Topic 15
Implementing and Using Stacks

"stack n.
The set of things a person has to do in the future. "I haven't done it yet because every time I pop my stack something new gets pushed." If you are interrupted several times in the middle of a conversation, "My stack overflowed" means "I forget what we were talking about."

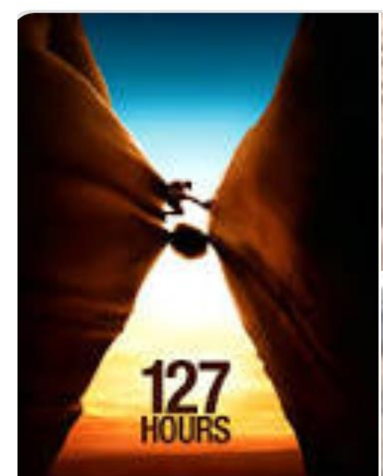
-The Hacker's Dictionary

Friedrich L. Bauer
German computer scientist who proposed "stack method of expression evaluation" in 1955.

Sharper Tools

Stacks

Lists

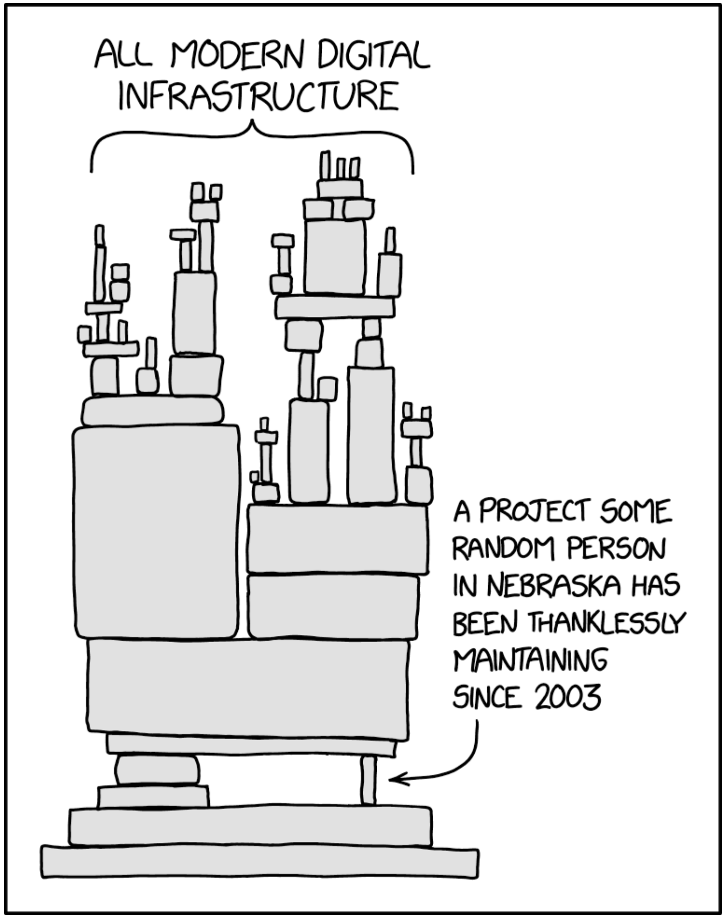


Stacks

- Access is allowed only at one point of the structure, normally termed the *top* of the stack
  - access to the most recently added item only
- Operations are limited:
  - push (add item to stack)
  - pop (remove top item from stack)
  - top (get top item without removing it)
  - isEmpty
- Described as a "Last In First Out" (LIFO) data structure

Implementing a stack

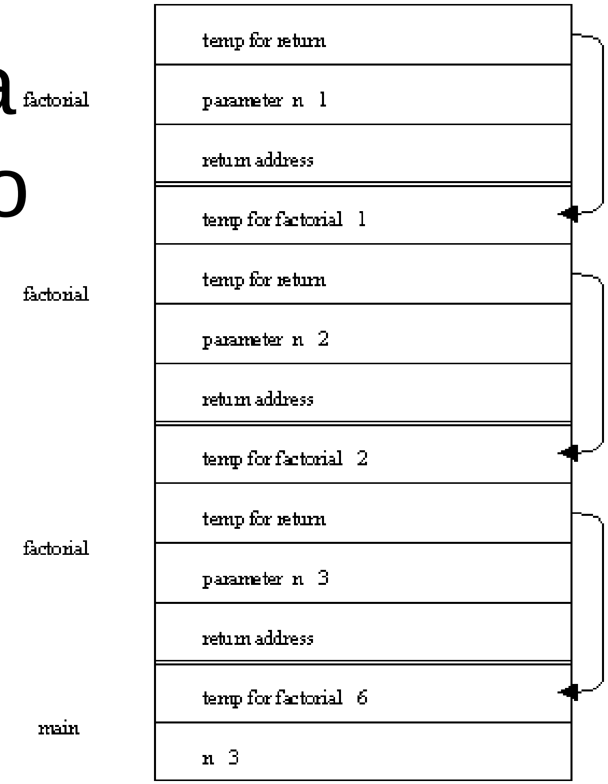
- need an underlying collection to hold the elements of the stack
- 3 obvious choices?
  - native array
  - linked structure of nodes
  - a list!!!
- Adding a *layer of abstraction*. A HUGE idea.
- array implementation
- linked list implementation

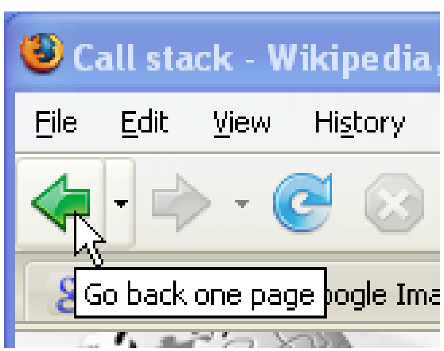


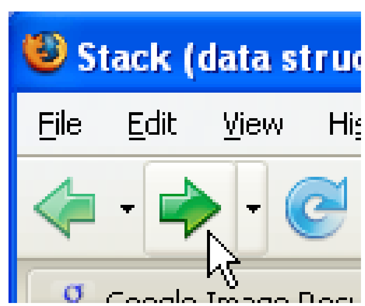
https://xkcd.com/2347/

Uses of Stacks

- The runtime stack used by a process (running program) to keep track of methods in progress
- Search problems
- Undo, redo, back, forward







Stack Operations

Assume a simple stack for integers.

```
Stack<Integer> s = new Stack<>();
s.push(12);
s.push(4);
s.push( s.top() + 2 );
s.pop();
s.push( s.top() );
//what are contents of stack?
```

Clicker 1 - What is Output?

```
Stack<Integer> s = new Stack<>();
// put stuff in stack
for (int i = 0; i < 5; i++)
    s.push(i);
// Print out contents of stack.
// Assume there is a size method.
for (int i = 0; i < s.size(); i++)
    System.out.print(s.pop() + " ");
```

A `0 1 2 3 4`

B `4 3 2 1 0`

C `4 3 2`

D `2 3 4`

E No output due to runtime error

Corrected Version

```
Stack<Integer> s = new Stack<Integer>();
// put stuff in stack
for (int i = 0; i < 5; i++)
    s.push(i);
// print out contents of stack
// while emptying it
final int LIMIT = s.size();
for (int i = 0; i < LIMIT; i++)
    System.out.print(s.pop() + " ");
//or
//  while (!s.isEmpty())
//      System.out.println(s.pop());
```

Stack Operations

Write a method to print out contents of stack in reverse order.

Applications of Stacks

Mathematical Calculations

- What does 3 + 2 * 4 equal?
  2 * 4 + 3?   3 * 2 + 4?
- The precedence of operators affects the order of operations.
- A mathematical expression cannot simply be evaluated left to right.
- A challenge when evaluating a program.
- *Lexical analysis* is the process of interpreting a program.

What about 1 - 2 - 4 ^ 5 * 3 * 6 / 7 ^ 2 ^ 3

Infix and Postfix Expressions

- The way we are use to writing expressions is known as infix notation
- Postfix expression does not
- require any precedence rules
- 3 2 * 1 + is postfix of 3 * 2 + 1
- evaluate the following postfix expressions and write out a corresponding infix expression:

2 3 2 4 * + *　　　　1 2 3 4 ^ * +

1 2 - 3 2 ^ 3 * 6 / +　　　　2 5 ^ 1 -

Clicker Question 2

- What does the following postfix expression evaluate to?

  6 3 2 + *

A. 11

B. 18

C. 24

D. 30

E. 36

Evaluation of Postfix Expressions

- Easy to do with a stack
- given a proper postfix expression:
  - get the next token
  - if it is an operand push it onto the stack
  - else if it is an operator
    - pop the stack for the right hand operand
    - pop the stack for the left hand operand
    - apply the operator to the two operands
    - push the result onto the stack
  - when the expression has been exhausted the result is the top (and only element) of the stack

Infix to Postfix

- Convert the following equations from infix to postfix:

  2 ^ 3 ^ 3 + 5 * 1

  11 + 2 - 1 * 3 / 3 + 2 ^ 2 / 3

  Problems:

    Negative numbers?

    parentheses in expression

Infix to Postfix Conversion

- Requires operator precedence parsing algorithm
  - parse v. To determine the syntactic structure of a sentence or other utterance

Operands: add to expression

Close parenthesis: pop stack symbols until an open parenthesis appears

Operators:

Have an on stack and off stack precedence

Pop all stack symbols until a symbol of lower precedence appears. Then push the operator

End of input: Pop all remaining stack symbols and add to the expression

Simple Example

Infix Expression: 3 + 2 * 4

PostFix Expression:

Operator Stack:

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression: + 2 * 4

PostFix Expression: 3

Operator Stack:

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression: 2 * 4

PostFix Expression: 3

Operator Stack: +

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression: * 4

PostFix Expression: 3 2

Operator Stack: +

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression: 4

PostFix Expression: 3 2

Operator Stack: + *

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression:

PostFix Expression: 3 2 4

Operator Stack: + *

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression:

PostFix Expression: 3 2 4 *

Operator Stack: +

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Simple Example

Infix Expression:

PostFix Expression: 3 2 4 * +

Operator Stack:

Precedence Table

Symbol	Off Stack Precedence	On Stack Precedence
+	1	1
-	1	1
*	2	2
/	2	2
^	10	9
(	20	0

Example

11 + 2 ^ 4 ^ 3 - ((4 + 5) * 6) ^ 2

Show algorithm in action on above equation

Balanced Symbol Checking

- In processing programs and working with computer languages there are many instances when symbols must be balanced

  { } , [ ] , ( )

  A stack is useful for checking symbol balance. When a closing symbol is found it must match the most recent opening symbol of the same type.

- Applicable to checking html and xml tags!

Algorithm for Balanced Symbol Checking

- Make an empty stack
- read symbols until end of file
  - if the symbol is an opening symbol push it onto the stack
  - if it is a closing symbol do the following
    - if the stack is empty report an error
    - otherwise pop the stack. If the symbol popped does not match the closing symbol report an error
- At the end of the file if the stack is not empty report an error

Algorithm in practice

- list[i] = 3 * (44 - method(foo(list[2 * (i + 1) + foo(list[i - 1])) / 2 *) - list[method(list[0])];
- Complications
  - when is it not an error to have non matching symbols?
- Processing a file
  - *Tokenization*: the process of scanning an input stream. Each independent chunk is a token.
- Tokens may be made up of 1 or more characters