"stack n. The set of things a person has to do in the future. "I haven't done it yet because every time I pop my stack something new gets pushed." If you are interrupted several times in the middle of a conversation, "My stack overflowed" means "I forget what we were talking about."

-The Hacker's Dictionary

Friedrich L. Bauer
German computer scientist who proposed "stack method of expression evaluation" in 1955.
Sharper Tools

Lists

Stacks

Stacks
Stacks

- Access is allowed only at one point of the structure, normally termed the *top* of the stack
  - access to the most recently added item only

- Operations are limited:
  - push (add item to stack)
  - pop (remove top item from stack)
  - top (get top item without removing it)
  - isEmpty

- Described as a "Last In First Out" (LIFO) data structure
Stack Operations

Assume a simple stack for integers.

```java
Stack<Integer> s = new Stack<Integer>();
s.push(12);
s.push(4);
s.push(s.top() + 2);
s.pop();
s.pop();
s.push(s.top());
```

//what are contents of stack?
Stack Operations

Write a method to print out contents of stack in reverse order.
Uses of Stacks

- The runtime stack used by a process (running program) to keep track of methods in progress
- Search problems
- Undo, redo, back, forward
Clicker 1 - What is Output?

Stack<Integer> s = new Stack<>();
// put stuff in stack
for(int i = 0; i < 5; i++)
    s.push(i);
// Print out contents of stack
// while emptying it.
// Assume there is a size method.
for(int i = 0; i < s.size(); i++)
    System.out.print(s.pop() + " ");

A 0 1 2 3 4  D 2 3 4
B 4 3 2 1 0  E  No output due
to runtime error
C 4 3 2
Stack<Integer> s = new Stack<Integer>();
// put stuff in stack
for(int i = 0; i < 5; i++)
    s.push(i);
// print out contents of stack
// while emptying it
int limit = s.size();
for(int i = 0; i < limit; i++)
    System.out.print(s.pop() + " ");
// or
// while(!s.isEmpty())
//    System.out.println(s.pop());
Implementing a stack

- need an underlying collection to hold the elements of the stack
- 2 obvious choices
  - native array
  - a list!!!
- Adding a *layer of abstraction*. A HUGE idea.
- array implementation
- linked list implementation

https://xkcd.com/2347/
Applications of Stacks
Mathematical Calculations

- What does $3 + 2 \times 4$ equal? $2 \times 4 + 3$? $3 \times 2 + 4$?
- The precedence of operators affects the order of operations.
- A mathematical expression cannot simply be evaluated left to right.
- A challenge when evaluating a program.
- *Lexical analysis* is the process of interpreting a program.

What about $1 - 2 - 4 ^ 5 \times 3 \times 6 / 7 ^ 2 ^ 3$
Infix and Postfix Expressions

- The way we are use to writing expressions is known as infix notation
- Postfix expression does not require any precedence rules
- $3 \ 2 \ * \ 1 \ +$ is postfix of $3 \ * \ 2 \ + \ 1$
- evaluate the following postfix expressions and write out a corresponding infix expression:
  
  1 2 3 4 ^ * + 
  2 3 2 4 * + * 
  1 2 - 3 2 ^ 3 * 6 / + 
  2 5 ^ 1 -
Clicker Question 2

What does the following postfix expression evaluate to?

6 3 2 + *

A. 11  
B. 18  
C. 24  
D. 30  
E. 36
Evaluation of Postfix Expressions

- Easy to do with a stack
- given a proper postfix expression:
  - get the next token
  - if it is an operand push it onto the stack
  - else if it is an operator
    - pop the stack for the right hand operand
    - pop the stack for the left hand operand
    - apply the operator to the two operands
    - push the result onto the stack
  - when the expression has been exhausted the result is the top (and only element) of the stack
Convert the following equations from infix to postfix:

2 ^ 3 ^ 3 + 5 * 1
11 + 2 - 1 * 3 / 3 + 2 ^ 2 / 3

Problems:
- Negative numbers?
- Parentheses in expression
Infix to Postfix Conversion

- Requires operator precedence parsing algorithm
  - parse v. To determine the syntactic structure of a sentence or other utterance

Operands: add to expression
Close parenthesis: pop stack symbols until an open parenthesis appears

Operators:
  - Have an on stack and off stack precedence
  - Pop all stack symbols until a symbol of lower precedence appears. Then push the operator

End of input: Pop all remaining stack symbols and add to the expression
Simple Example

Infix Expression: 3 + 2 * 4

PostFix Expression:

Operator Stack:

Precedence Table

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Simple Example

Infix Expression: \(+ 2 * 4\)

PostFix Expression: \(3\)

Operator Stack:

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Simple Example

Infix Expression: $2 \times 4$

PostFix Expression: $3$

Operator Stack: $+$

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Simple Example

Infix Expression: * 4
PostFix Expression: 3 2
Operator Stack: +

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Simple Example

Infix Expression:  4
PostFix Expression:  3 2
Operator Stack:  + *

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Simple Example

Infix Expression:

PostFix Expression: 3 2 4

Operator Stack: + *

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Simple Example

Infix Expression:
PostFix Expression: \[ 3 \ 2 \ 4 \ * \]
Operator Stack: \[ + \]

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Simple Example

Infix Expression:

PostFix Expression: 3 2 4 * +

Operator Stack:

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Example

$$11 + 2^4^3 - ((4 + 5) \times 6)^2$$

Show algorithm in action on above equation
Balanced Symbol Checking

- In processing programs and working with computer languages there are many instances when symbols must be balanced: { }, [ ], ( )

  A stack is useful for checking symbol balance. When a closing symbol is found it must match the most recent opening symbol of the same type.

- Applicable to checking html and xml tags!
Algorithm for Balanced Symbol Checking

- Make an empty stack
- read symbols until end of file
  - if the symbol is an opening symbol push it onto the stack
  - if it is a closing symbol do the following
    - if the stack is empty report an error
    - otherwise pop the stack. If the symbol popped does not match the closing symbol report an error
- At the end of the file if the stack is not empty report an error
Algorithm in practice

- list[i] = 3 * (44 - method(foo(list[2 * (i + 1) + foo(list[i - 1])]) / 2) - list[method(list[0])];

- Complications
  - when is it not an error to have non matching symbols?

- Processing a file
  - *Tokenization*: the process of scanning an input stream. Each independent chunk is a token.

- Tokens may be made up of 1 or more characters