Name:________________________

CS 367: Numerical Methods
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Midterm Exam
Fall 1994

Rules of the game: Closed book, closed notes. Show all work. Any person found to have cheated on this exam will be persecuted to the fullest extend possible. If you have any questions, feel free to ask for help (although we may choose not to answer your question).

1. Consider a floating point system that uses three digits for the mantissa, one digit for the exponent, and base 10, i.e., all numbers look like

$$\pm.d_1 d_2 d_3 \times 10^{\pm e_1}$$

   (a) Define the machine epsilon in general, and give the machine epsilon for this specific case.

   (b) Pretend to be a computer that uses this floating point number system.

      i. Evaluate

$$\sqrt{x^2 + y^2}$$

      with $x = 5.78$ and $y = 1.231$.

      ii. Find the largest root of $x^2 - 1.12x + 0.012$ *using a stable formula.*

2. Consider the following program for solving a lower triangular linear system, with the matrix stored in `A(i,j)` *in column major order* and the right hand side stored in B:

```
for i=1,n                              (loop over rows)
   X(i) = B(i)                         (initialize solution to rhs)
   for j=1,i-1
      X(i) = X(i) - A(i,j) * X(j)      (update)
   endfor
   X(i) = X(i) / A(i,i)                (divide by the diagonal)
endfor
```

(a) Give a formula for the approximate cost, in floating point operations, of this algorithm. Give all intermediate steps (i.e., memorizing the cost of a triangular solve is not enough).

(b) What can you say about the expected efficiency of this loop?

(c) *In words*, what could you do to improve performance?

3. Consider the data points

$$(0, 3)$$
$$(1, 0)$$
$$(3, 0)$$
$$(4, 1)$$

(a) What is the highest degree of a *unique* polynomial that will interpolate these points?

(b) Derive a formula for this polynomial *using Lagrange Interpolation*.

NOTE: Lagrange polynomials are NOT covered by the exam, but some other approaches are.

4. **the following is a question that will NOT be covered by the exam (since we haven't covered this material yet), but was on the exam last time I taught it.** Consider the function

$$f(x) = 3 + 2x - 4x^2 + x^3$$

(a) Using your knowledge of calculus, compute

$$\int_0^1 f(x)dx$$

(b) Estimate the above integral using the Trapezoidal Rule.

(c) Estimate the above integral using the Midpoint Rule.

(d) Estimate the above integral using Simpson's Rule.

(e) What answer would you expect if you used a two-point Gauss quadrature rule? Why? (Do not compute!)