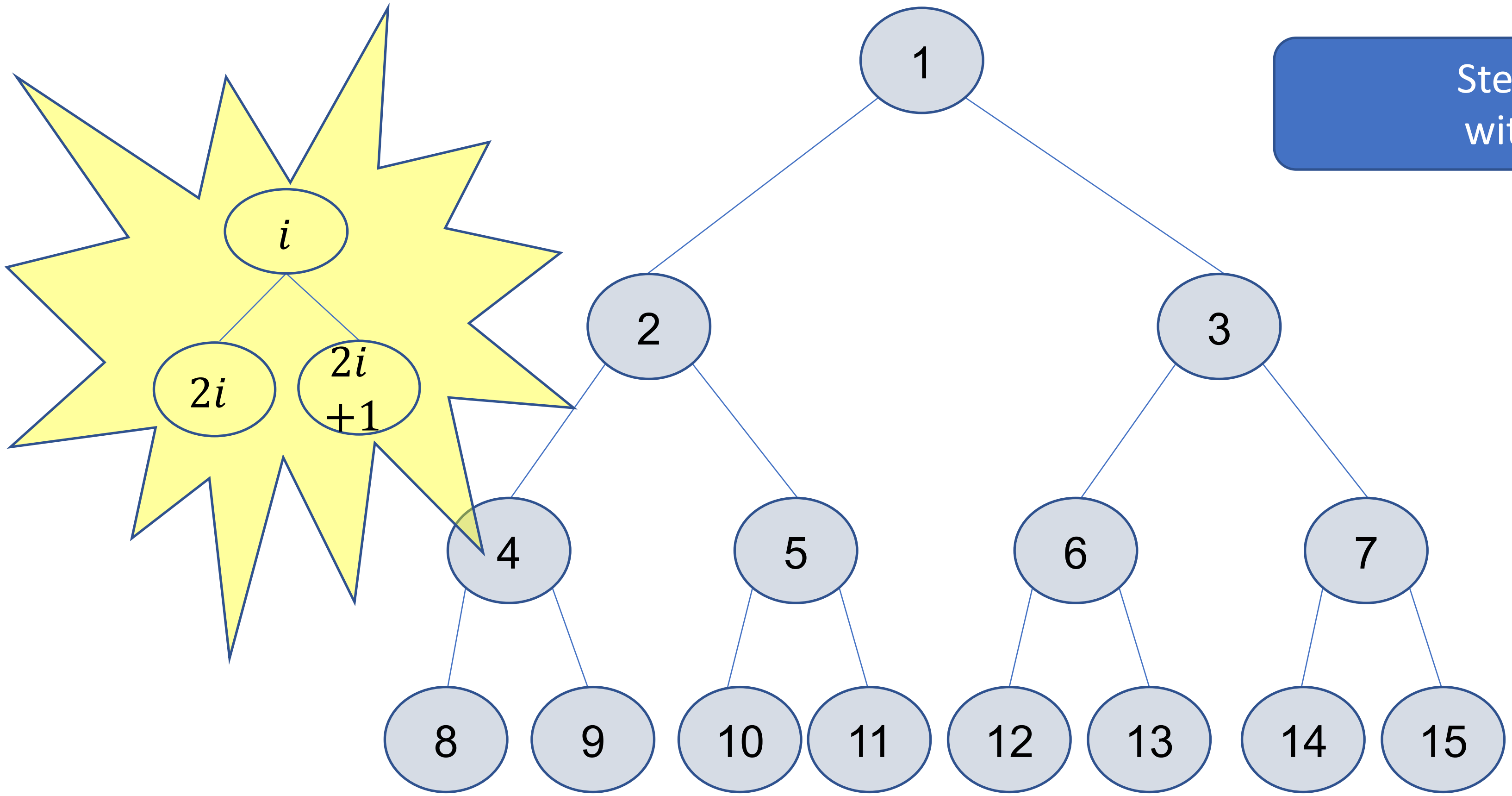
A Tree-based Barrier: indexing

Assume $n = 2^k - 1$



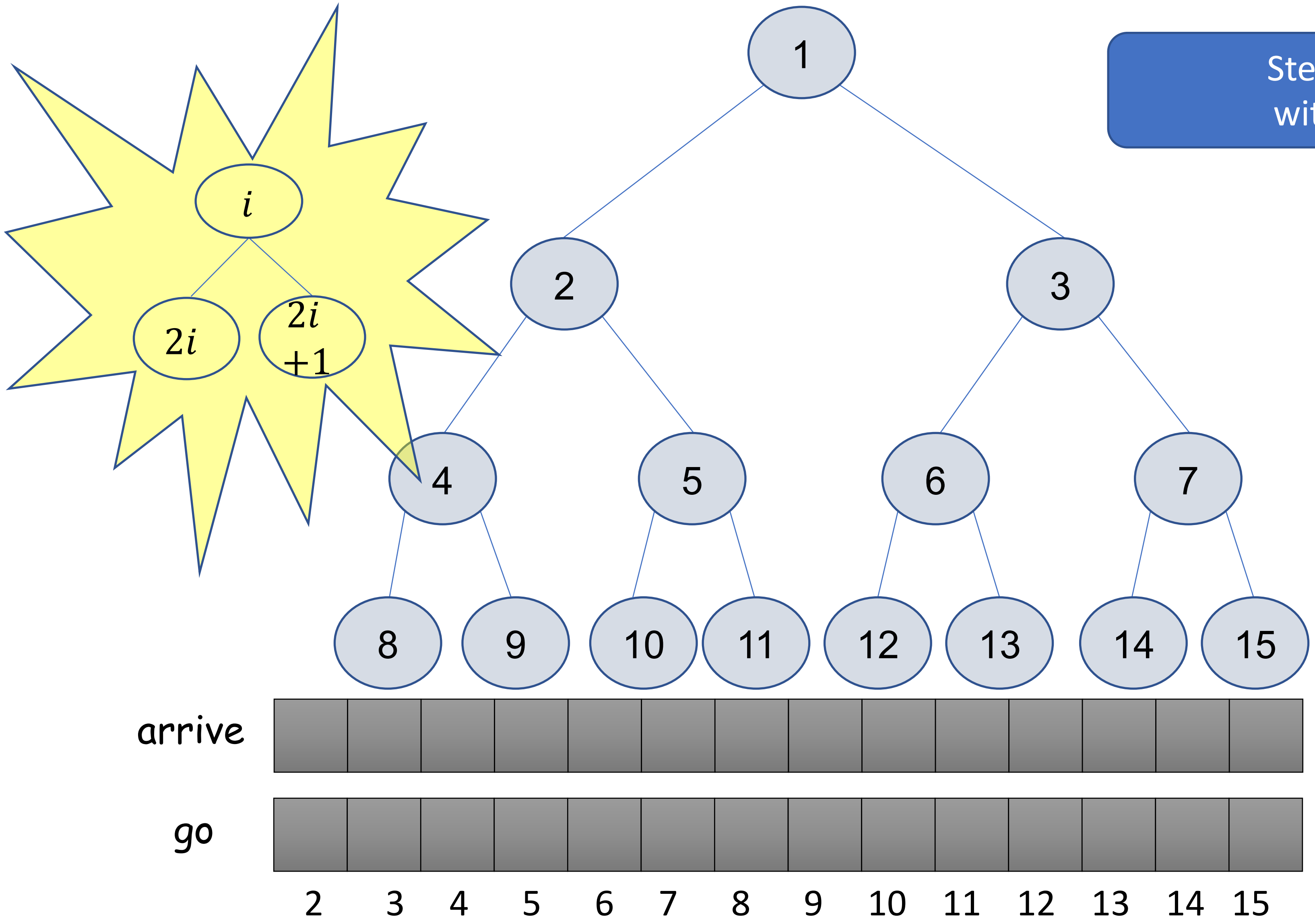
Step 1: label numerically with depth-first traveral

A Tree-based Barrier: indexing



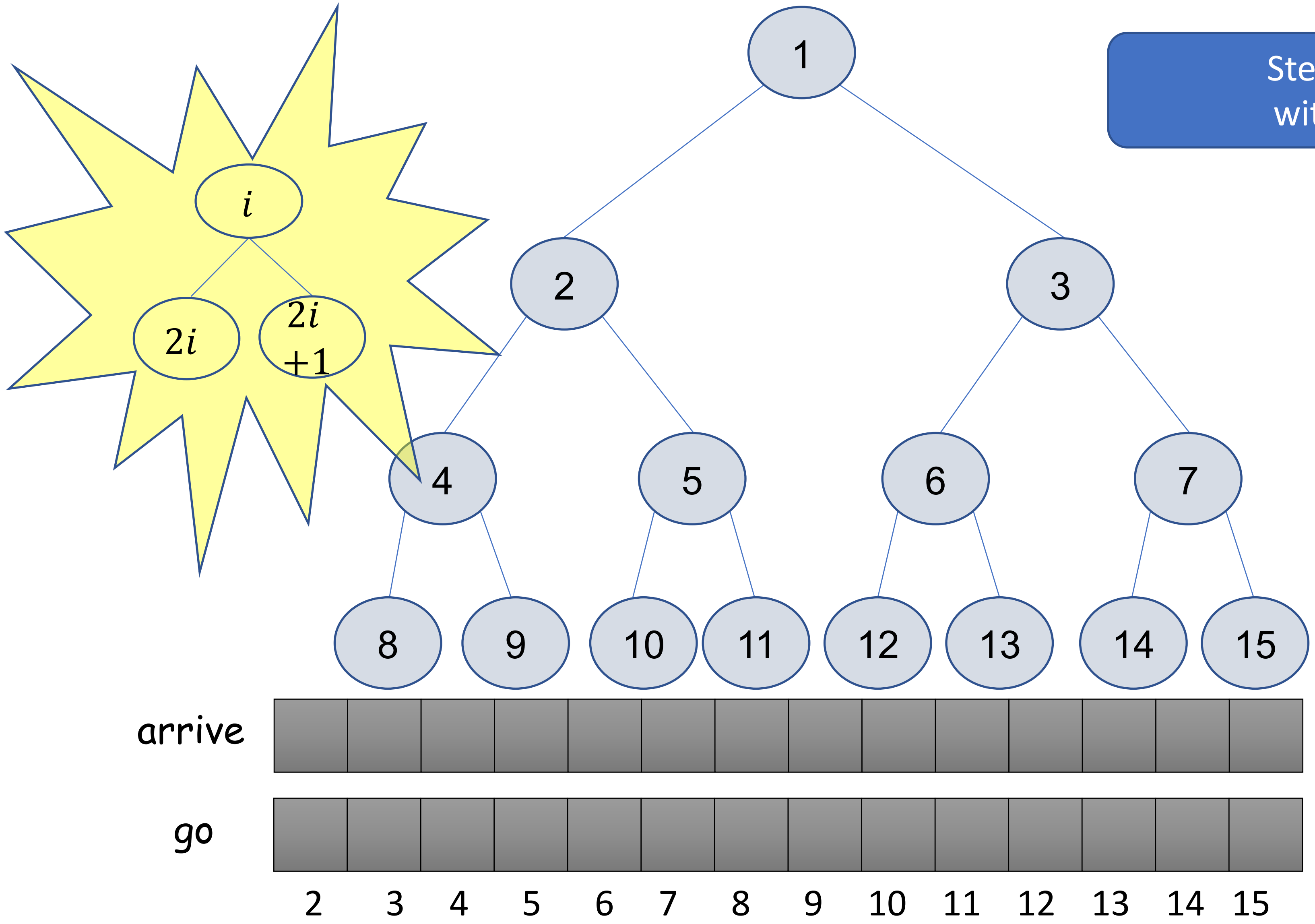
Step 1: label numerically with depth-first traveral

A Tree-based Barrier: indexing



Step 1: label numerically with depth-first traveral

A Tree-based Barrier: indexing



Step 1: label numerically
with depth-first traveral

Indexing starts from 2
Root → 1, doesn't need wait objects

A Tree-based Barrier

program of thread i

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                              // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                 // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                     // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier

program of thread i

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

Root

```
if i=1 then                                            // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
```

Internal

```
else if i ≤ (n-1)/2 then                               // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
```

Leaf

```
else                                                   // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier

program of thread i

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                              // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                 // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                     // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

Root

Internal

Leaf

Root:
- Wait for arriving children
- Tell children to go

A Tree-based Barrier

program of thread i

```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

Root

Internal

Leaf

```
if i=1 then                                               // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                  // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                      // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

Root:
- Wait for arriving children
- Tell children to go

Internal:
- Wait for arriving children
- Wait for parent go signal
- Tell children to go

A Tree-based Barrier

program of thread i

```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

Root

```
if i=1 then                                              // root
    await(arrive[2] = 1); arrive[2] := 0
    await(arrive[3] = 1); arrive[3] := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                 // internal node
    await(arrive[2i] = 1); arrive[2i] := 0
    await(arrive[2i+1] = 1); arrive[2i+1] := 0
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                                     // leaf
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

Internal

Leaf

Root:
- Wait for arriving children
- Tell children to go

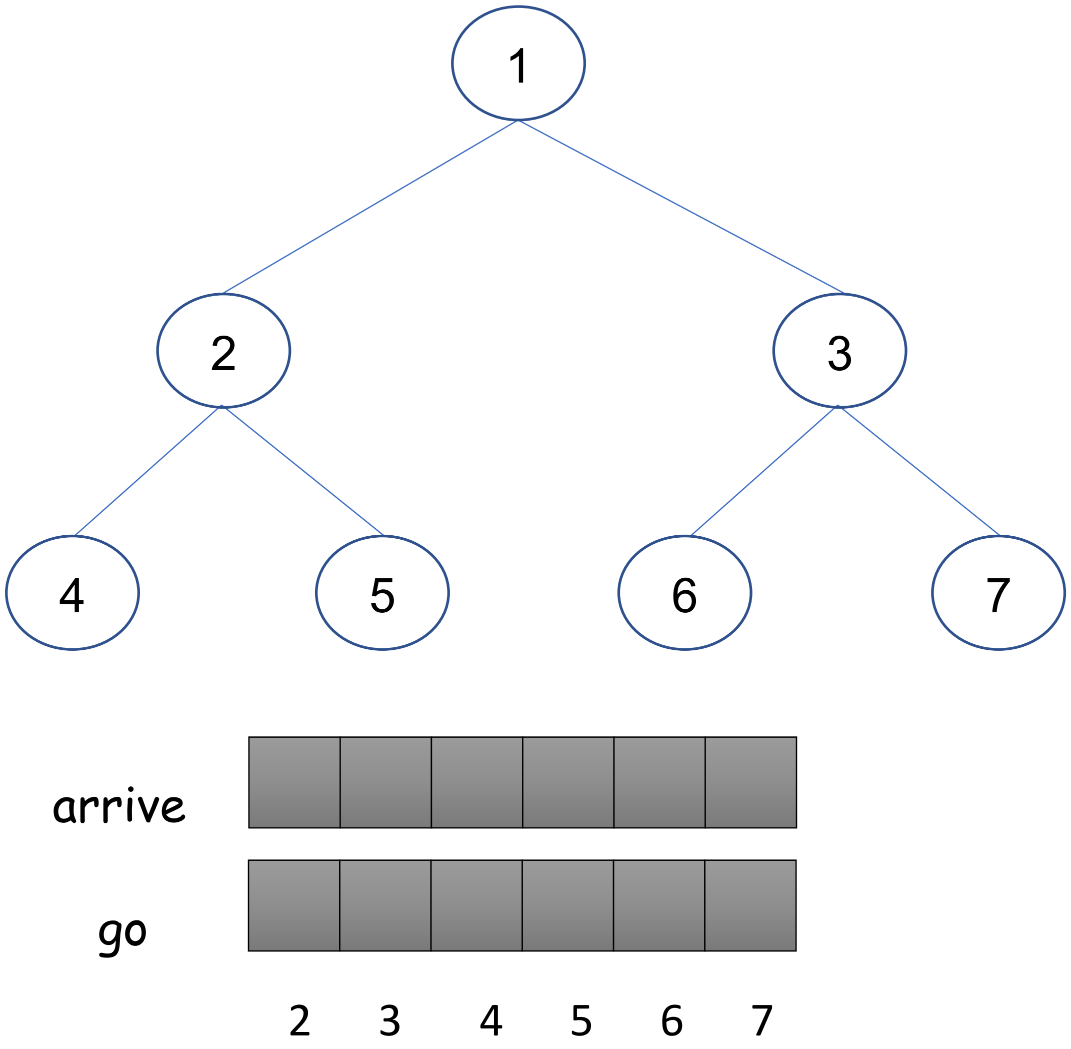
Internal:
- Wait for arriving children
- Wait for parent go signal
- Tell children to go

Child:
- arrive
- Wait for parent go signal

A Tree-based Barrier
Example Run for n=7 threads

```
shared   arrive[2..n]:  array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0

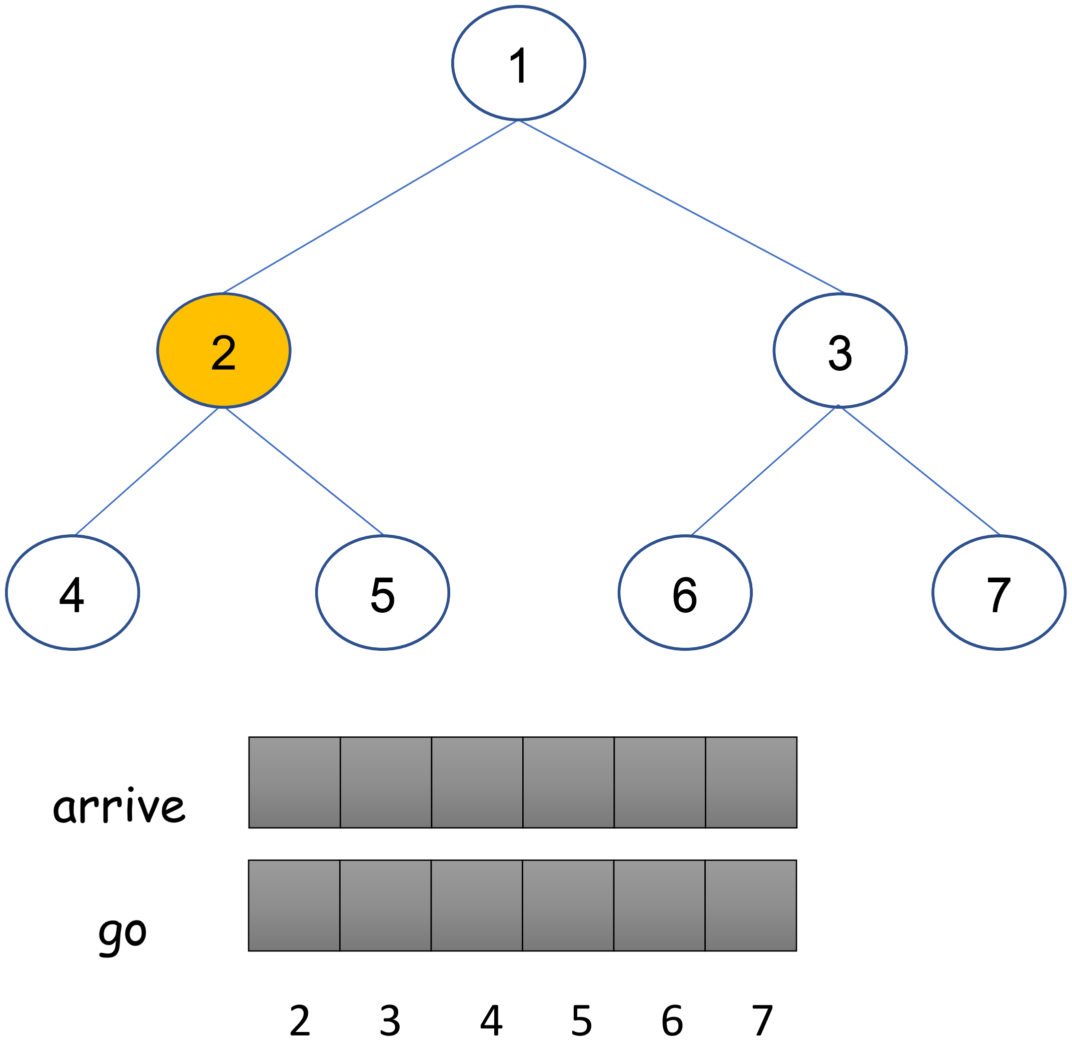
if i=1 then                                          // root
         await(arrive[2]  = 1); arrive[2]  := 0
         await(arrive[3]  = 1); arrive[3]  := 0
         go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
         await(arrive[2i]  = 1); arrive[2i]  := 0
         await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
         arrive[i]  := 1
         await(go[i]  = 1); go[i] := 0
         go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
         arrive[i]  := 1
         await(go[i]  = 1); go[i] := 0 fi
fi
```



A Tree-based Barrier
Example Run for n=7 threads

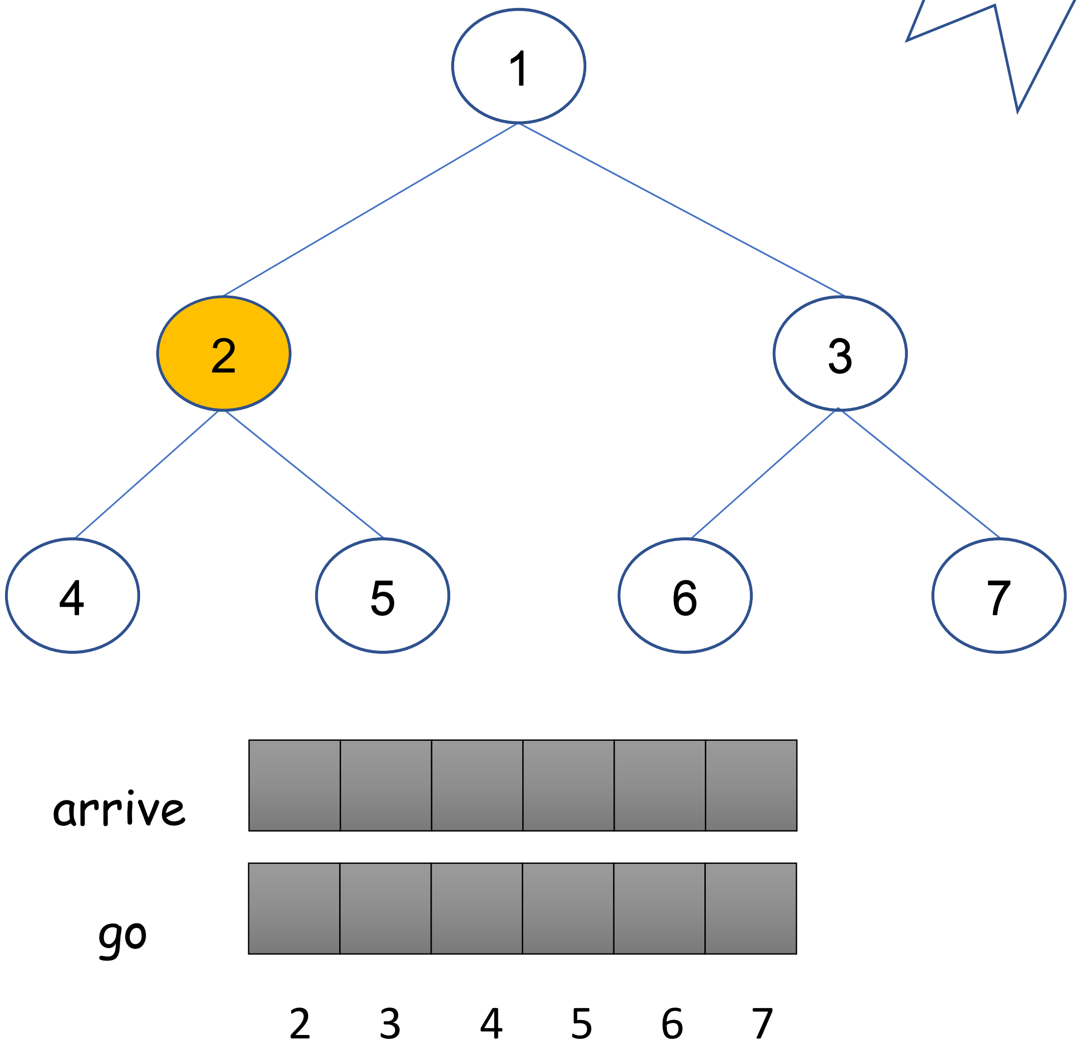
```
shared      arrive[2..n]:  array of atomic bits, initial values = 0
            go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                                // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                   // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                       // leaf
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0 fi
fi
```



A Tree-based Barrier
Example Run for n=7 threads

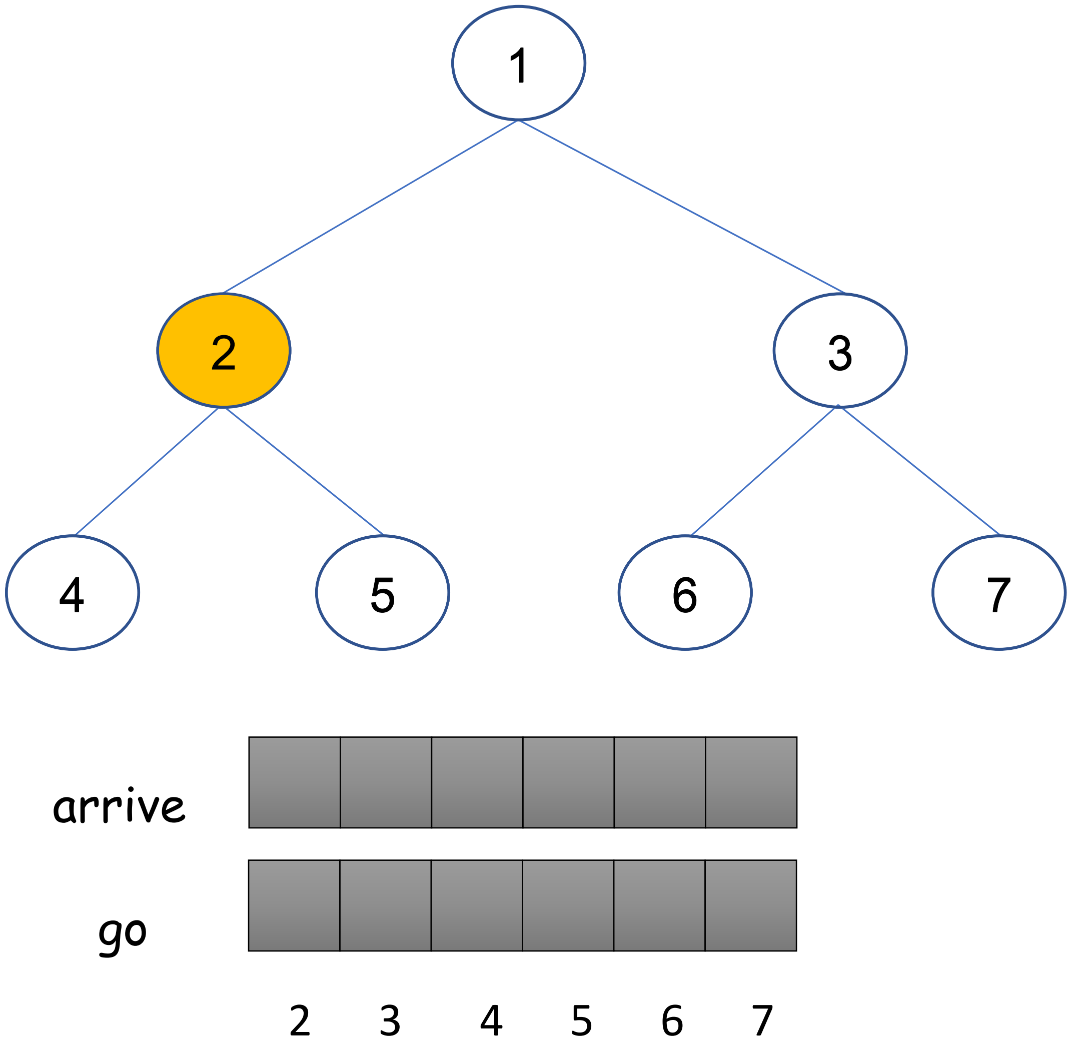
arrive[2]=1
?



```
shared   arrive[2..n]:  array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

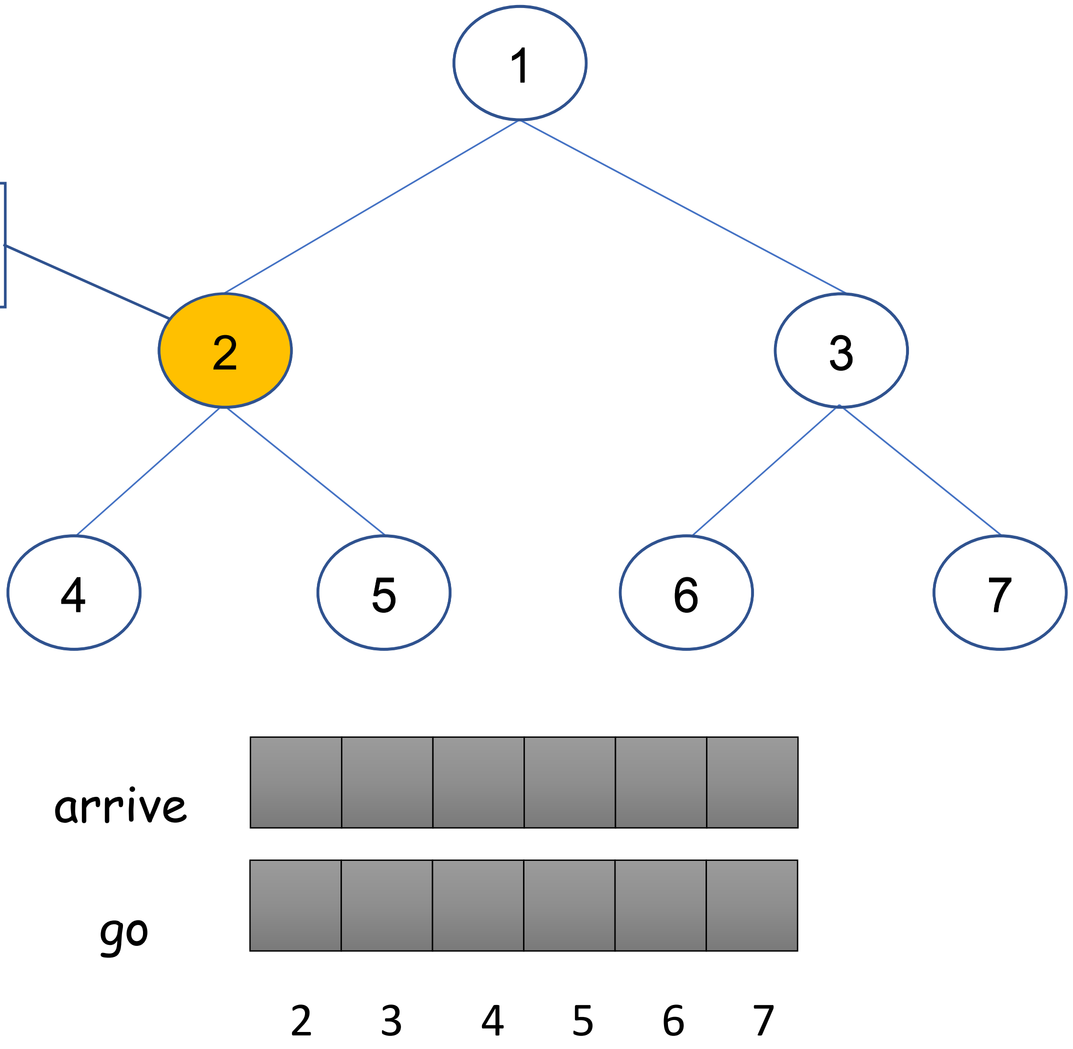


```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                         // root
    await(arrive[2]  = 1); arrive[2]  := 0
    await(arrive[3]  = 1); arrive[3]  := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                            // internal node
    await(arrive[2i]  = 1); arrive[2i]  := 0
    await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
    arrive[i]  := 1
    await(go[i]  = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                                // leaf
    arrive[i]  := 1
    await(go[i]  = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

Waiting for p_4 to arrive



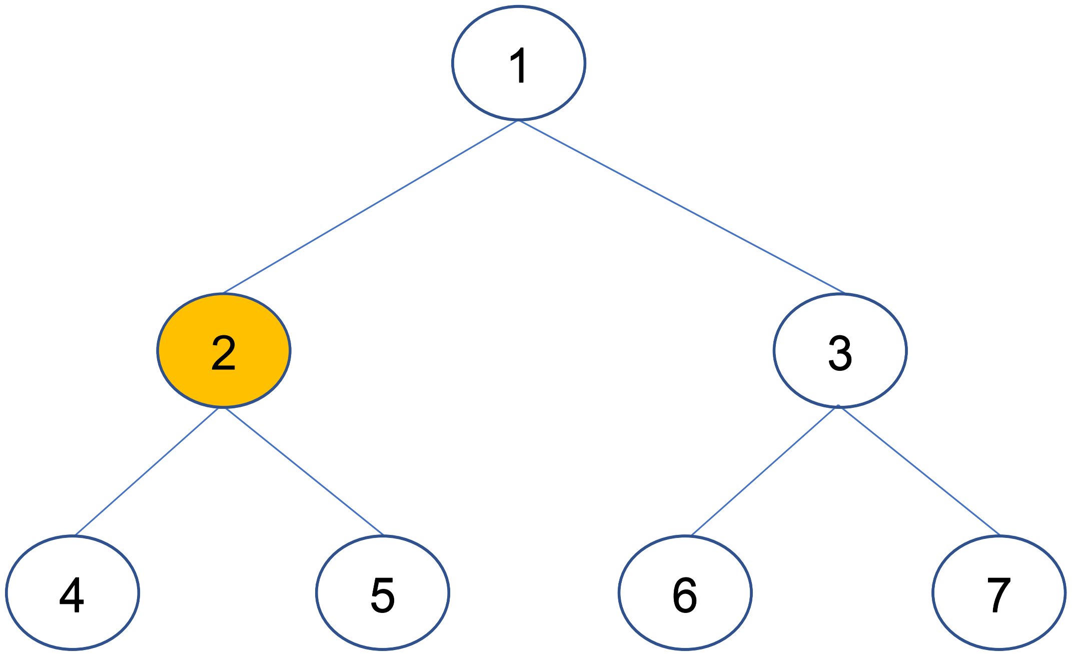
```
shared   arrive[2..n]:  array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                          // root
         await(arrive[2]  = 1); arrive[2] := 0
         await(arrive[3]  = 1); arrive[3] := 0
         go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
         await(arrive[2i]  = 1); arrive[2i]  := 0
         await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
         arrive[i]  := 1
         await(go[i]  = 1); go[i] := 0
         go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
         arrive[i]  := 1
         await(go[i]  = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

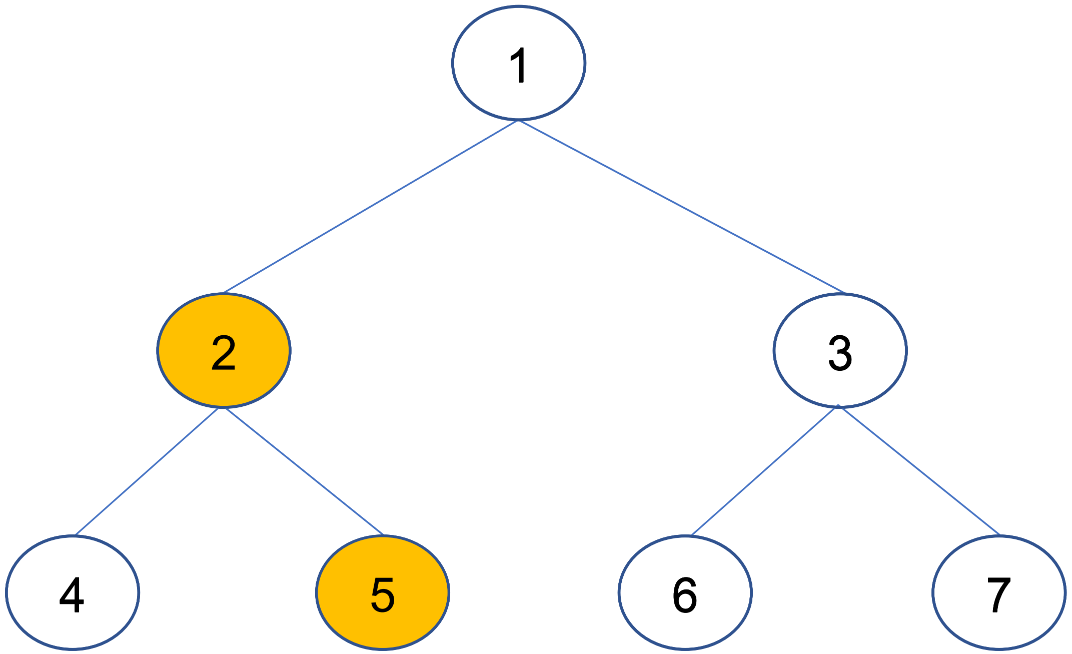


	2	3	4	5	6	7
arrive						
go						

A Tree-based Barrier
Example Run for n=7 threads

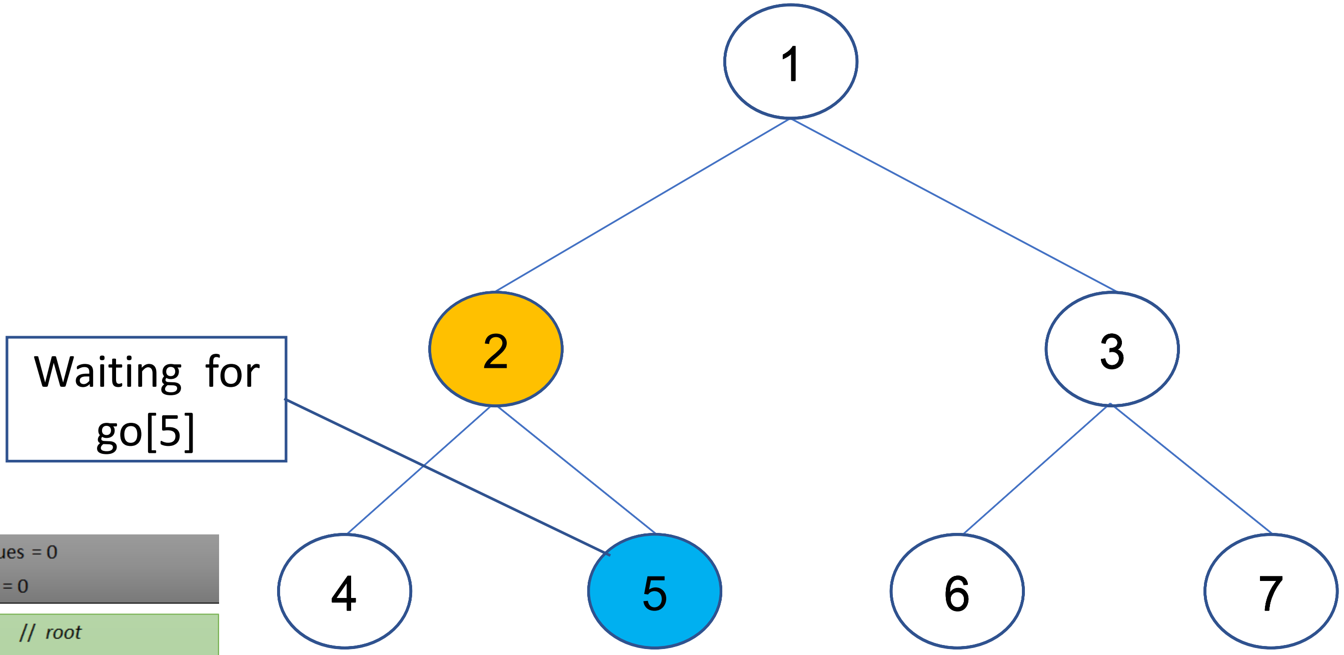
```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                         // root
      await(arrive[2] = 1); arrive[2] := 0
      await(arrive[3] = 1); arrive[3] := 0
      go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                            // internal node
      await(arrive[2i] = 1); arrive[2i] := 0
      await(arrive[2i+1] = 1); arrive[2i+1] := 0
      arrive[i] := 1
      await(go[i] = 1); go[i] := 0
      go[2i] = 1; go[2i+1] := 1
else                                                // leaf
      arrive[i] := 1
      await(go[i] = 1); go[i] := 0 fi
fi
```



arrive				1		
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



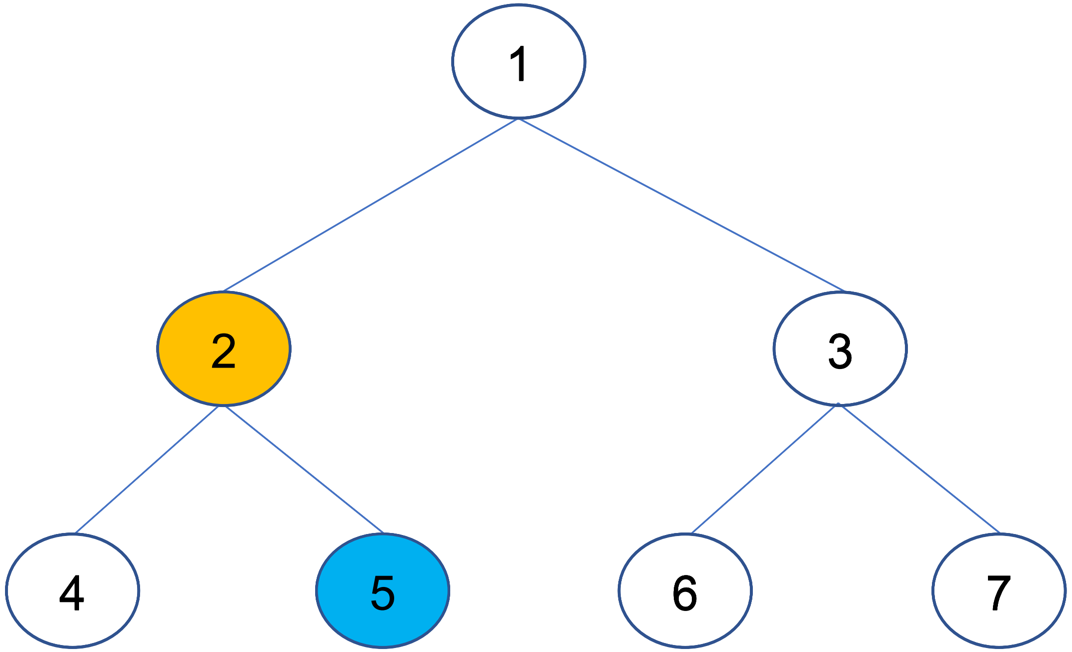
Waiting for go[5]

```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                   // root
    await(arrive[2] = 1); arrive[2] := 0
    await(arrive[3] = 1); arrive[3] := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                      // internal node
    await(arrive[2i] = 1); arrive[2i] := 0
    await(arrive[2i+1] = 1); arrive[2i+1] := 0
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                          // leaf
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

	2	3	4	5	6	7
arrive				1		
go						

A Tree-based Barrier
Example Run for n=7 threads

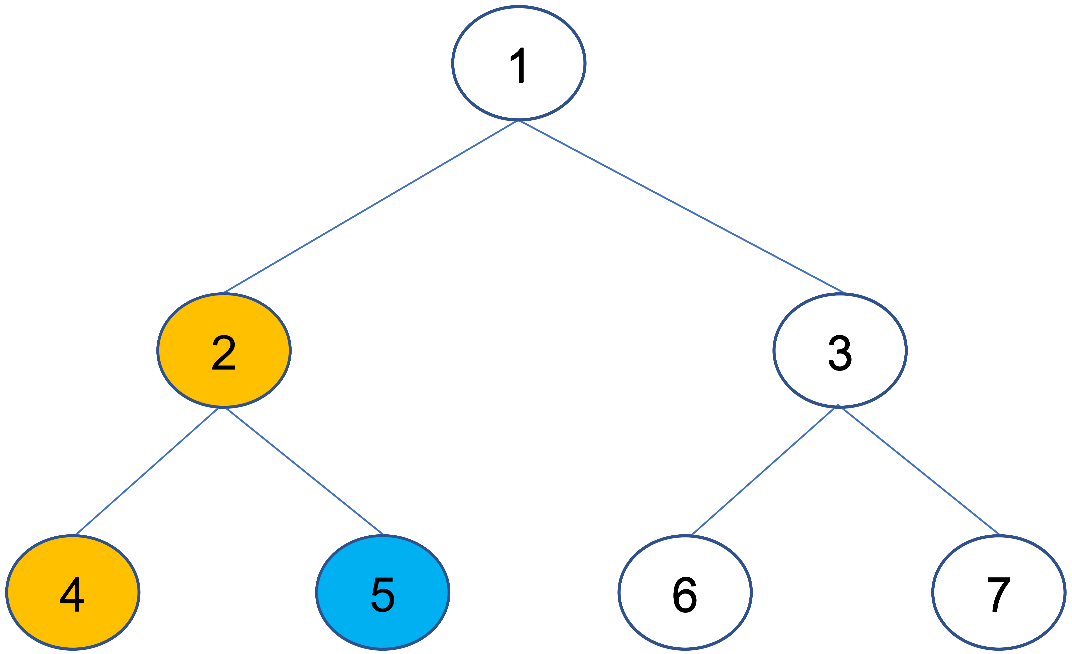


```
shared   arrive[2..n]:  array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                              // root
    await(arrive[2]  = 1); arrive[2]  := 0
    await(arrive[3]  = 1); arrive[3]  := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                 // internal node
    await(arrive[2i]  = 1); arrive[2i]  := 0
    await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
    arrive[i]  := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                                     // leaf
    arrive[i]  := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

arrive				1		
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

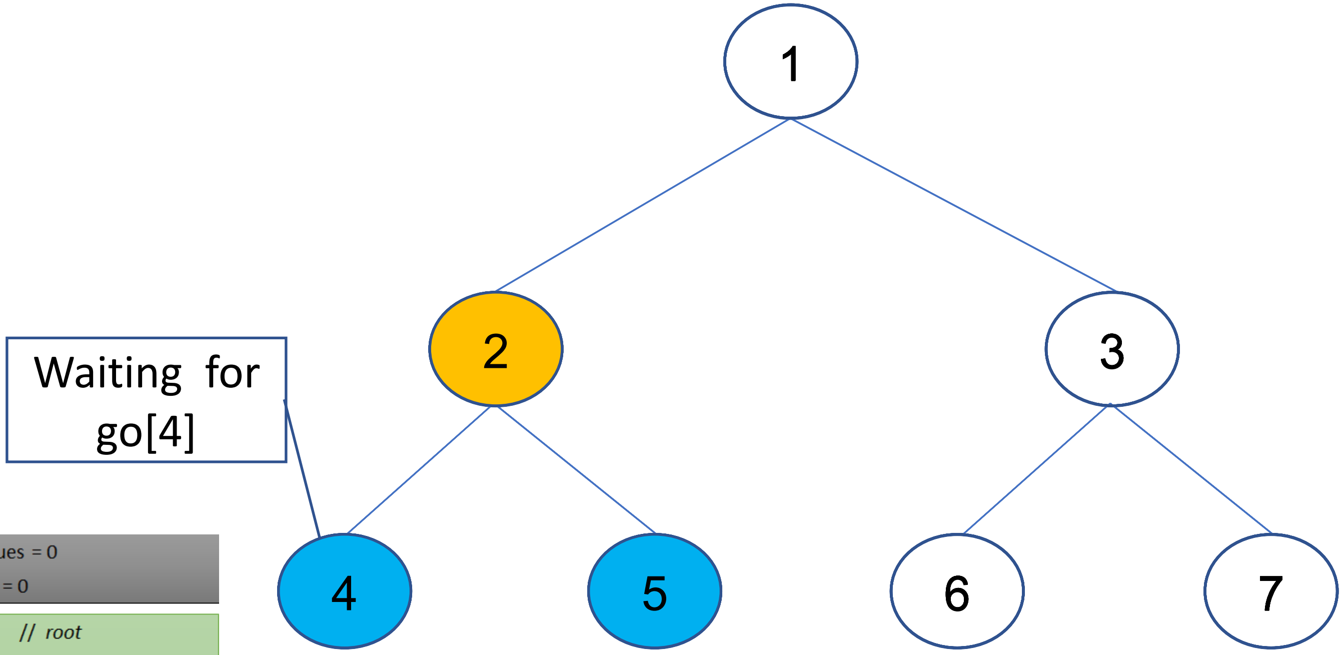


	2	3	4	5	6	7
arrive			1	1		
go						

```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                              // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                 // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                     // leaf
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads



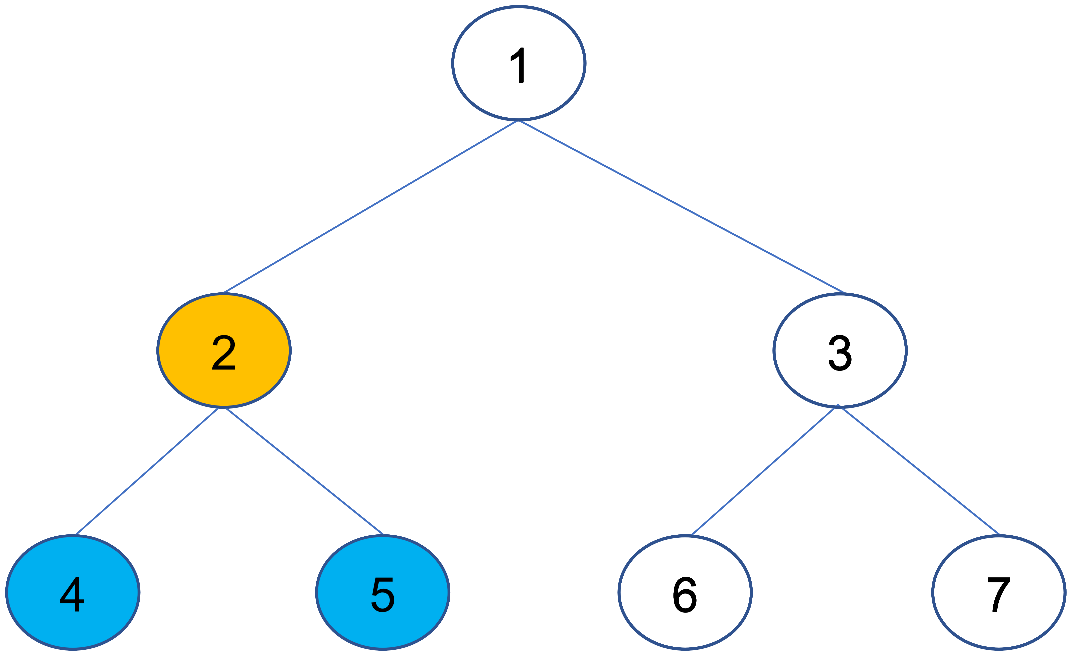
Waiting for go[4]

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                   // root
    await(arrive[2] = 1); arrive[2] := 0
    await(arrive[3] = 1); arrive[3] := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                      // internal node
    await(arrive[2i] = 1); arrive[2i] := 0
    await(arrive[2i+1] = 1); arrive[2i+1] := 0
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                          // leaf
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

arrive			1	1		
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



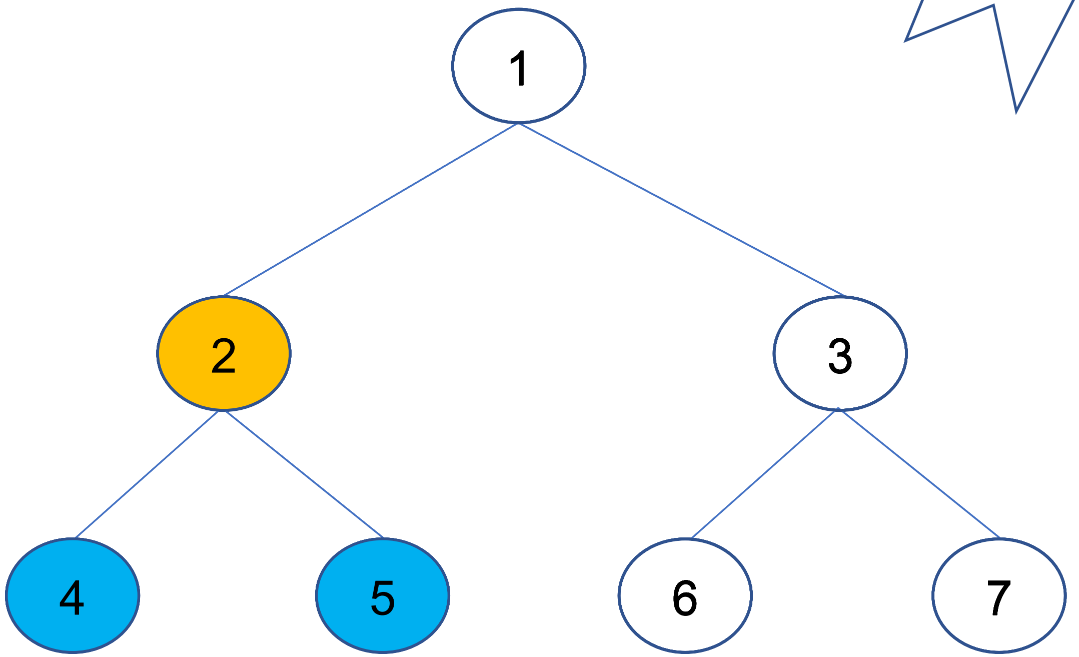
```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                          // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

	2	3	4	5	6	7
arrive			1	1		
go						

A Tree-based Barrier
Example Run for n=7 threads

P_2 zeros arrive[4,5]

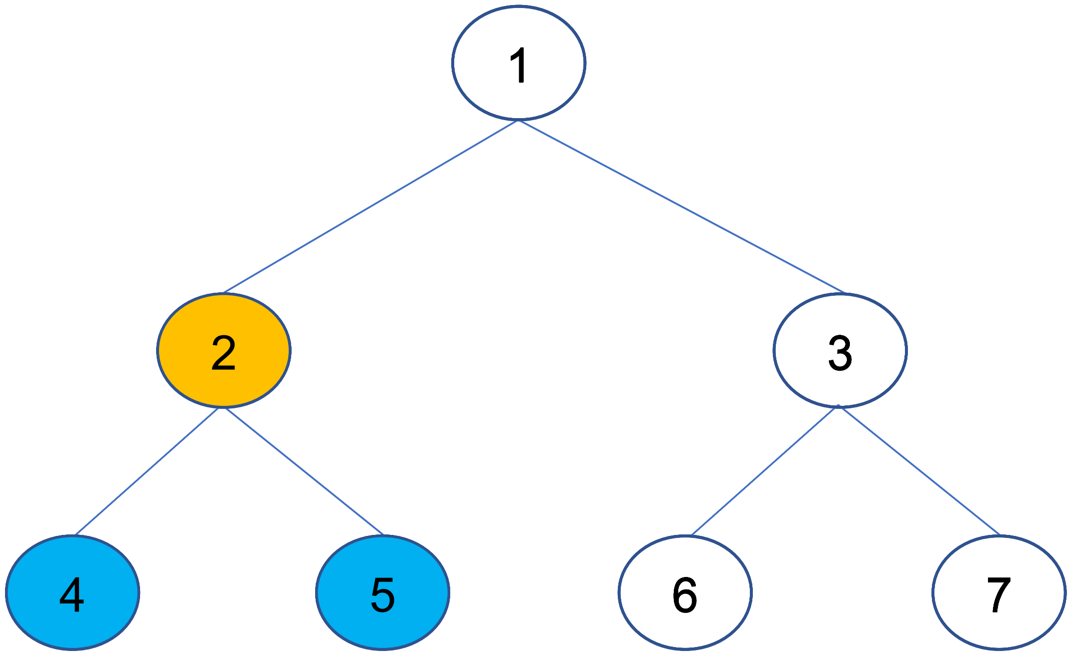


	2	3	4	5	6	7
arrive			0	0		
go						

```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                          // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

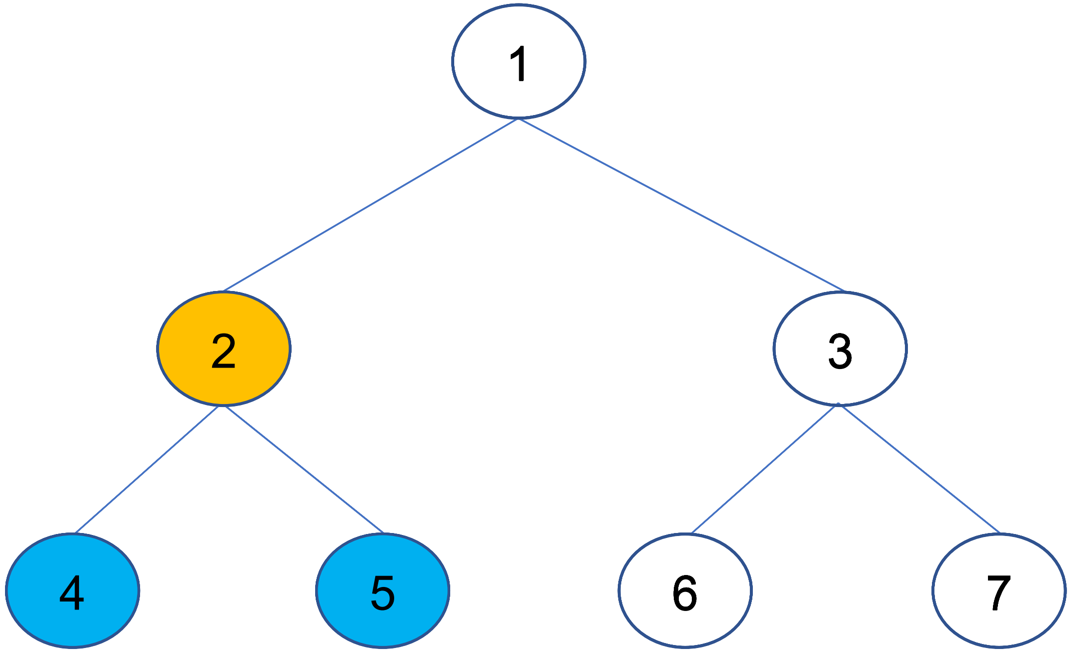


	2	3	4	5	6	7
arrive			0	0		
go						

```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                          // root
          await(arrive[2]  = 1); arrive[2]  := 0
          await(arrive[3]  = 1); arrive[3]  := 0
          go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
          await(arrive[2i]  = 1); arrive[2i]  := 0
          await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
          arrive[i]  := 1
          await(go[i] = 1); go[i] := 0
          go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
          arrive[i]  := 1
          await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

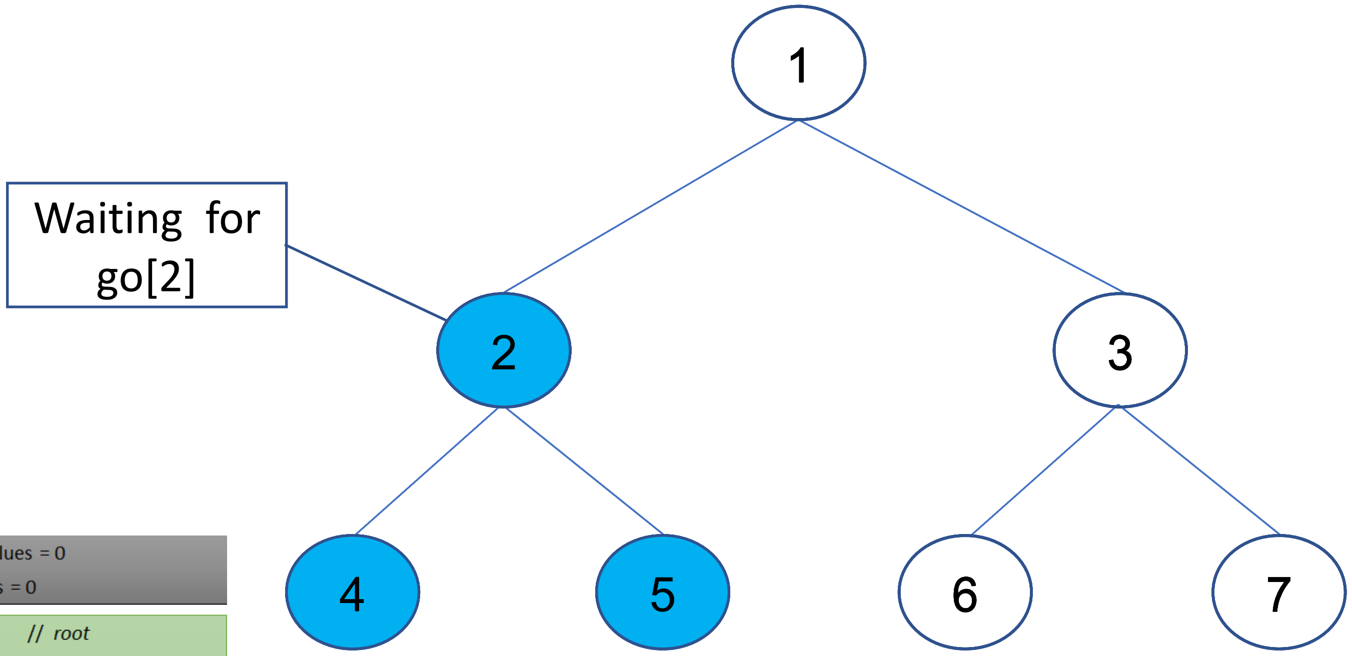


```
shared  arrive[2..n]: array of atomic bits, initial values = 0
        go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	1		0	0		
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

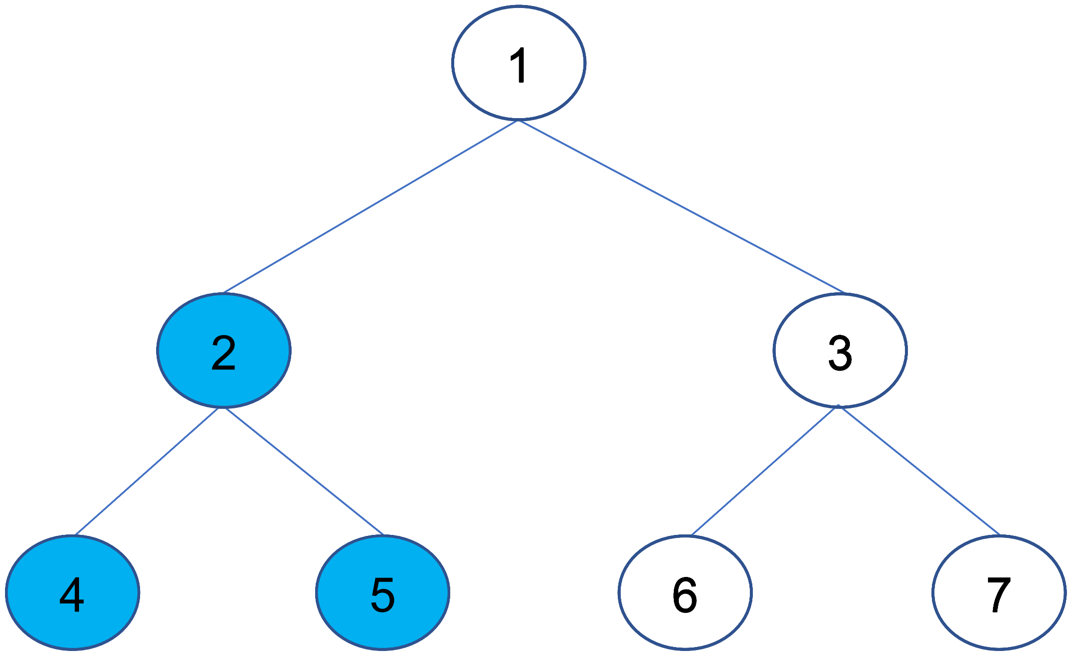


```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                         // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                            // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

	2	3	4	5	6	7
arrive	1		0	0		
go						

A Tree-based Barrier
Example Run for n=7 threads

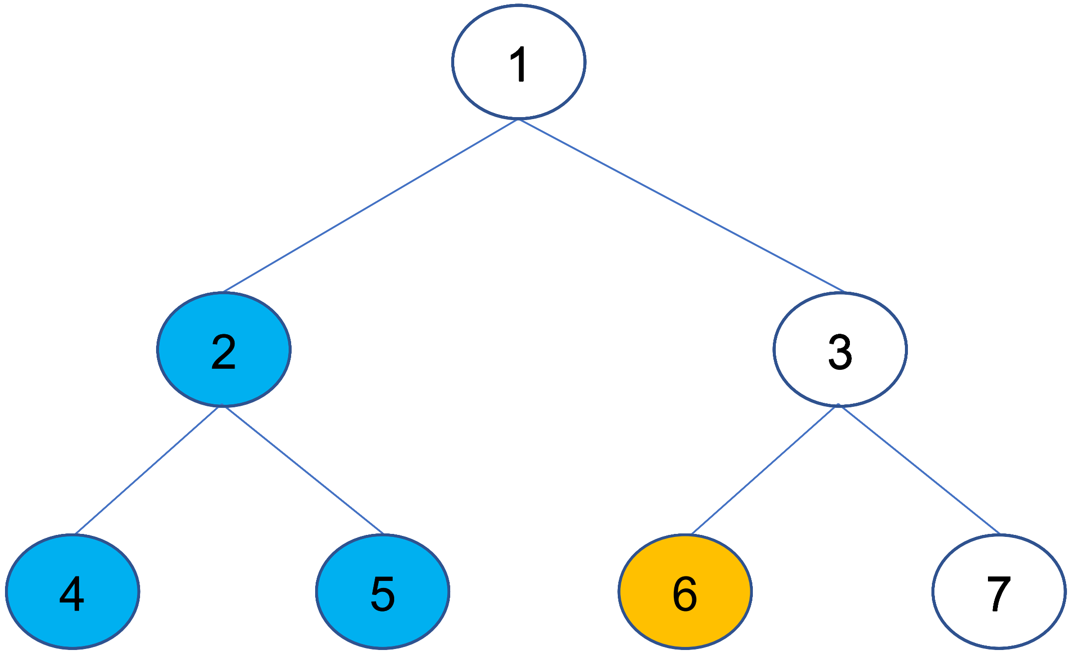


	2	3	4	5	6	7
arrive	1		0	0		
go						

```
shared      arrive[2..n]:  array of atomic bits, initial values = 0
            go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                              // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                                 // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                     // leaf
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

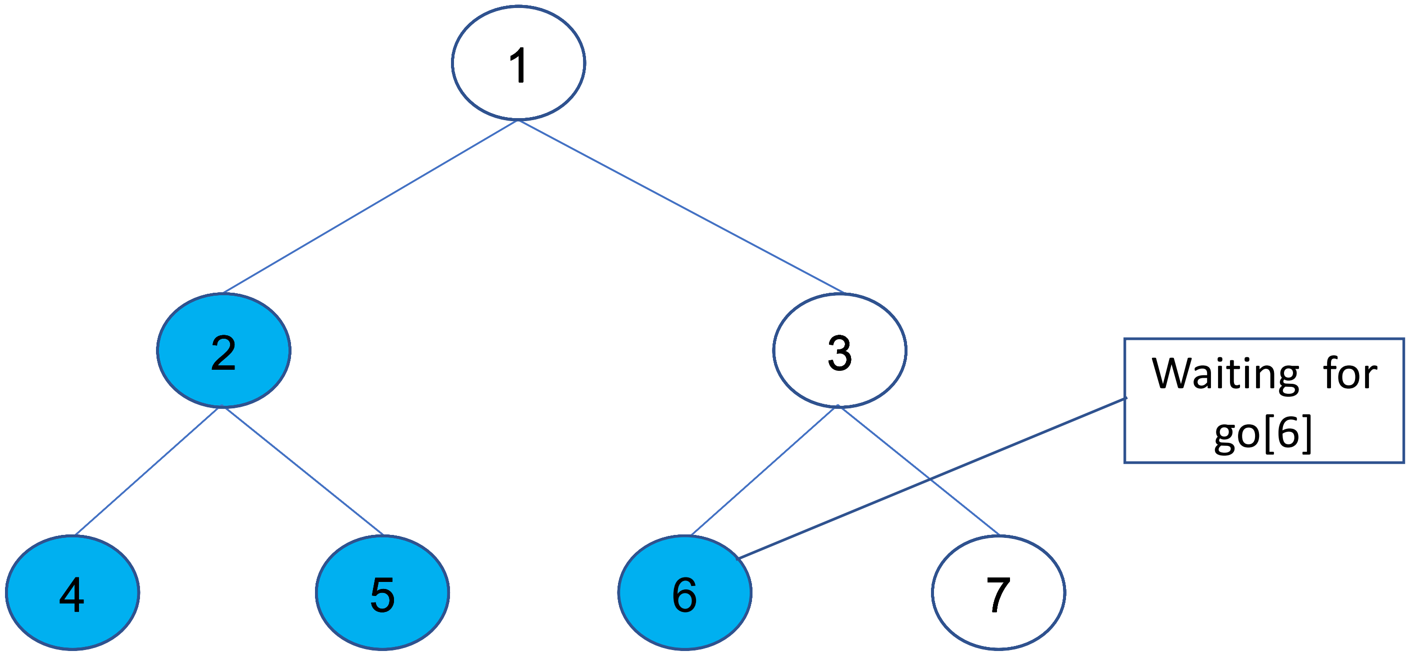


```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                      // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                         // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                             // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	1		0	0	1	
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



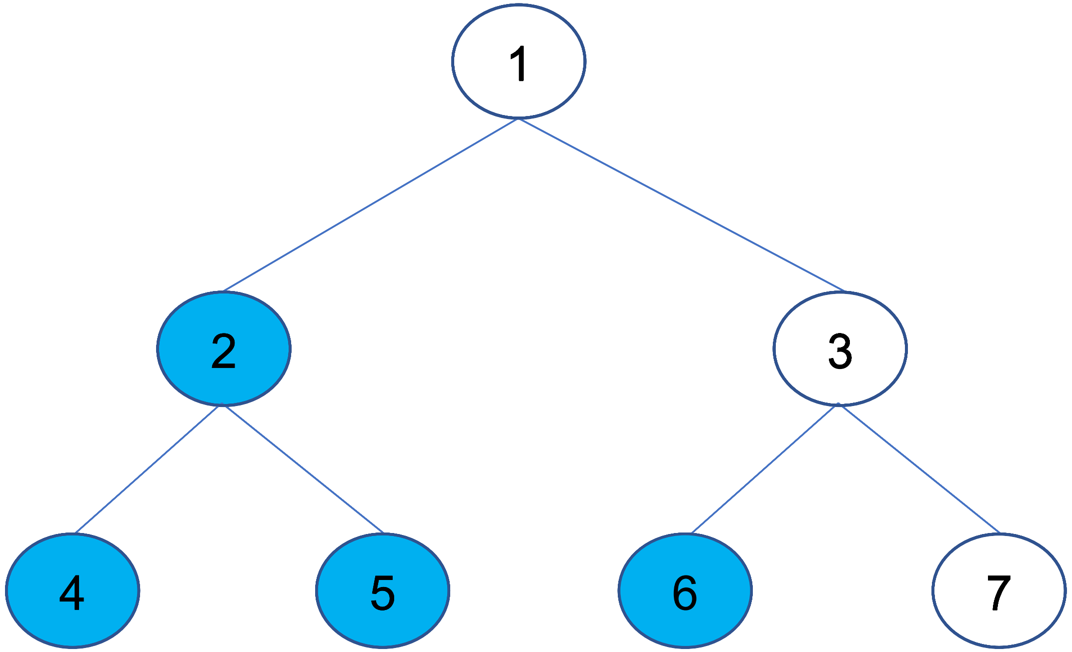
Waiting for go[6]

	2	3	4	5	6	7
arrive	1		0	0	1	
go						

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

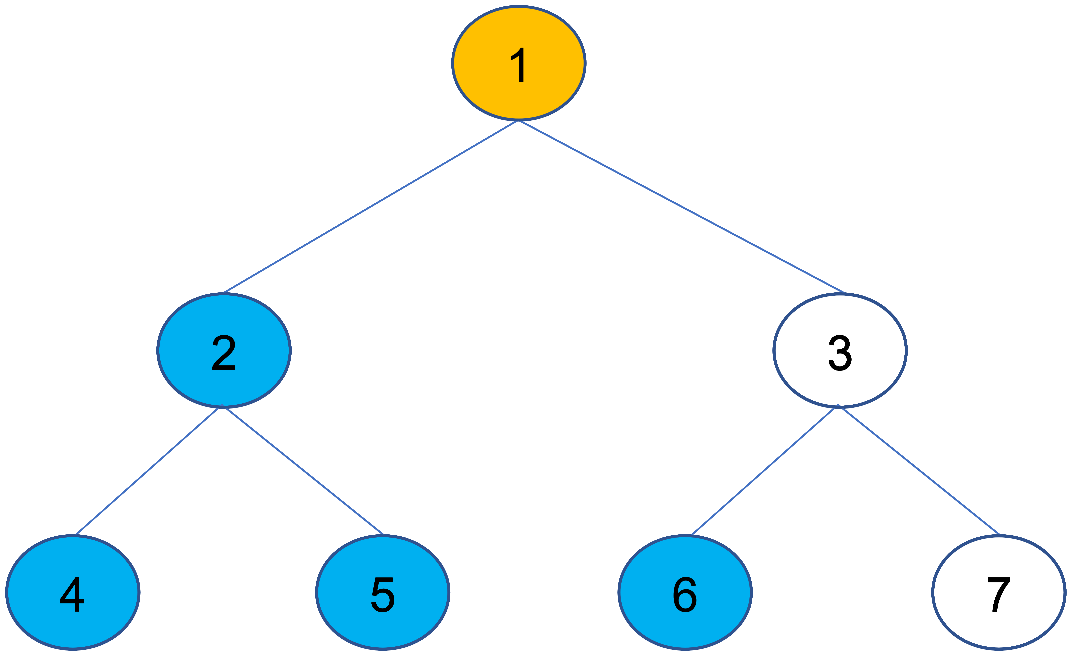


		2	3	4	5	6	7
arrive		1		0	0	1	
go							

shared arrive[2..n]: array of atomic bits, initial values = 0
go[2..n]: array of atomic bits, initial values = 0

```
if i=1 then                                        // root
        await(arrive[2]  = 1); arrive[2] := 0
        await(arrive[3]  = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
        await(arrive[2i]  = 1); arrive[2i] := 0
        await(arrive[2i+1]  = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                               // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads



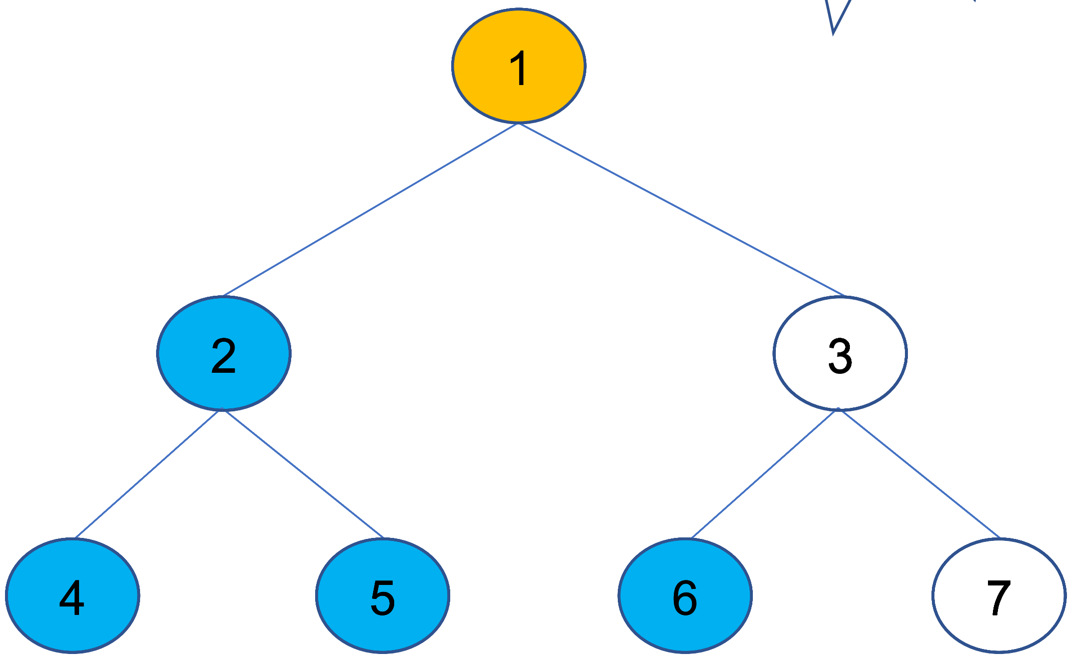
```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                         // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                            // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	1		0	0	1	
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

Arrive[2]=1
?

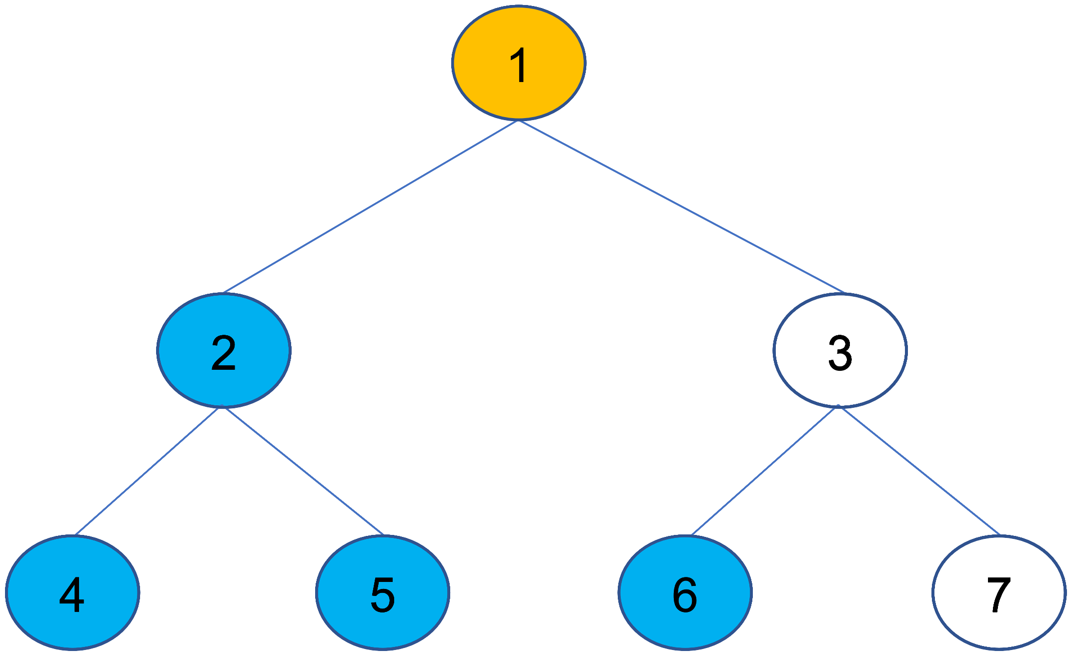


	2	3	4	5	6	7
arrive	1		0	0	1	
go						

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
      await(arrive[2] = 1); arrive[2] := 0
      await(arrive[3] = 1); arrive[3] := 0
      go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
      await(arrive[2i] = 1); arrive[2i] := 0
      await(arrive[2i+1] = 1); arrive[2i+1] := 0
      arrive[i] := 1
      await(go[i] = 1); go[i] := 0
      go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
      arrive[i] := 1
      await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

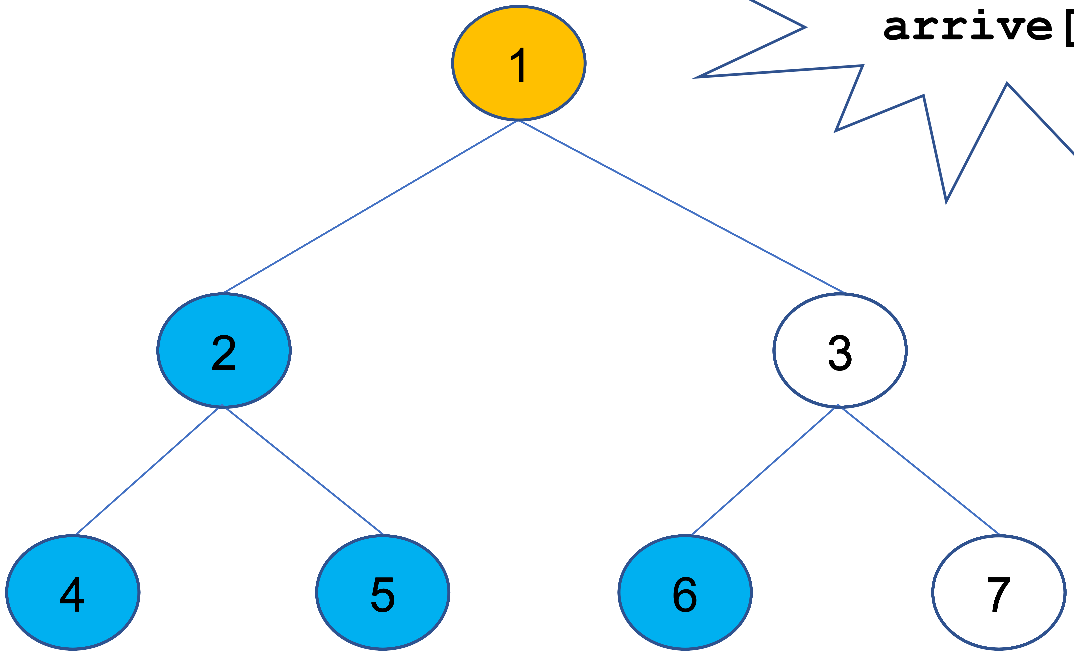


```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                          // root
      await(arrive[2]  = 1); arrive[2]  := 0
      await(arrive[3]  = 1); arrive[3]  := 0
      go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
      await(arrive[2i]  = 1); arrive[2i]  := 0
      await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
      arrive[i]  := 1
      await(go[i]  = 1); go[i] := 0
      go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
      arrive[i]  := 1
      await(go[i]  = 1); go[i] := 0 fi
fi
```

	2	3	4	5	6	7
arrive	1		0	0	1	
go						

A Tree-based Barrier
Example Run for n=7 threads



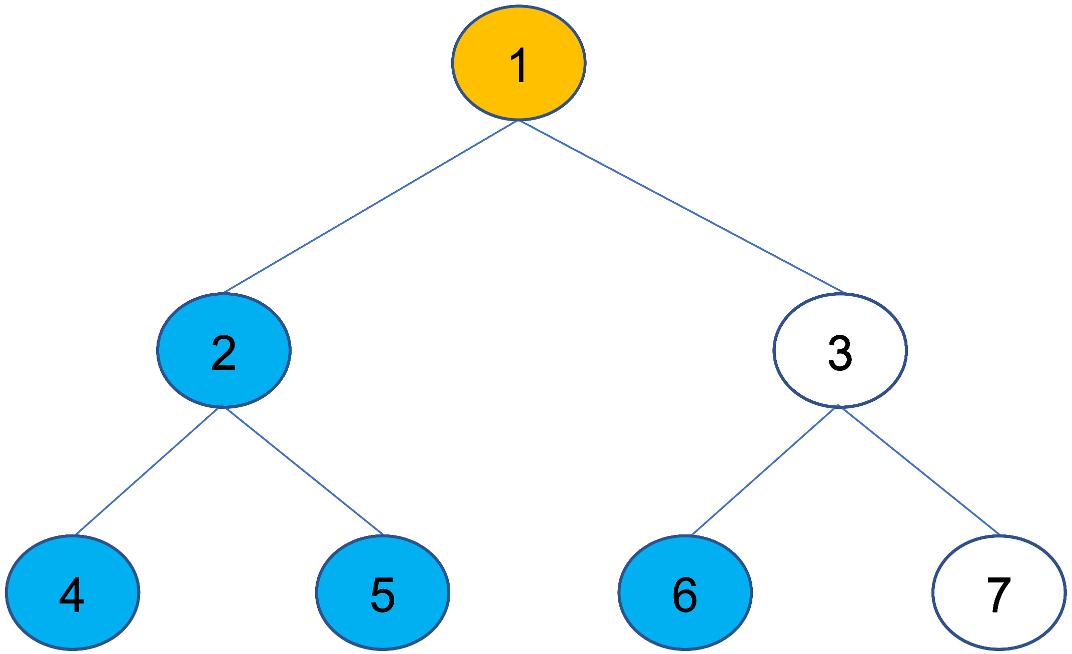
P_1 zeros arrive[2]

	2	3	4	5	6	7
arrive	0		0	0	1	
go						

shared arrive[2..n]: array of atomic bits, initial values = 0
go[2..n]: array of atomic bits, initial values = 0

```
if i=1 then                                          // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads



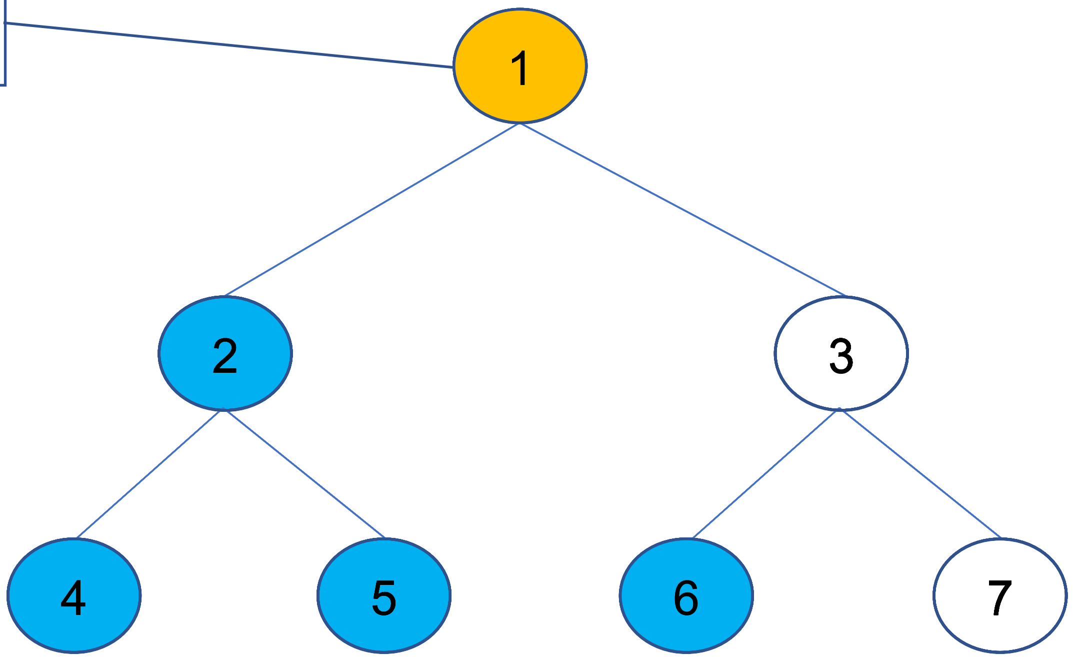
	2	3	4	5	6	7
arrive	0		0	0	1	
go						

```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                      // root
        await(arrive[2]  = 1); arrive[2] := 0
        await(arrive[3]  = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                         // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                             // leaf
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

Waiting for p_3 to arrive

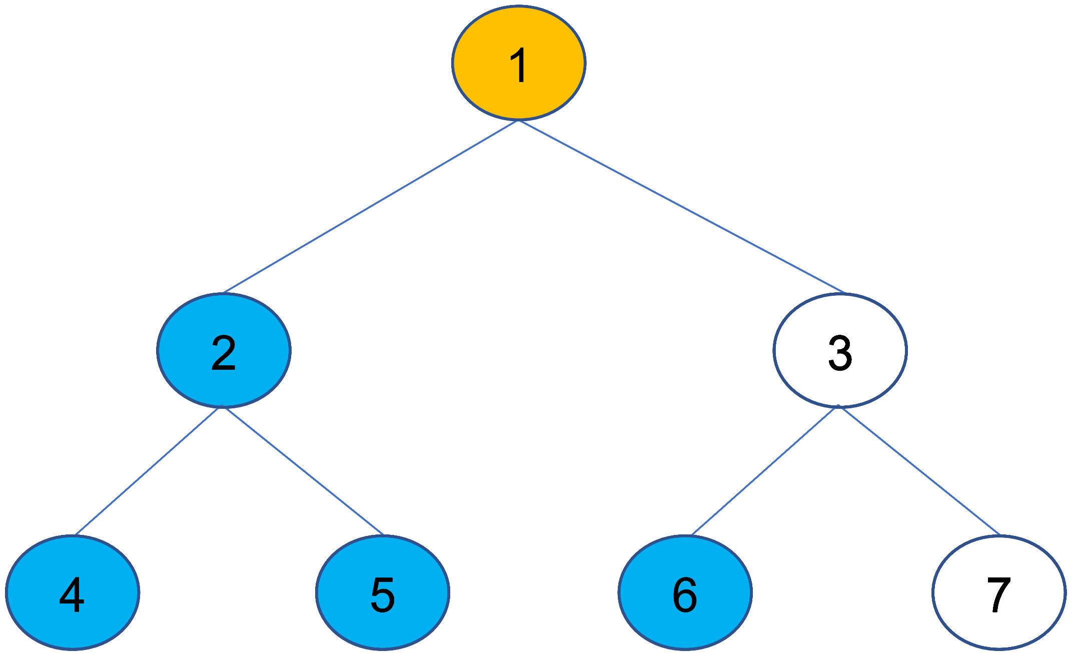


```
shared  arrive[2..n]: array of atomic bits, initial values = 0
        go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                               // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0		0	0	1	
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

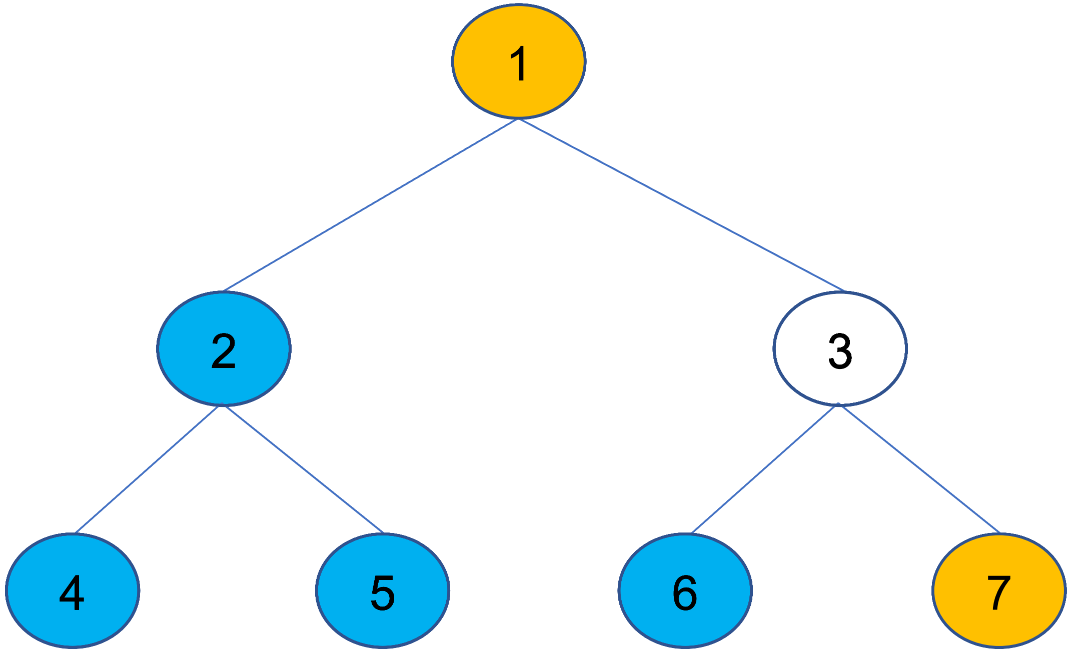


	2	3	4	5	6	7
arrive	0		0	0	1	
go						

```
shared      arrive[2..n]:  array of atomic bits, initial values = 0
            go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

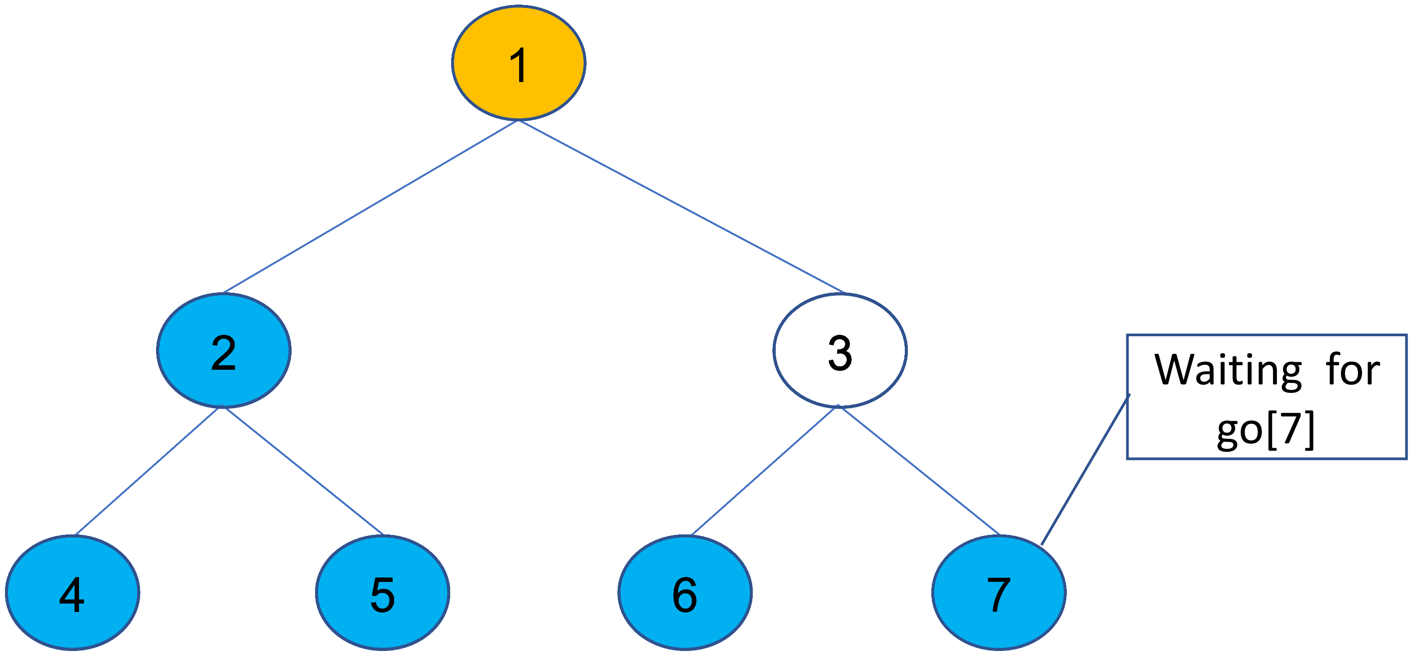


	2	3	4	5	6	7
arrive	0		0	0	1	1
go						

```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
    await(arrive[2]  = 1); arrive[2]  := 0
    await(arrive[3]  = 1); arrive[3]  := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
    await(arrive[2i]  = 1); arrive[2i]  := 0
    await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
    arrive[i]  := 1
    await(go[i]  = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
    arrive[i]  := 1
    await(go[i]  = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads



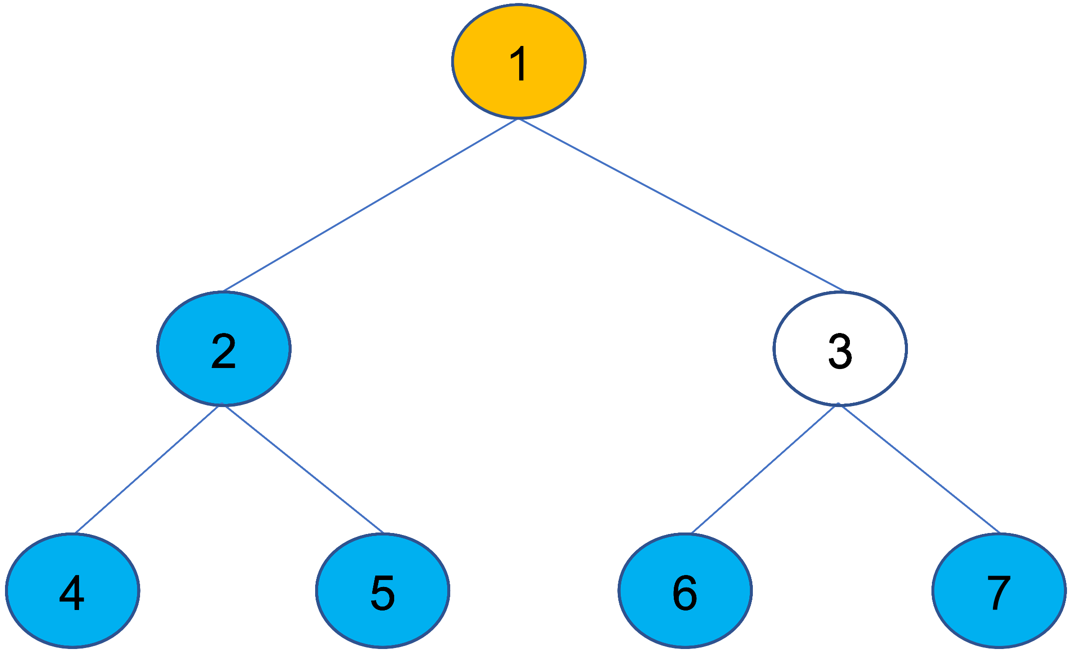
Waiting for go[7]

	2	3	4	5	6	7
arrive	0		0	0	1	1
go						

```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                               // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

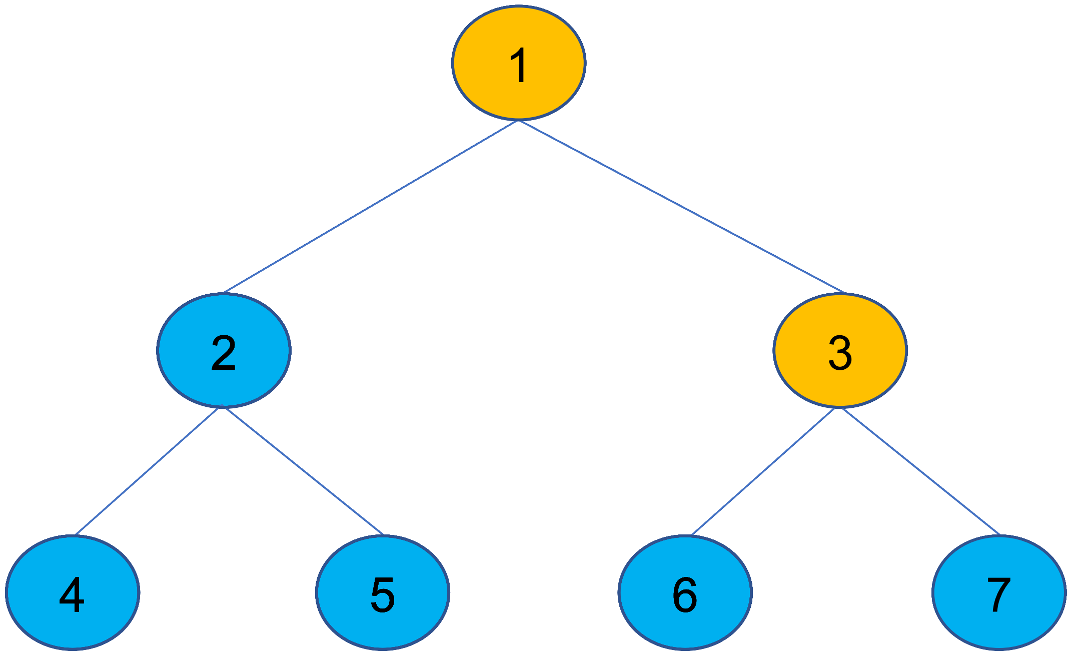


```
shared  arrive[2..n]: array of atomic bits, initial values = 0
        go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
    await(arrive[2] = 1); arrive[2] := 0
    await(arrive[3] = 1); arrive[3] := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
    await(arrive[2i] = 1); arrive[2i] := 0
    await(arrive[2i+1] = 1); arrive[2i+1] := 0
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                               // leaf
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0		0	0	1	1
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



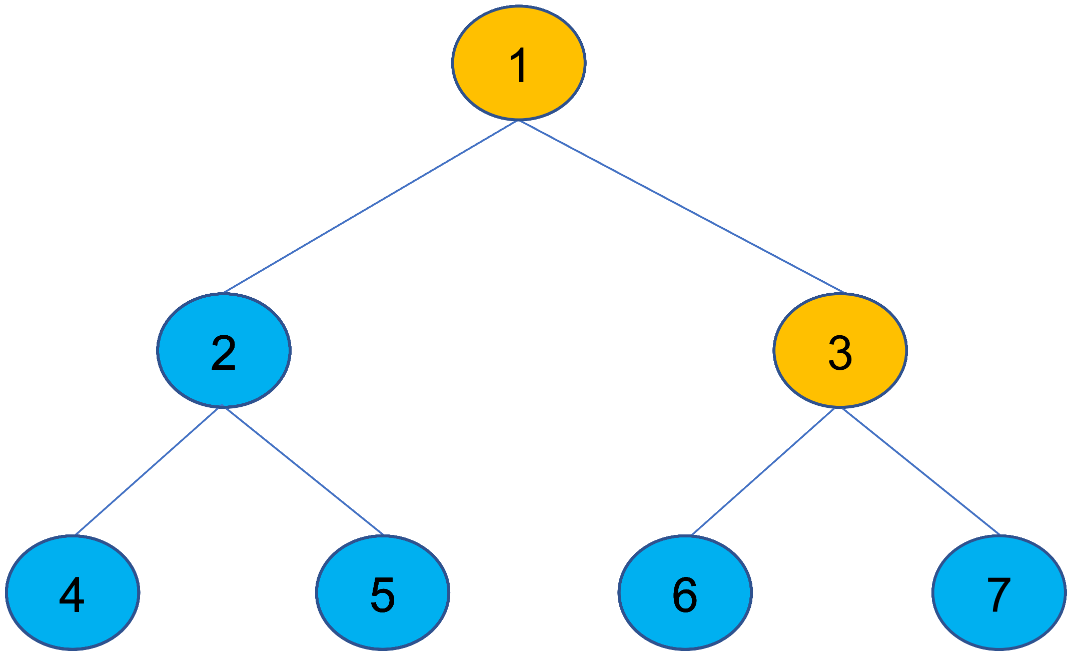
```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                   // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                      // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                          // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0		0	0	1	1
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

P_3 **zeros arrive[6,7]**

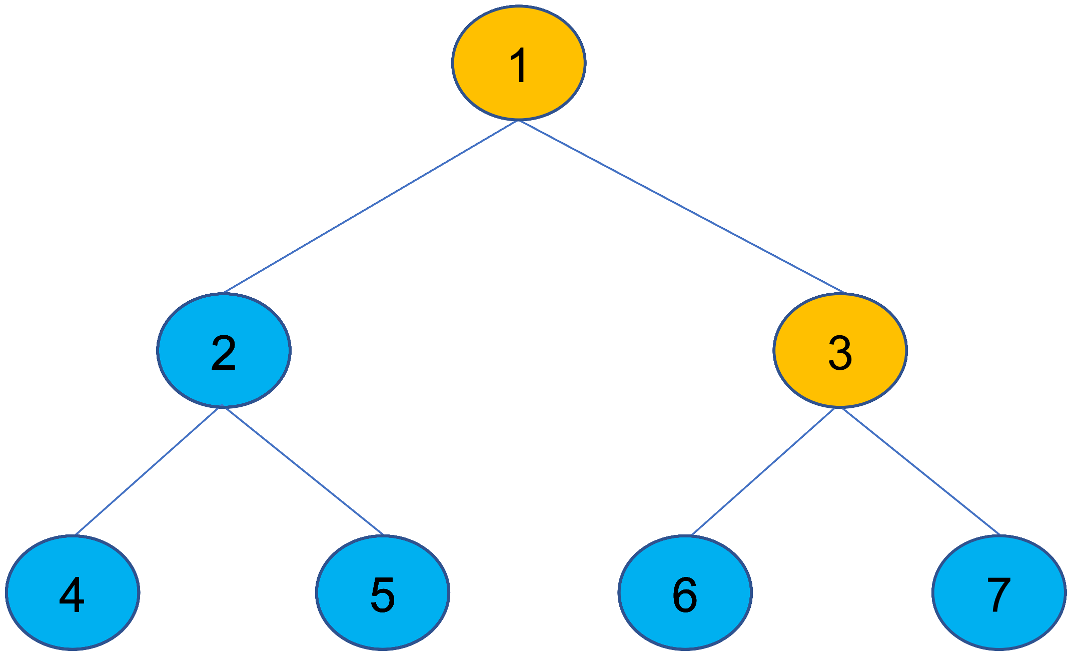


	2	3	4	5	6	7
arrive	0		0	0	0	0
go						

```
shared   arrive[2..n]: array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0

if i=1 then                                          // root
         await(arrive[2]  = 1); arrive[2]  := 0
         await(arrive[3]  = 1); arrive[3]  := 0
         go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
         await(arrive[2i]  = 1); arrive[2i]  := 0
         await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
         arrive[i]  := 1
         await(go[i] = 1); go[i] := 0
         go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
         arrive[i]  := 1
         await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

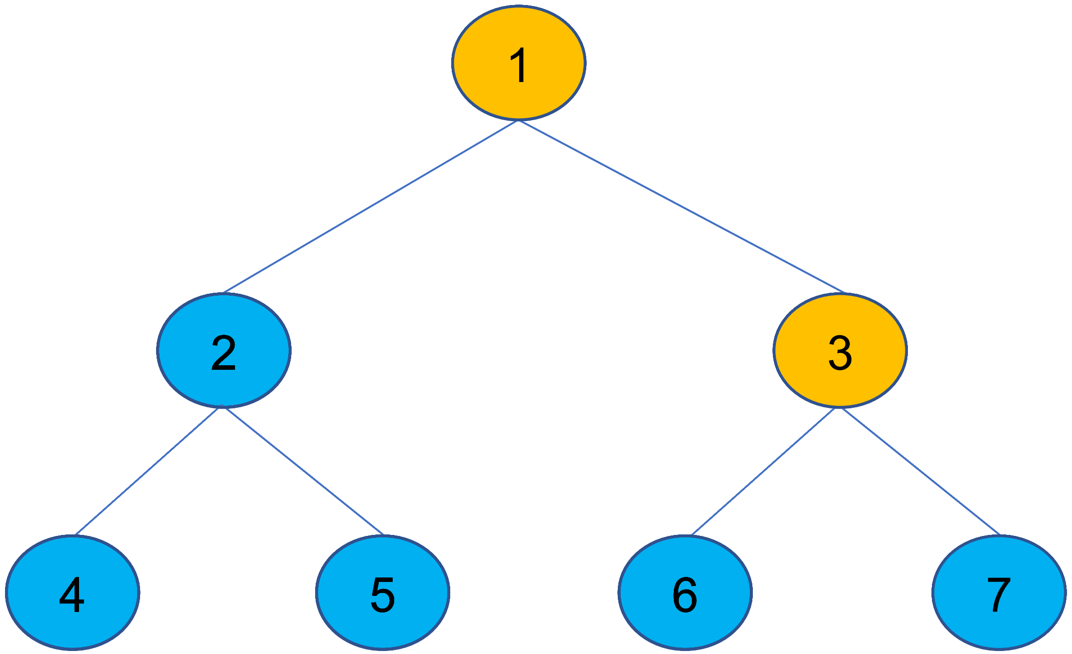


```
shared    arrive[2..n]:  array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                               // leaf
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0 fi
fi
```

arrive	0		0	0	0	0
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

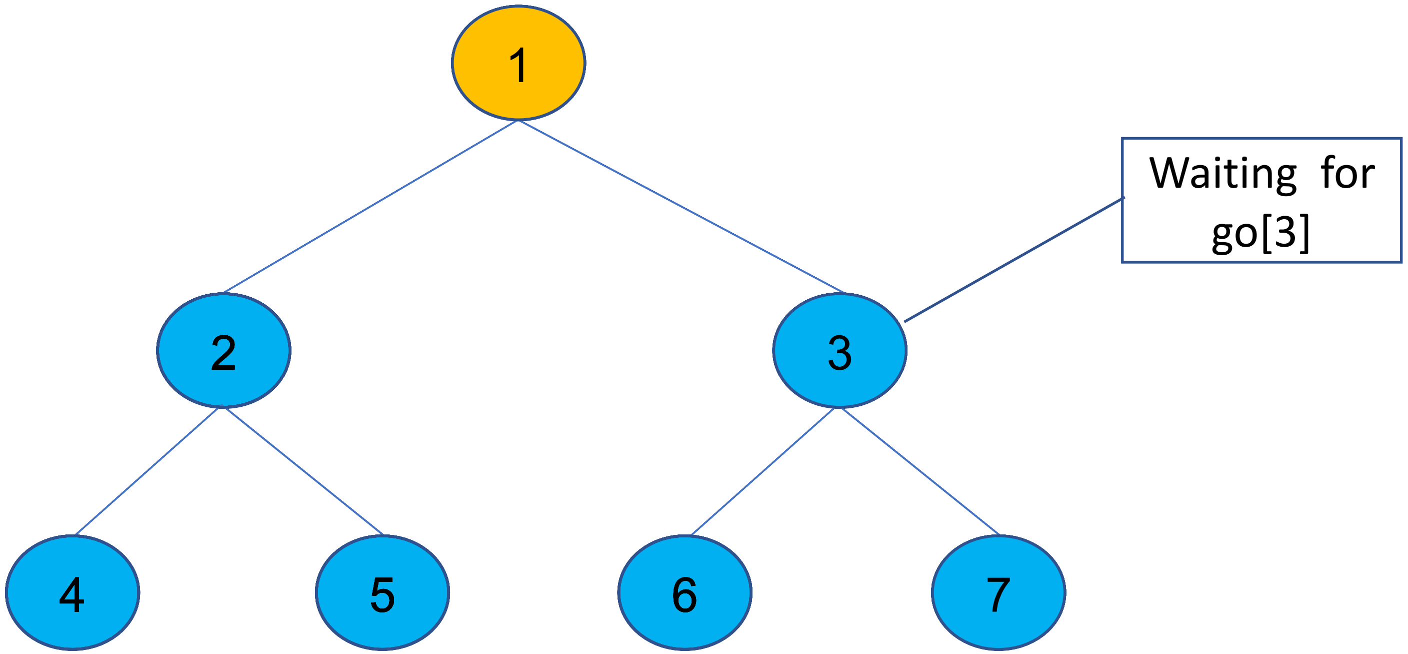


```
shared   arrive[2..n]:  array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                               // leaf
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	1	0	0	0	0
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



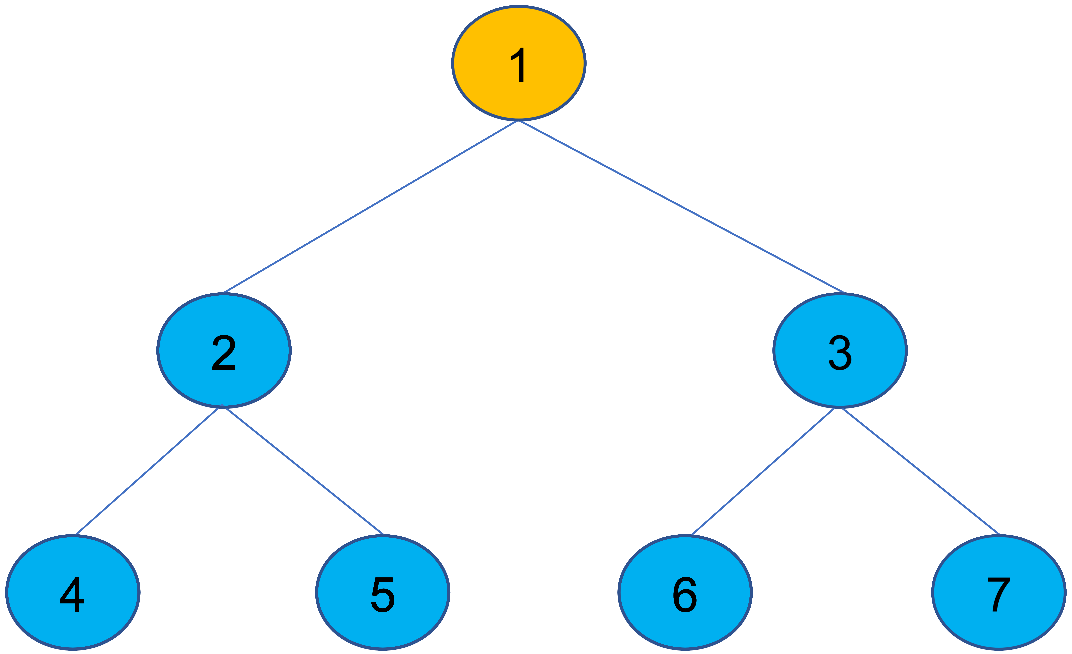
Waiting for go[3]

	2	3	4	5	6	7
arrive	0	1	0	0	0	0
go						

```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                      // root
      await(arrive[2] = 1); arrive[2] := 0
      await(arrive[3] = 1); arrive[3] := 0
      go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                         // internal node
      await(arrive[2i] = 1); arrive[2i] := 0
      await(arrive[2i+1] = 1); arrive[2i+1] := 0
      arrive[i] := 1
      await(go[i] = 1); go[i] := 0
      go[2i] = 1; go[2i+1] := 1
else                                             // leaf
      arrive[i] := 1
      await(go[i] = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads



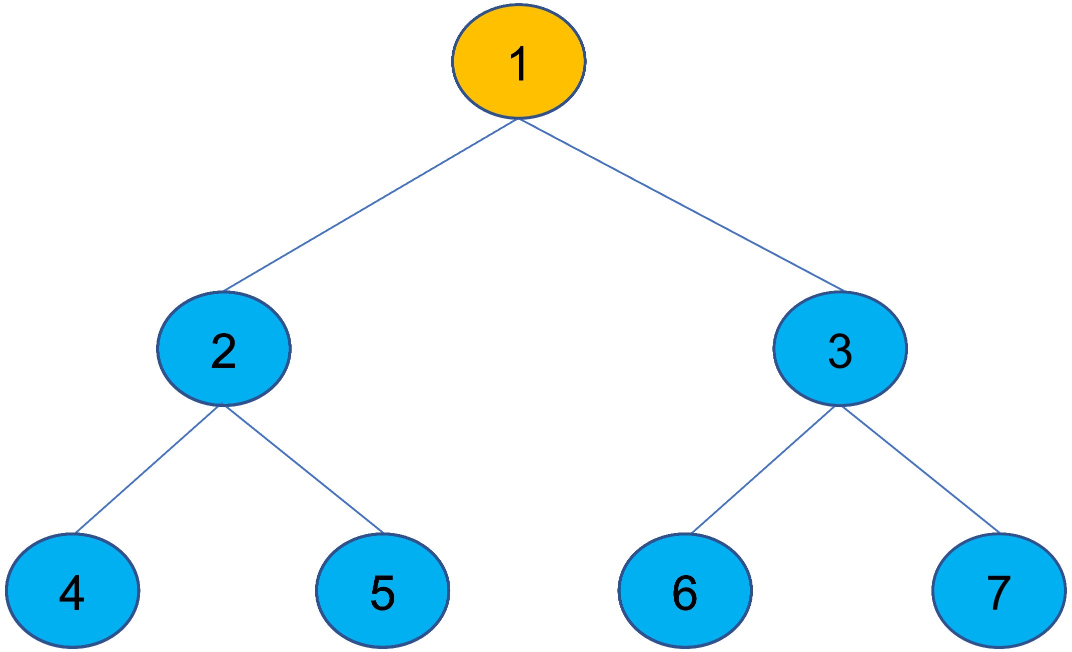
```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                               // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	1	0	0	0	0
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

P_1 **zeros arrive[3]**

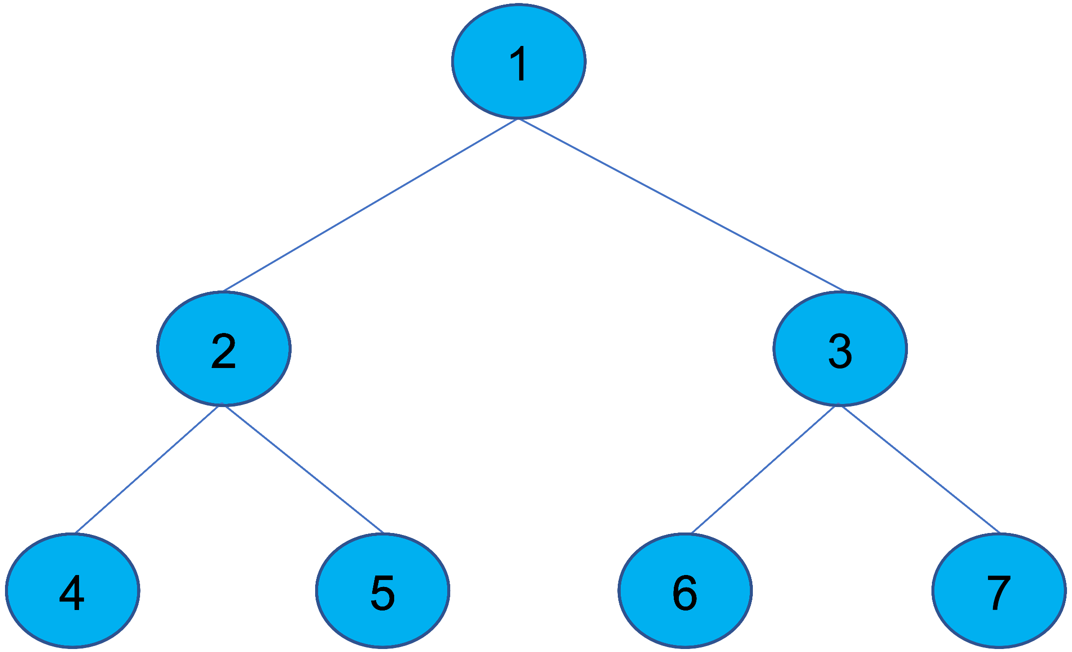


	2	3	4	5	6	7
arrive	0	0	0	0	0	0
go						

```
shared     arrive[2..n]:  array of atomic bits, initial values = 0
           go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i]  := 1
        await(go[i]  = 1); go[i] := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads

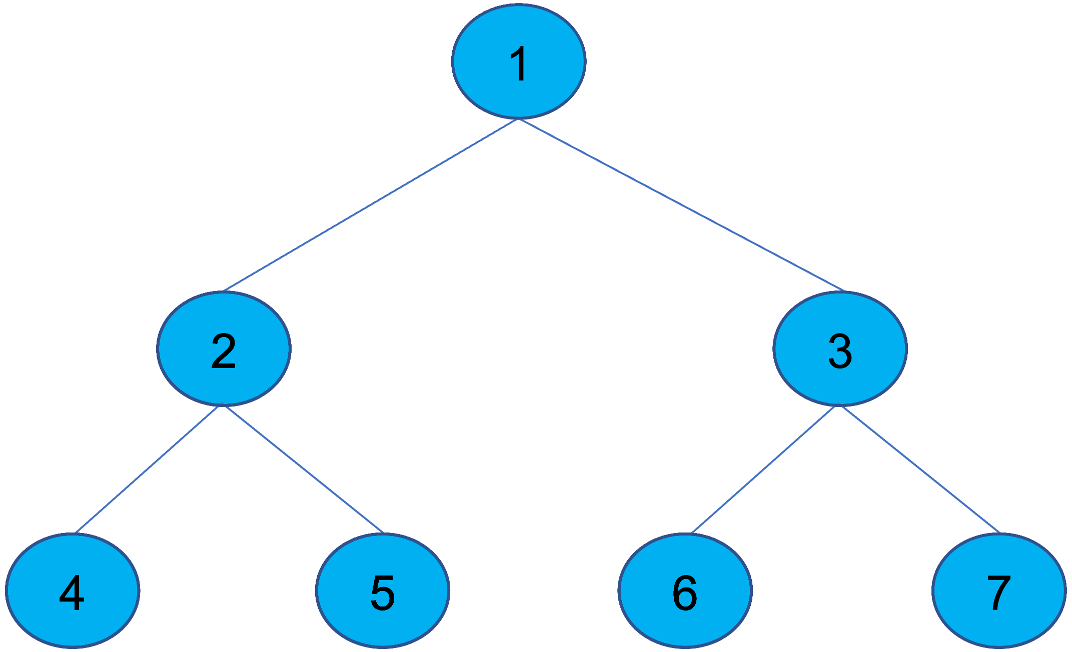


```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                        // root
    await(arrive[2] = 1); arrive[2] := 0
    await(arrive[3] = 1); arrive[3] := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                           // internal node
    await(arrive[2i] = 1); arrive[2i] := 0
    await(arrive[2i+1] = 1); arrive[2i+1] := 0
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                               // leaf
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	0	0	0	0	0
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



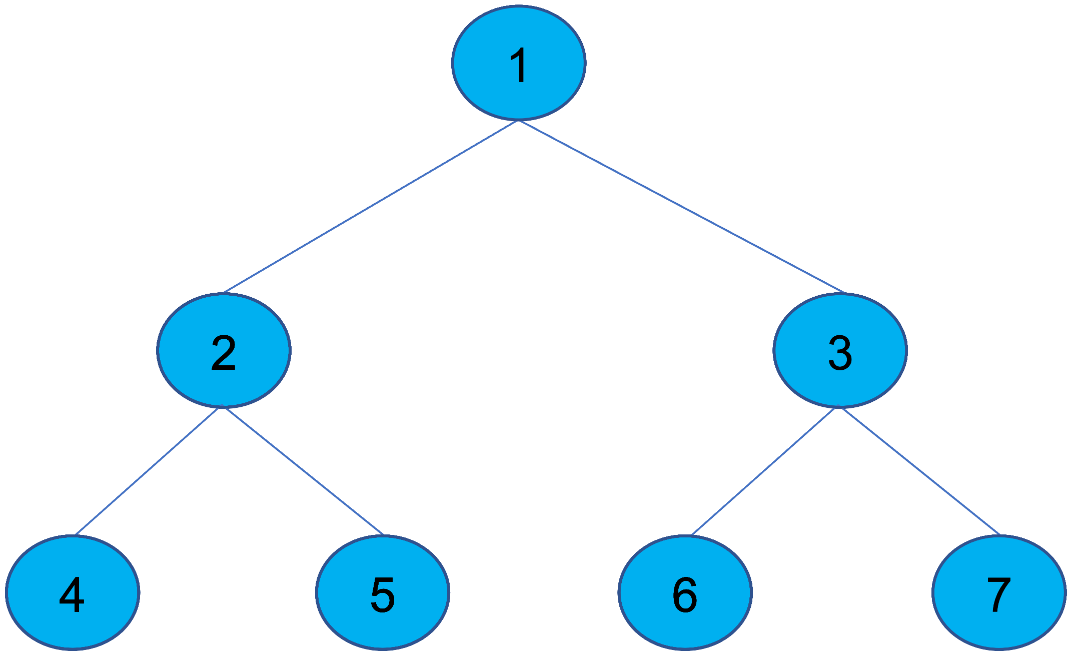
```
shared      arrive[2..n]: array of atomic bits, initial values = 0
            go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	0	0	0	0	0
go						
	2	3	4	5	6	7

At this point all non-root threads in some await(go) case

A Tree-based Barrier
Example Run for n=7 threads

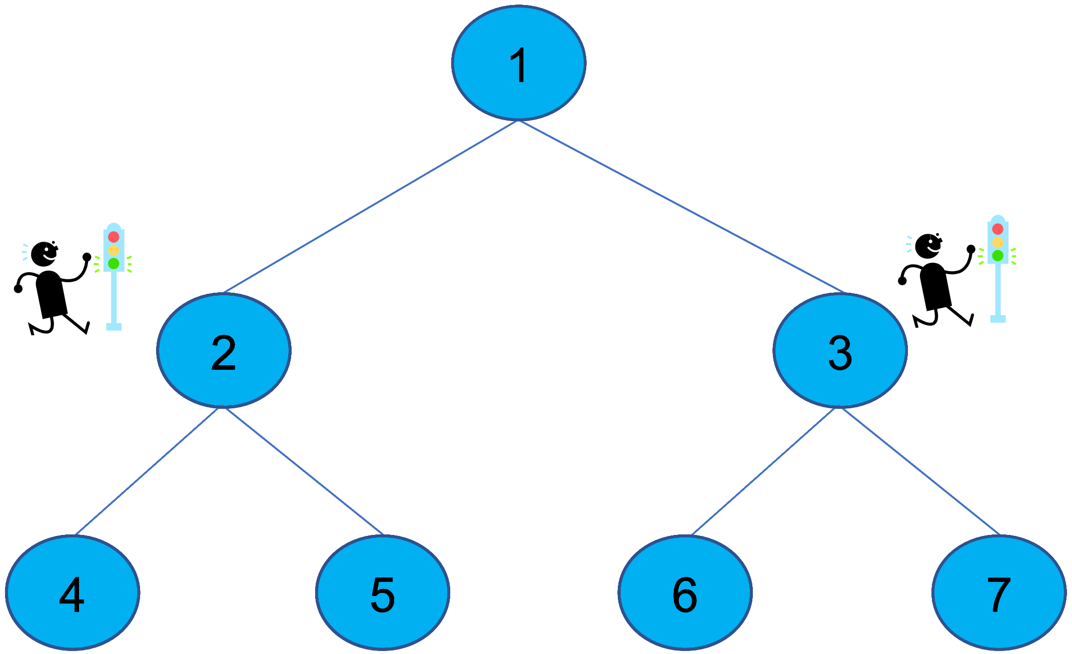


```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                    // root
    await(arrive[2] = 1); arrive[2] := 0
    await(arrive[3] = 1); arrive[3] := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                       // internal node
    await(arrive[2i] = 1); arrive[2i] := 0
    await(arrive[2i+1] = 1); arrive[2i+1] := 0
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0
    go[2i] = 1; go[2i+1] := 1
else                                           // leaf
    arrive[i] := 1
    await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	0	0	0	0	0
go						
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads

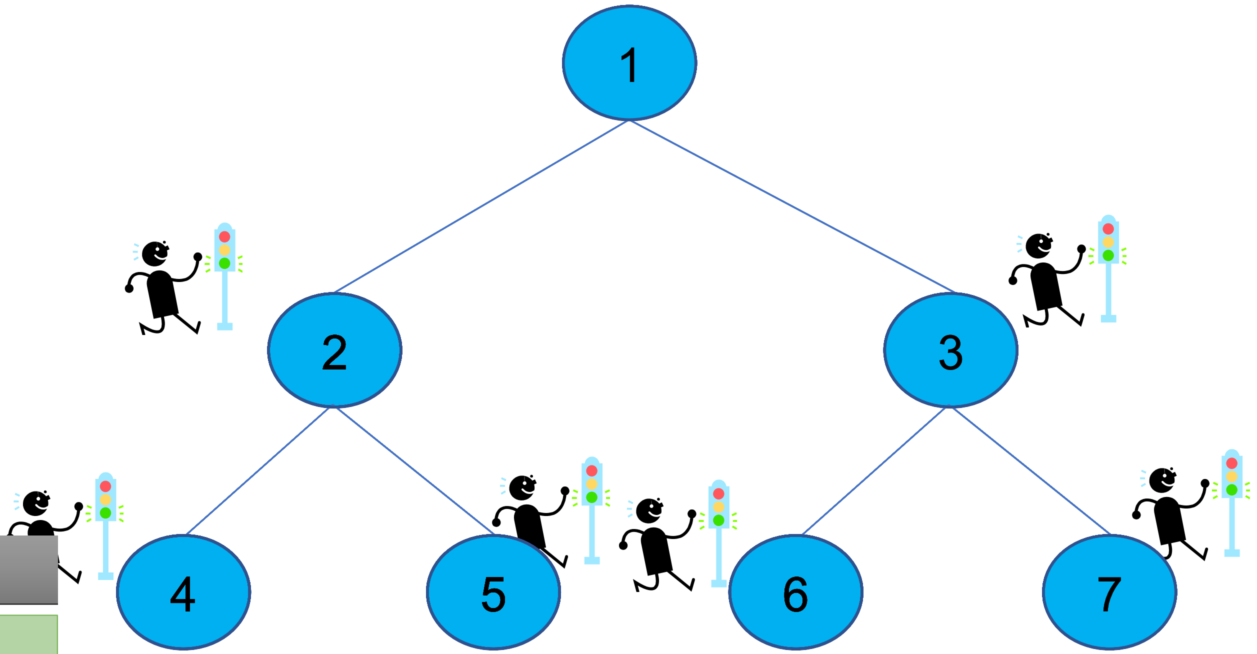


arrive	0	0	0	0	0	0
go	1	1				
	2	3	4	5	6	7

```
shared   arrive[2..n]:  array of atomic bits, initial values = 0
         go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
    await(arrive[2]  = 1); arrive[2]  := 0
    await(arrive[3]  = 1); arrive[3]  := 0
    go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
    await(arrive[2i]  = 1); arrive[2i]  := 0
    await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
    arrive[i]  := 1
    await(go[i]  = 1); go[i]  := 0
    go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
    arrive[i]  := 1
    await(go[i]  = 1); go[i]  := 0 fi
fi
```

A Tree-based Barrier
Example Run for n=7 threads



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shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```
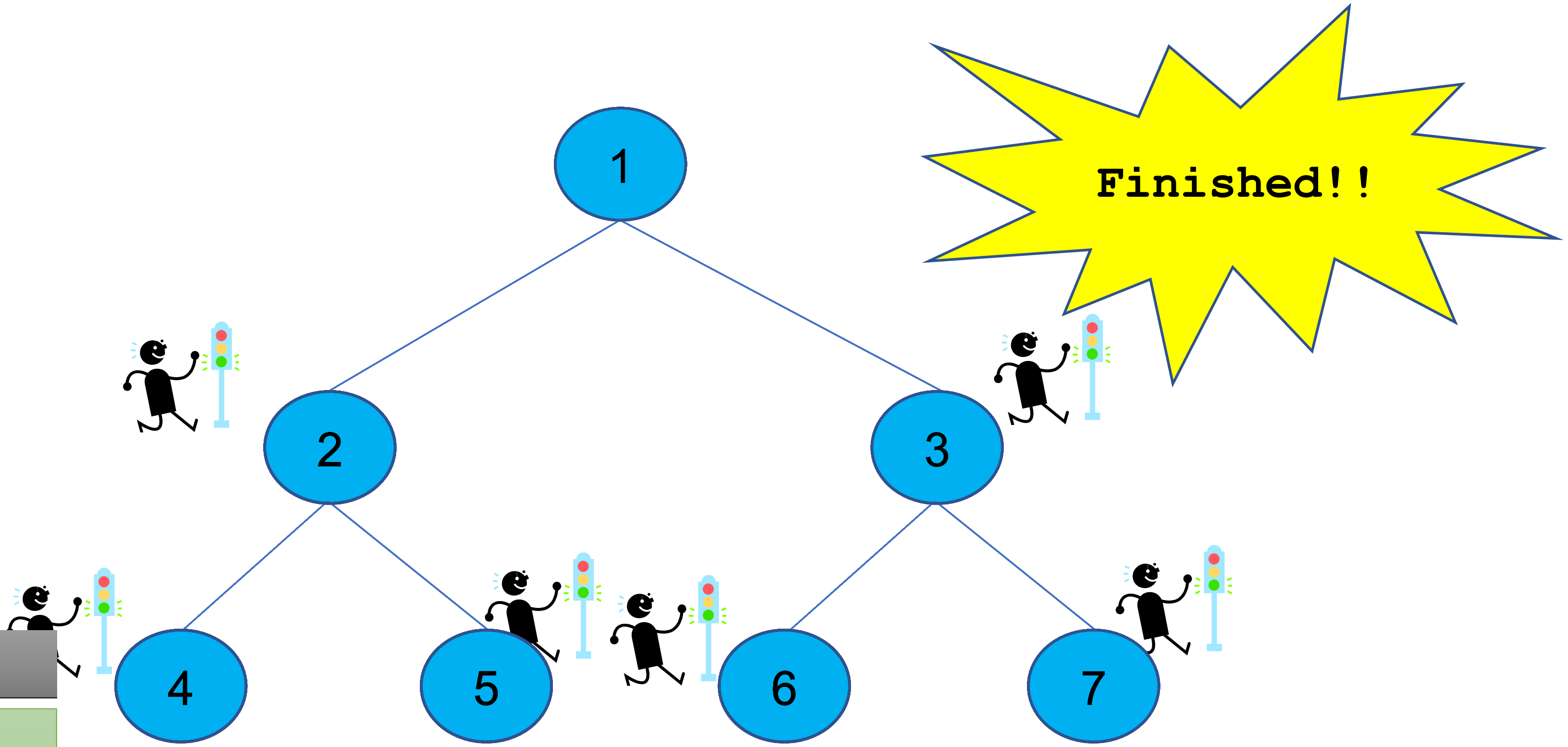

```
if i=1 then                                  // root
        await(arrive[2] = 1); arrive[2] := 0
        await(arrive[3] = 1); arrive[3] := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                     // internal node
        await(arrive[2i] = 1); arrive[2i] := 0
        await(arrive[2i+1] = 1); arrive[2i+1] := 0
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                         // leaf
        arrive[i] := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	0	0	0	0	0
go	1	1	1	1	1	1
	2	3	4	5	6	7

A Tree-based Barrier
Example Run for n=7 threads



```
shared    arrive[2..n]: array of atomic bits, initial values = 0
          go[2..n]: array of atomic bits, initial values = 0
```

```
if i=1 then                                          // root
        await(arrive[2]  = 1); arrive[2]  := 0
        await(arrive[3]  = 1); arrive[3]  := 0
        go[2] = 1; go[3] = 1
else if i ≤ (n-1)/2 then                             // internal node
        await(arrive[2i]  = 1); arrive[2i]  := 0
        await(arrive[2i+1]  = 1); arrive[2i+1]  := 0
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0
        go[2i] = 1; go[2i+1] := 1
else                                                 // leaf
        arrive[i]  := 1
        await(go[i] = 1); go[i] := 0 fi
fi
```

arrive	0	0	0	0	0	0
go	1	1	1	1	1	1
	2	3	4	5	6	7

Tree Barrier Tradeoffs

- Pros:

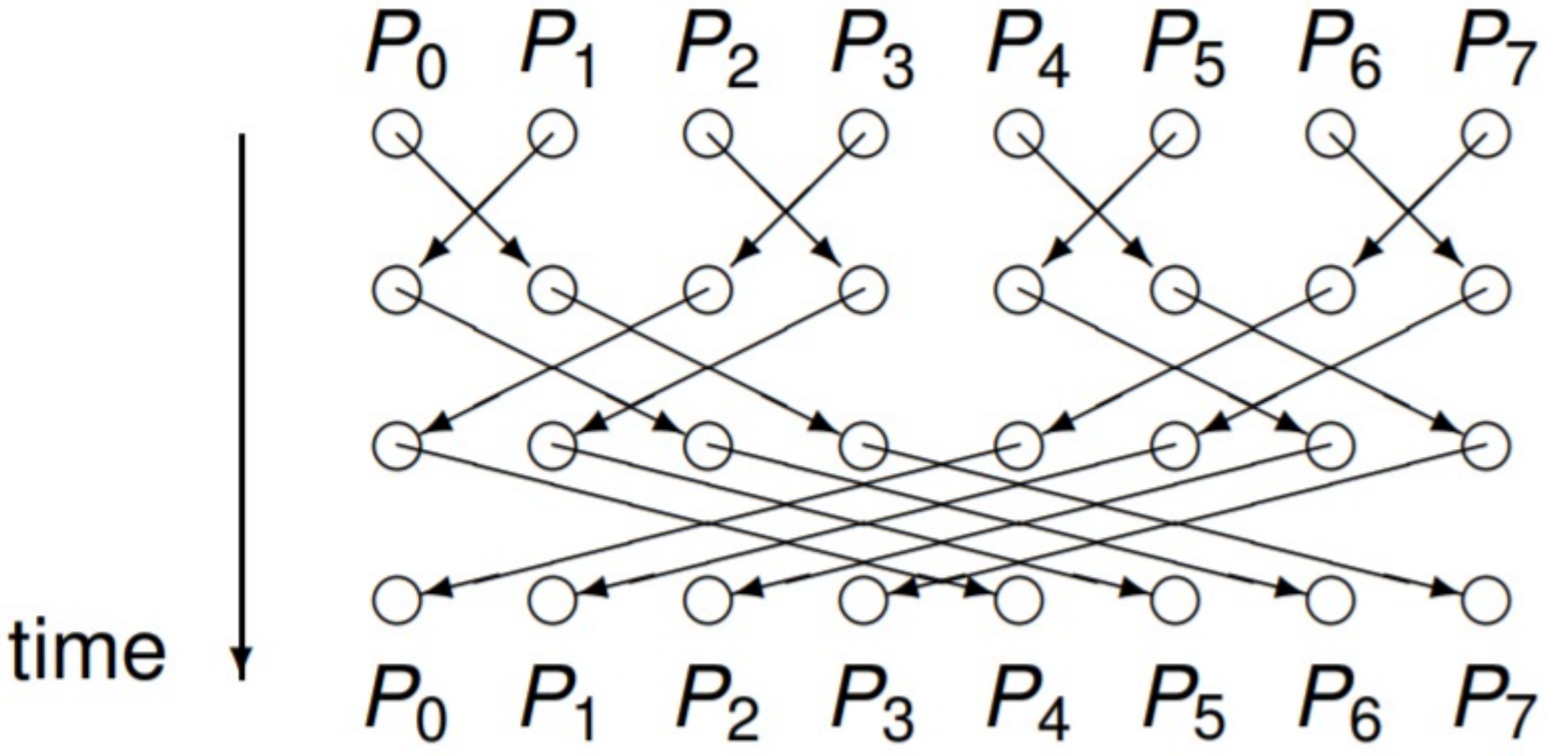
- Cons:

Tree Barrier Tradeoffs

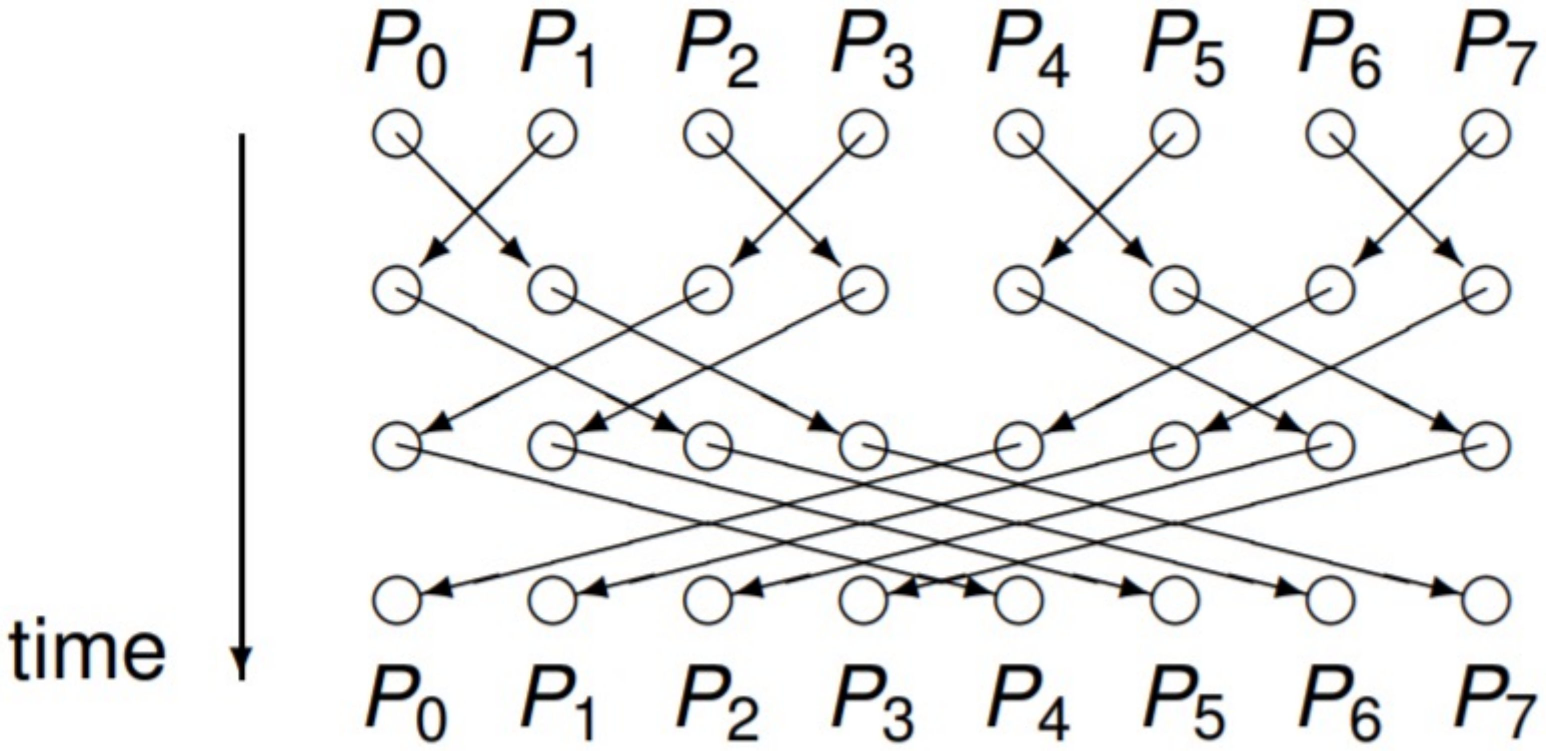
- Pros:
  - Low shared memory contention
    - No wait object is shared by more than 2 processes
    - Good for larger n
  - Fast – information from the root propagates after log(n) steps
  - Can use only atomic primitives (no special objects)
  - On some models:
    - each process spins on a locally accessible bit
    - # (remote memory ref.) = O(1) per process
- Cons:
  - Shared memory space complexity – O(n)
  - Asymmetric –all the processes don’t do the same amount of work

Butterfly Barrier

Butterfly Barrier

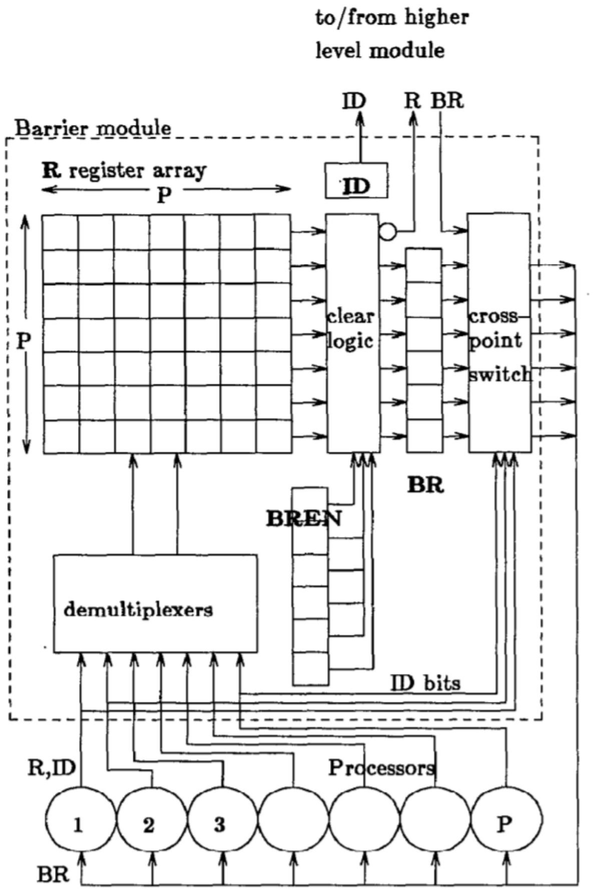

P_0 P_1 P_2 P_3 P_4 P_5 P_6 P_7
time
P_0 P_1 P_2 P_3 P_4 P_5 P_6 P_7

Butterfly Barrier

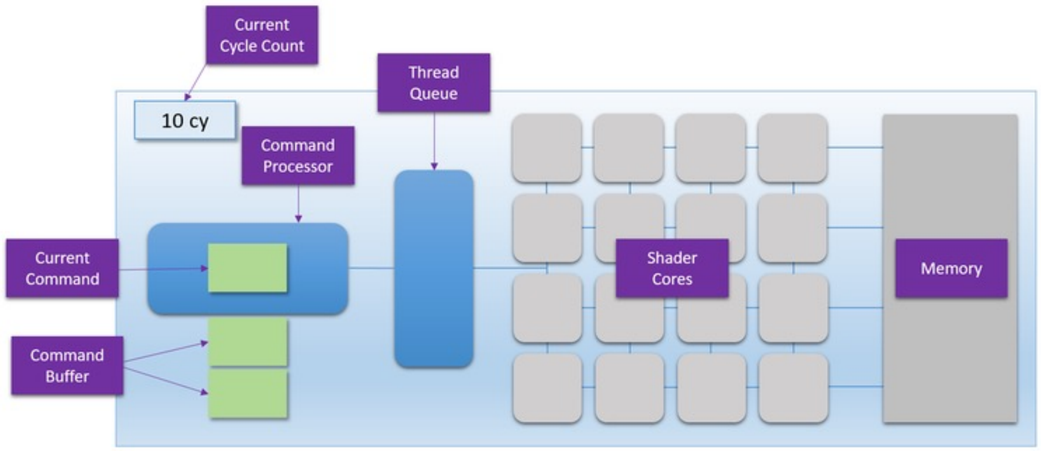

P_0 P_1 P_2 P_3 P_4 P_5 P_6 P_7
time
P_0 P_1 P_2 P_3 P_4 P_5 P_6 P_7

- When would this be preferable?

Hardware Supported Barriers



CPU



GPU

Barriers Summary

Seen:

- Semaphore-based barrier
- Simple barrier
    - Based on atomic fetch-and-increment counter
- Local spinning barrier
    - Based on atomic fetch-and-increment counter and go array
- Tree-based barrier

Not seen:

- Test-and-Set barriers
    - Based on test-and-test-and-set objects
    - One version without memory initialization
- See-Saw barrier

Questions?