Topic 19
Binary Search Trees

"Yes. Shrubberies are my trade. I am a shrubber. My name is 'Roger the Shrubber'. I arrange, design, and sell shrubberies."

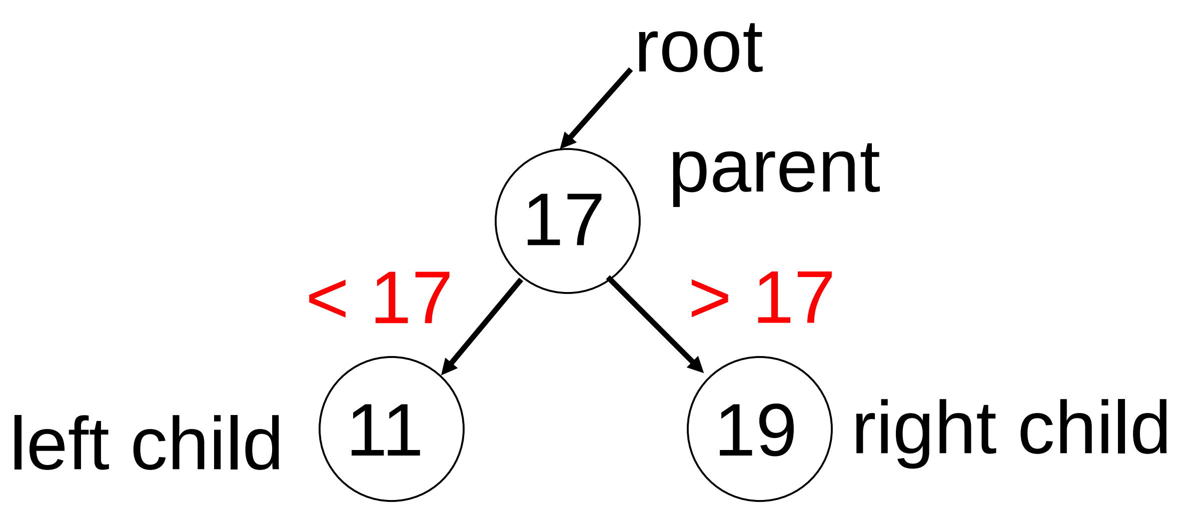
-Monty Python and The Holy Grail

The Problem with Linked Lists

- Accessing a item from a linked list takes O(N) time for an arbitrary element
- Binary trees can improve upon this and reduce access to O(log N) time for the average case
- Expands on the binary search technique and allows insertions and deletions
- Worst case degenerates to O(N) but this can be avoided by using balanced trees (AVL, Red-Black)

Binary Search Trees

- A binary search tree is a binary tree in which **every node's** left subtree holds values less than the node's value, and every right subtree holds values greater than the node's value.
- A new node is added as a leaf.



BST Insertion

- Add the following values one at a time to an initially empty binary search tree using the simple algorithm:

50 90 20 78 10 20 28 -25

- What is the resulting tree?

Traversals

- What is the result of an inorder traversal of the resulting tree?
- How could a preorder traversal be useful?

Clicker 1

- After adding N distinct elements in random order to a Binary Search Tree what is the expected height of the tree? (using the simple insertion algorithm)

A. $O(\log N)$
B. $O(N^{1/2})$
C. $O(N)$
D. $O(N\log N)$
E. $O(N^2)$

Clicker 2

- After adding N distinct elements to a Binary Search Tree what is the **worst case** height of the tree? (using the simple insertion algorithm)

A. $O(\log N)$

B. $O(N^{1/2})$

C. $O(N)$

D. $O(N\log N)$

E. $O(N^2)$ `

Worst Case Performance

- Insert the following values into an initially empty binary search tree using the simple, naïve algorithm:

2 3 5 7 11 13 17

- What is the height of the tree?
- What is the worst case height of a BST?

Node for Binary Search Trees

```
public class BSTNode<E extends Comparable<E> {
        private Comparable<E> myData;
        private BSTNode<E> myLeft;
        private BSTNode<E> myRightC;

        public BinaryNode(E item)
        {       myData = item;      }

        public E getValue()
        {       return myData;      }

        public BinaryNode<E> getLeft()
        {       return myLeft;      }

        public BinaryNode<E> getRight()
        {       return myRight;     }

        public void setLeft(BSTNode<E> b)
        {       myLeft = b; }
        // setRight not shown
}
```

More on Implementation

- Many ways to implement BSTs
- Using nodes is just one and even then many options and choices

```
public class BinarySearchTree<E extends Comparable<E>> {

      private BSTNode<E> root;
      private int size;
```

Add an Element, Recursive

Add an Element, Iterative

Clicker 3

- What are the best case and worst case order to add N distinct elements, one at a time, to an initially empty binary search tree using the simple add algorithm?

	Best	Worst
A.	O(N)	O(N)
B.	O(NlogN)	O(NlogN)
C.	O(N)	O(NlogN)
D.	O(NlogN)	$O(N^2)$
E.	O(N)	$O(N^2)$

```
// given int[] data
// no duplicates in
// data
BST<Integer> b =
  new BST<Integer>();
for(int x : data)
  b.add(x);
```

Performance of Binary Trees

- For the three core operations (add, access, remove) a binary search tree (BST) has an average case performance of O(log N)
- Even when using the *naïve insertion / removal algorithms*
  – no checks to maintain balance
  – balance achieved based on the randomness of the data inserted

Remove an Element

- Five (four?) cases
  - not present
  - node is a leaf, 0 children (easy)
  - node has 1 child, left or right (easy)
  - node has 2 children ("interesting")

Properties of a BST

- The minimum value is in the left most node
- The maximum value is in the right most node
  - useful when removing an element from the BST

Alternate Implementation

- In class examples of dynamic data structures have relied on *null terminated ends.*
  - Use null to show end of list or no children
- Alternative form
  - use structural recursion and polymorphism

BST Interface

```
public interface BST<E extends
                        Comparable<? super E>> {

    public int size();
    public boolean contains(E obj);
    public BST<E> add(E obj);
}
```

EmptyBST

```
public class EmptyBST<E extends Comparable<? super E>>
      implements BST<E> {

   private static final EmptyBST theOne = new EmptyBST();

   private EmptyBST() {}

   public static EmptyBST getEmptyBST(){ return theOne; }

   public  BST<E> add(E obj) { return new NEBST(obj); }

   public boolean contains(E obj) { return false; }

   public int size() { return 0; }
}
```

Non Empty BST – Part 1

```
public class NEBST <E extends Comparable<? super E>> implements BST<E> {

   private E data;
   private BST left;
   private BST right;

   public NEBST(E d) {
      data = d;
      right = EmptyBST.getEmptyBST();
      left = EmptyBST.getEmptyBST();
   }

   public BST add(E obj)  {
      int direction = obj.compareTo( data );
      if ( direction < 0 )
        left = left.add( obj );
      else if ( direction > 0 )
        right = right.add ( obj );
      return this;
   }
```

Non Empty BST – Part 2

```
public boolean contains(E obj){
        int dir = obj.compareTo(data);
        if( dir == 0 )
            return true;
        else if (dir < 0)
            return left.contains(obj);
        else
            return right.contains(obj);
    }

    public int size() {
        return 1 + left.size() + right.size();
    }
}
```