Graphs
Topic 21

" Hopefully, you've played around a bit with The Oracle of Bacon at Virginia and discovered how few steps are necessary to link just about anybody who has ever been in a movie to Kevin Bacon, but could there be some actor or actress who is even closer to the center of the Hollywood universe?.

By processing all of the almost half of a million people in the Internet Movie Database I discovered that there are currently 1160 people who are *better* centers than Kevin Bacon! … By computing the average of these numbers we see that the average (Sean) Connery Number is about 2.682 making Connery a better center than Bacon"

-**Who is the Center of the Hollywood Universe?,**
University of Virginia

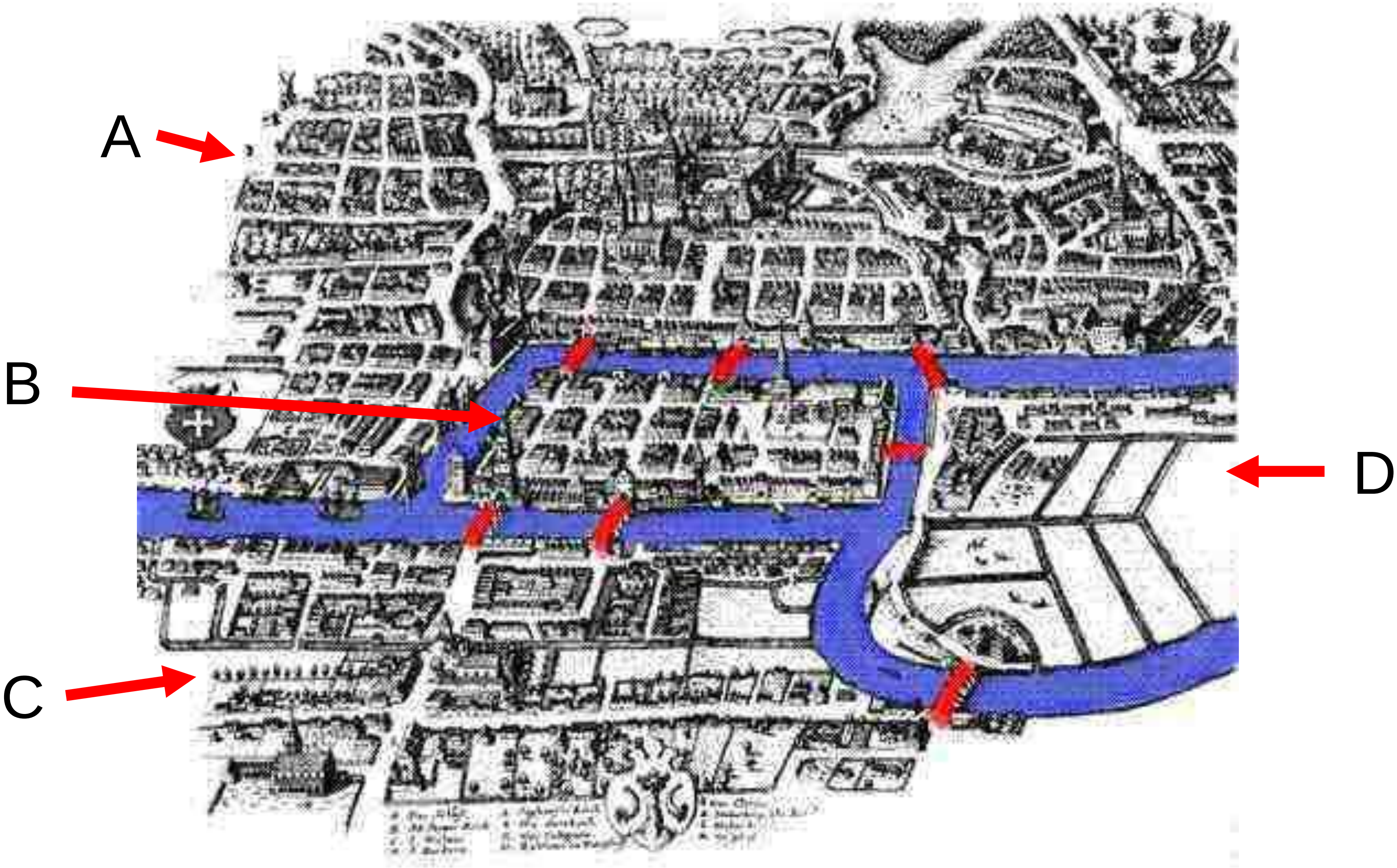
That was in 2001.

In 2013 Harvey Keitel has become the center of the Hollywood Universe. Connery is 136th.
Bacon has moved up to 370$^{th.}$

An Early Problem in Graph Theory

- Leonhard Euler (1707 - 1783)
  - One of the first mathematicians to study graphs
- The Seven Bridges of Konigsberg Problem
  - Konigsberg is now called Kaliningrad
- A puzzle for the residents of the city
- The river Pregel flows through the city
- 7 bridges crossed the river
- Can you cross all bridges while crossing each bridge only once? *An Eulerian Circuit*

Konigsberg and the River Pregel



Clicker 1

- How many solutions does the Seven Bridges of Konigsberg Problem have?

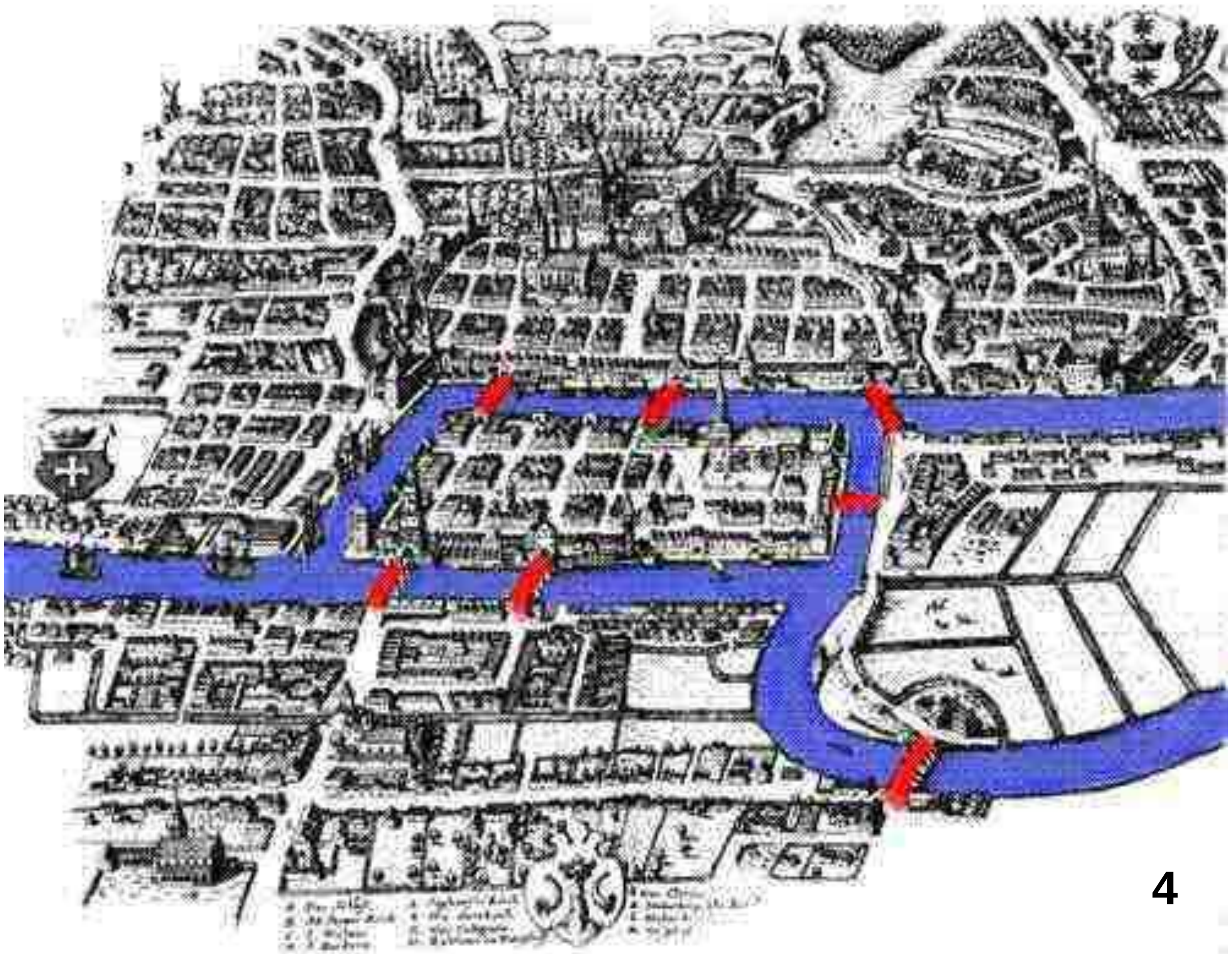
A. 0

B. 1

C. 2

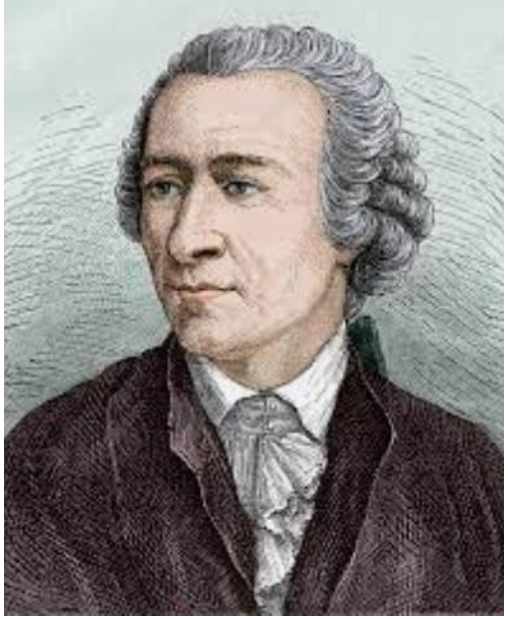
D. 3

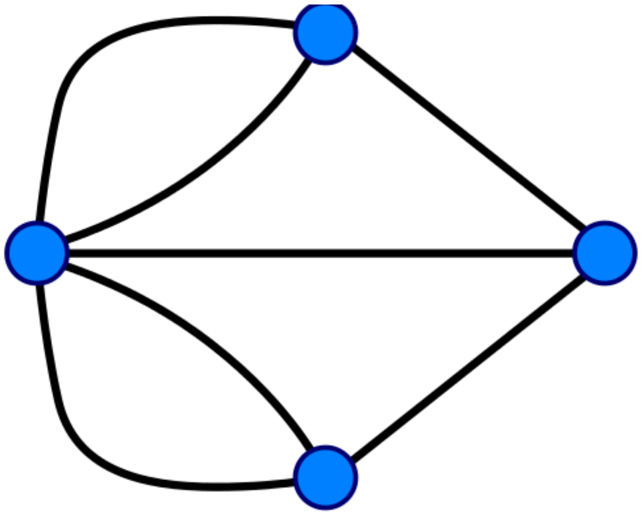
E. >= 4

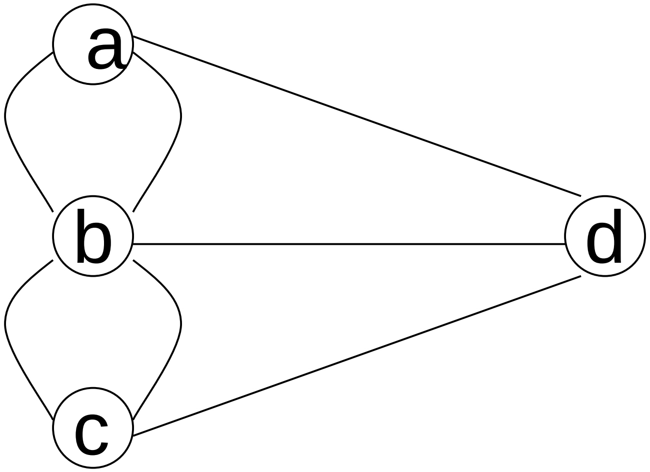


How to Solve

- Brute Force?
- Euler's Solution
  - Redraw the map as a graph (really a *multigraph as opposed to a simple graph, 1 or 0 edges per pair of vertices*)







Euler's Proposal

- A connected graph has an Euler tour (cross every edge exactly one time and end up at starting node) if and only if every vertex has an even number of edges
  - *Eulerian Circuit*
- Clicker 2 - What if we reduce the problem to only crossing each edge (bridge) exactly once?
  - Doesn't matter if we end up where we started
  - *Eulerian Trail*

A. 0 B. 1 C. 2 D. 3 E. >= 4

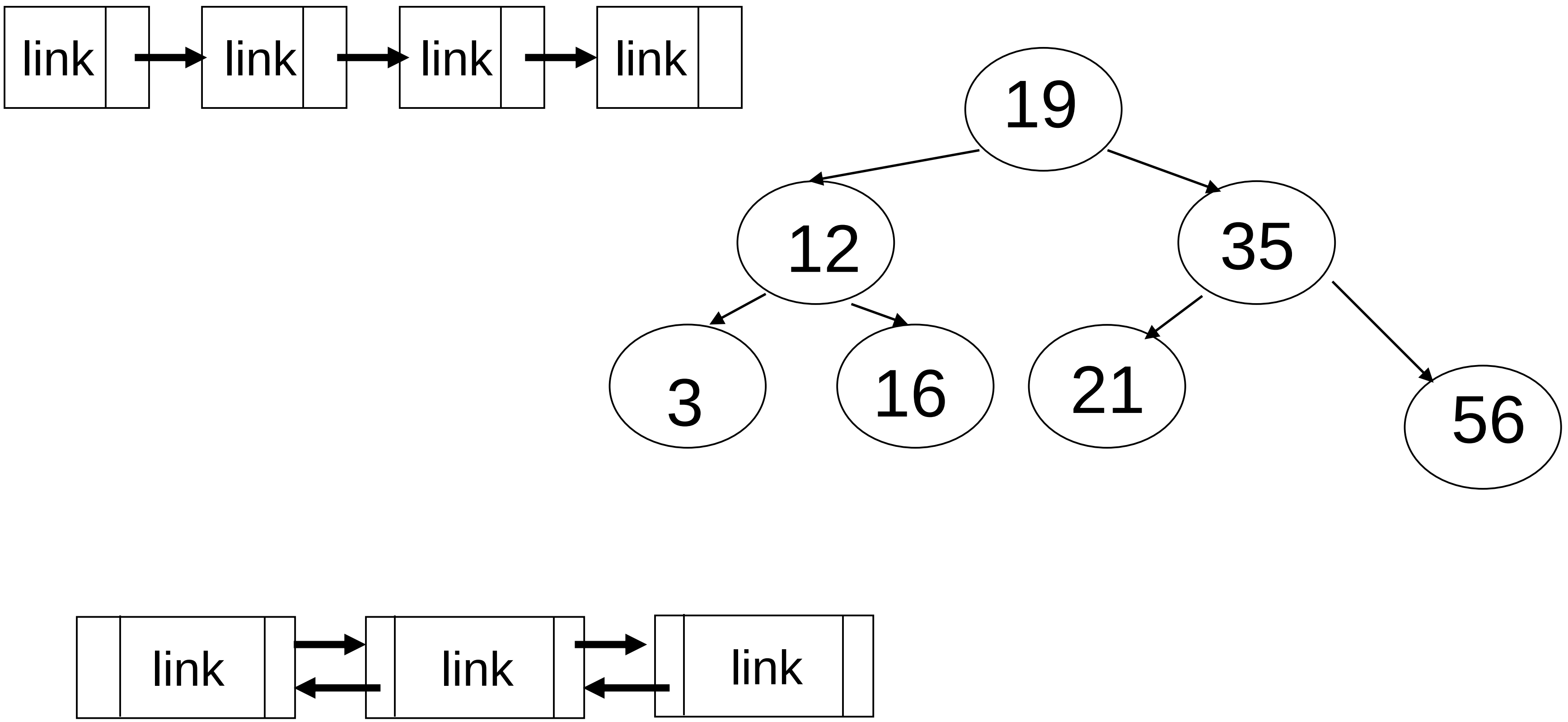
Graph Definitions

- A graph is comprised of a set of *vertices* (nodes) and a set of *edges* (links, arcs) connecting the vertices
  - An edge connects 2 vertices
- in a *directed* graph edges are one-way
  - movement allowed from first node to second, but not second to first
  - directed graphs also called *digraphs*
- in an *undirected* graph edges are two-way
  - movement allowed in either direction

Definitions

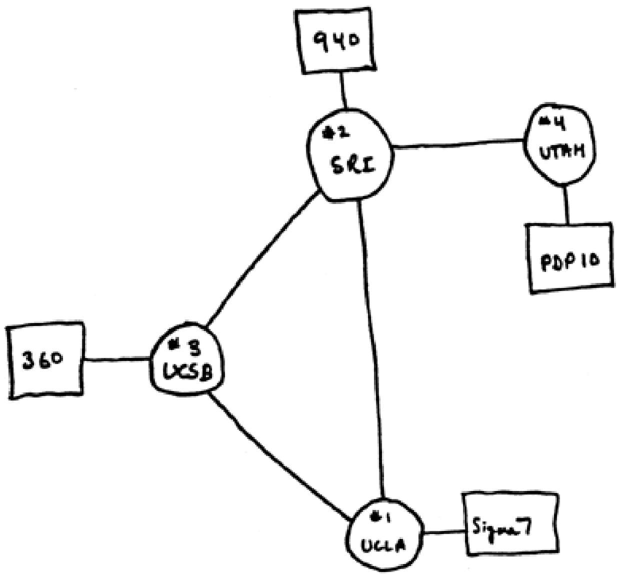
- In a *weighted* graph the edge has cost or weight that measures the cost of traveling along the edge
- A *path* is a sequence of vertices connected by edges
  – The *path length* is the number of edges
  – The *weighted* path length is the sum of the cost of the edges in a path
- A *cycle* is a path of length 1 or more that starts and ends at the same vertex without repeating any other vertices
  – a *directed acyclic graph* is a directed graph with no cycles

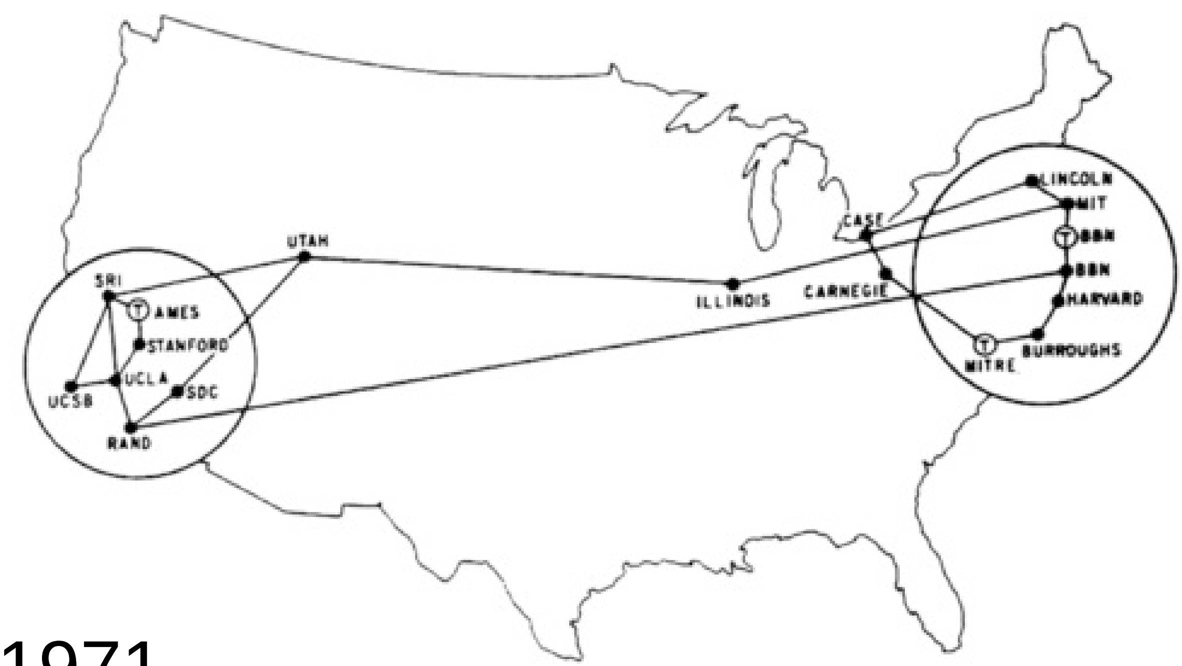
Graphs We've Seen



Example Graph

- Scientists (and academics of ALL kinds) use graphs to model all kinds of things.

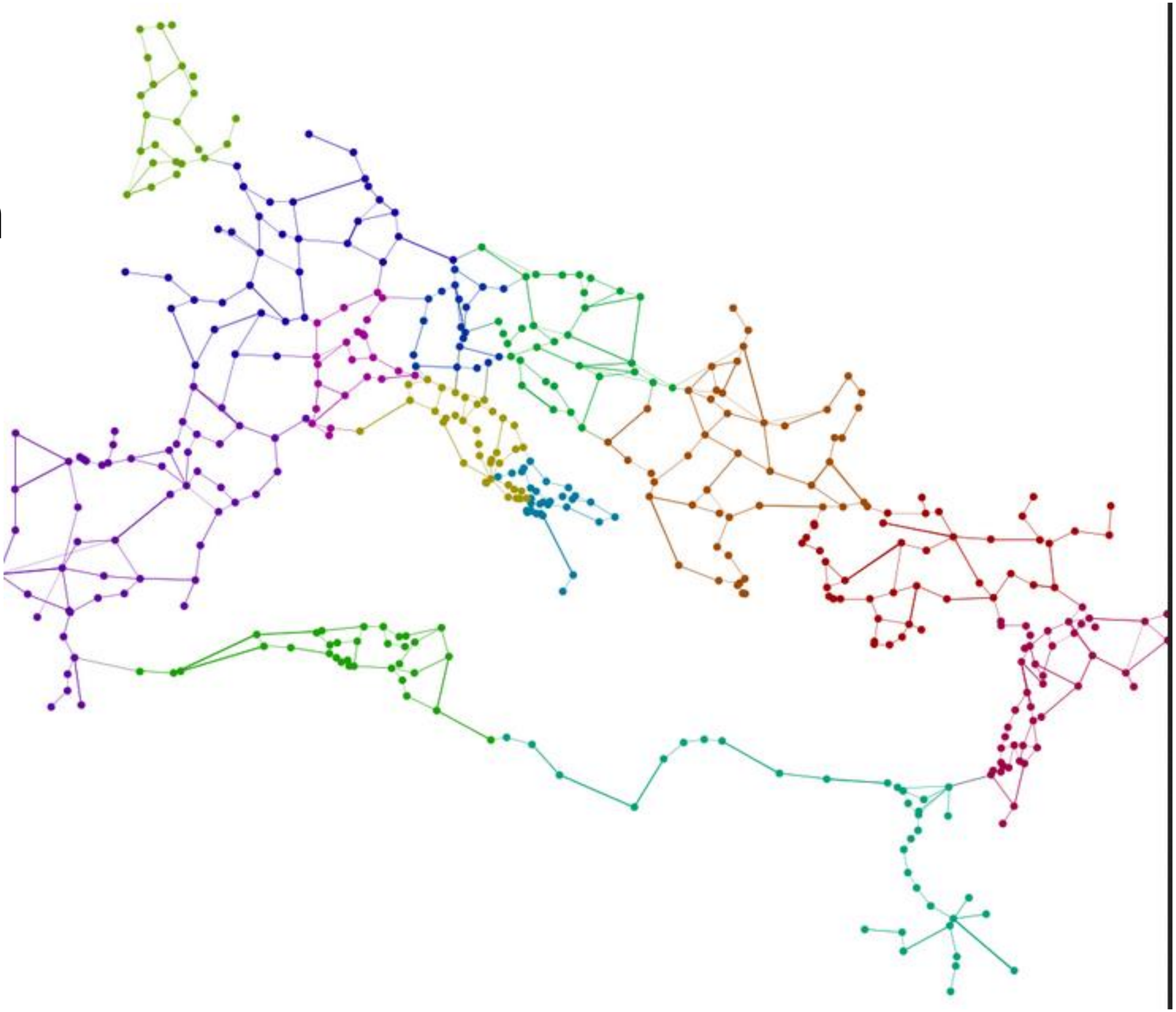




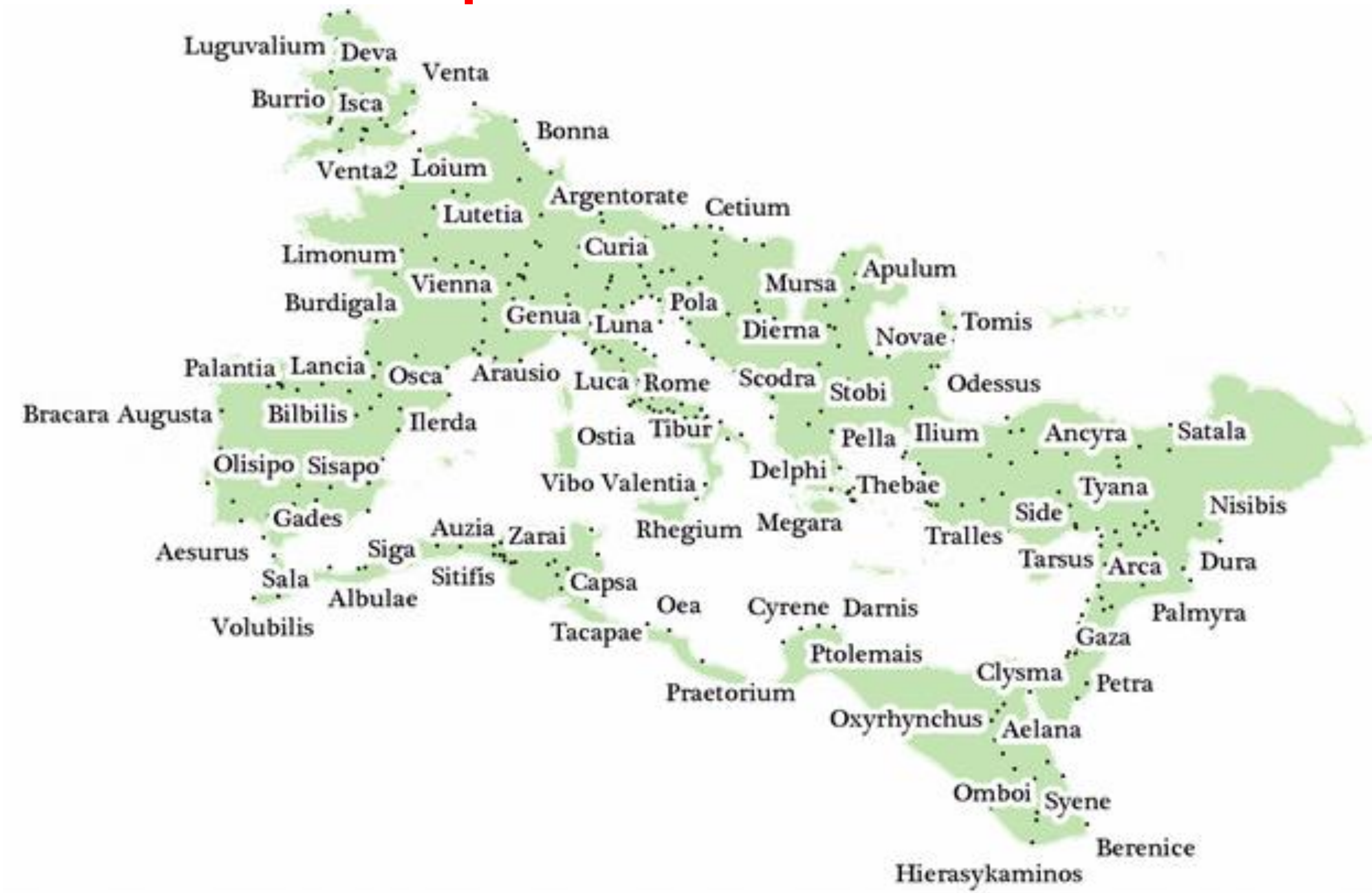
Arpanet 1969, 1971

Example Graph

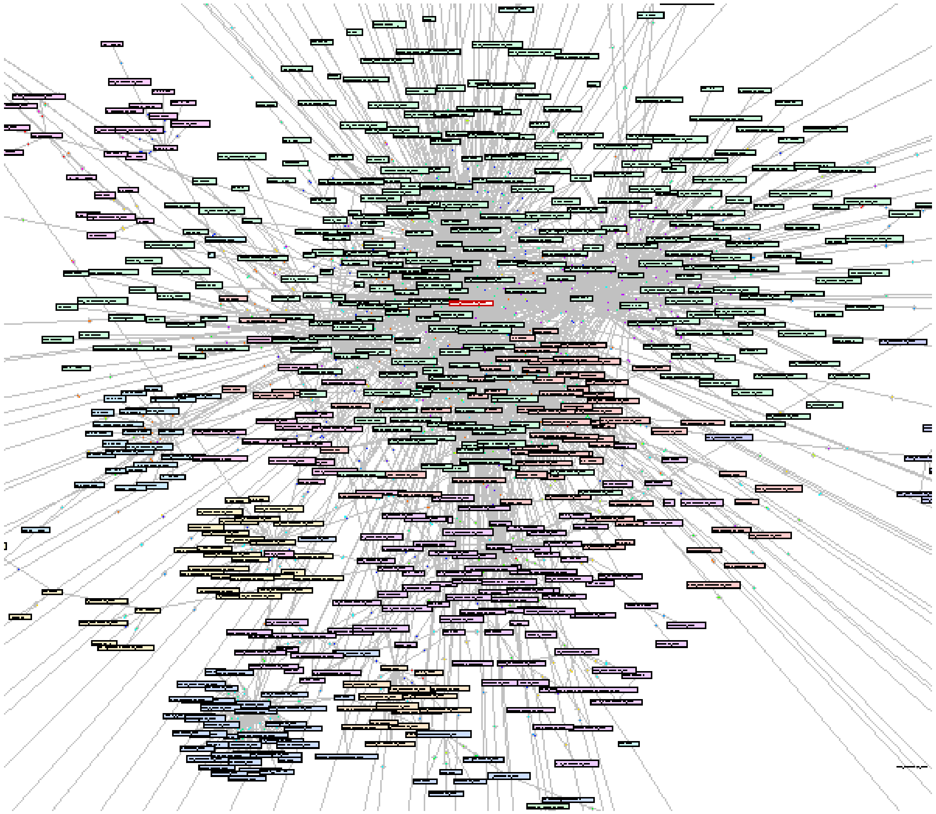
Roman
Transportation
Network



Roman Transportation Network

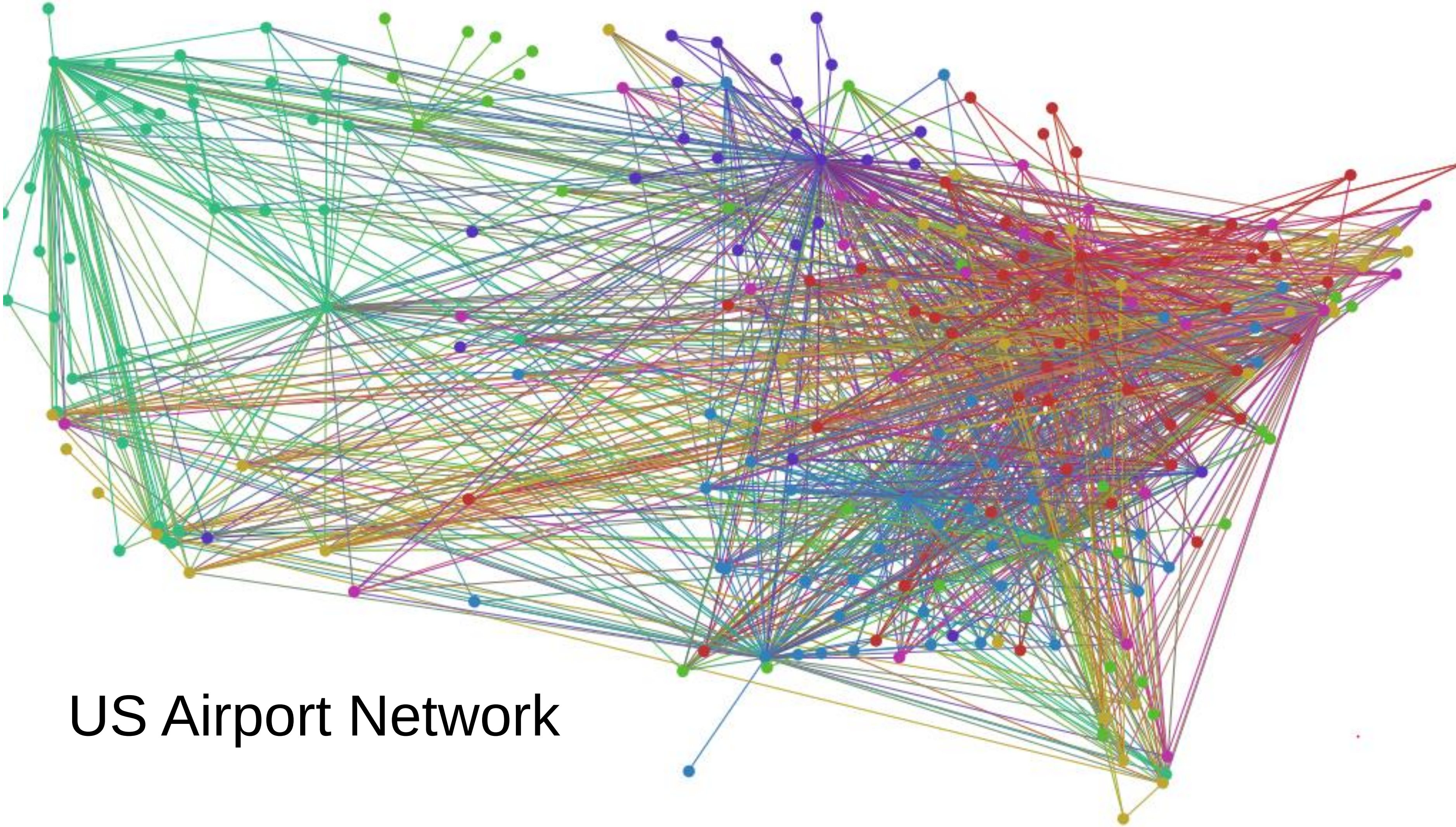


Example Graph



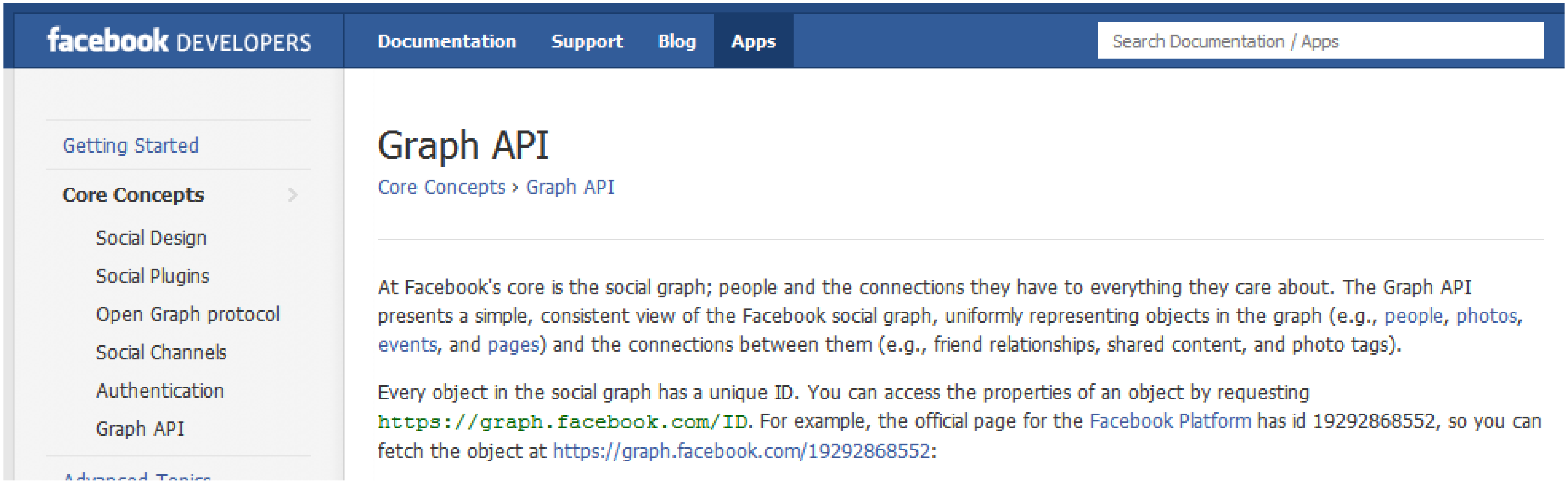
Enron emails 2001

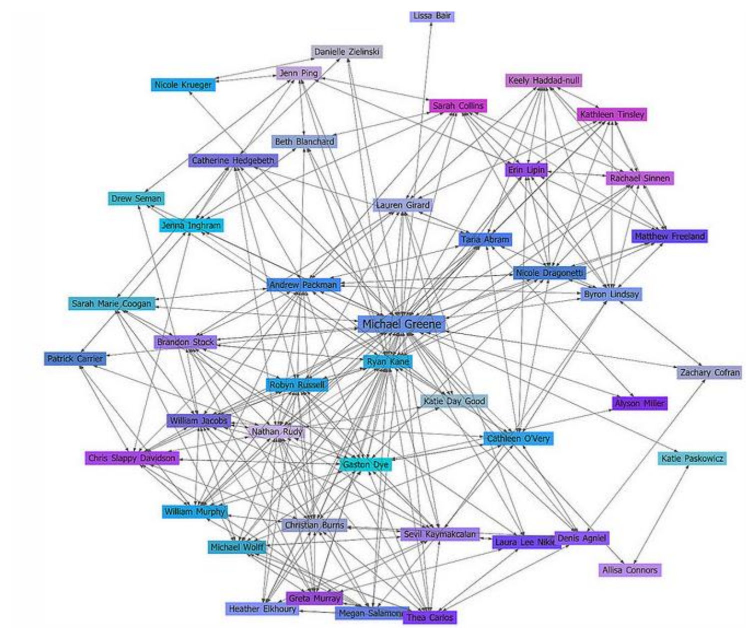
Example Graph



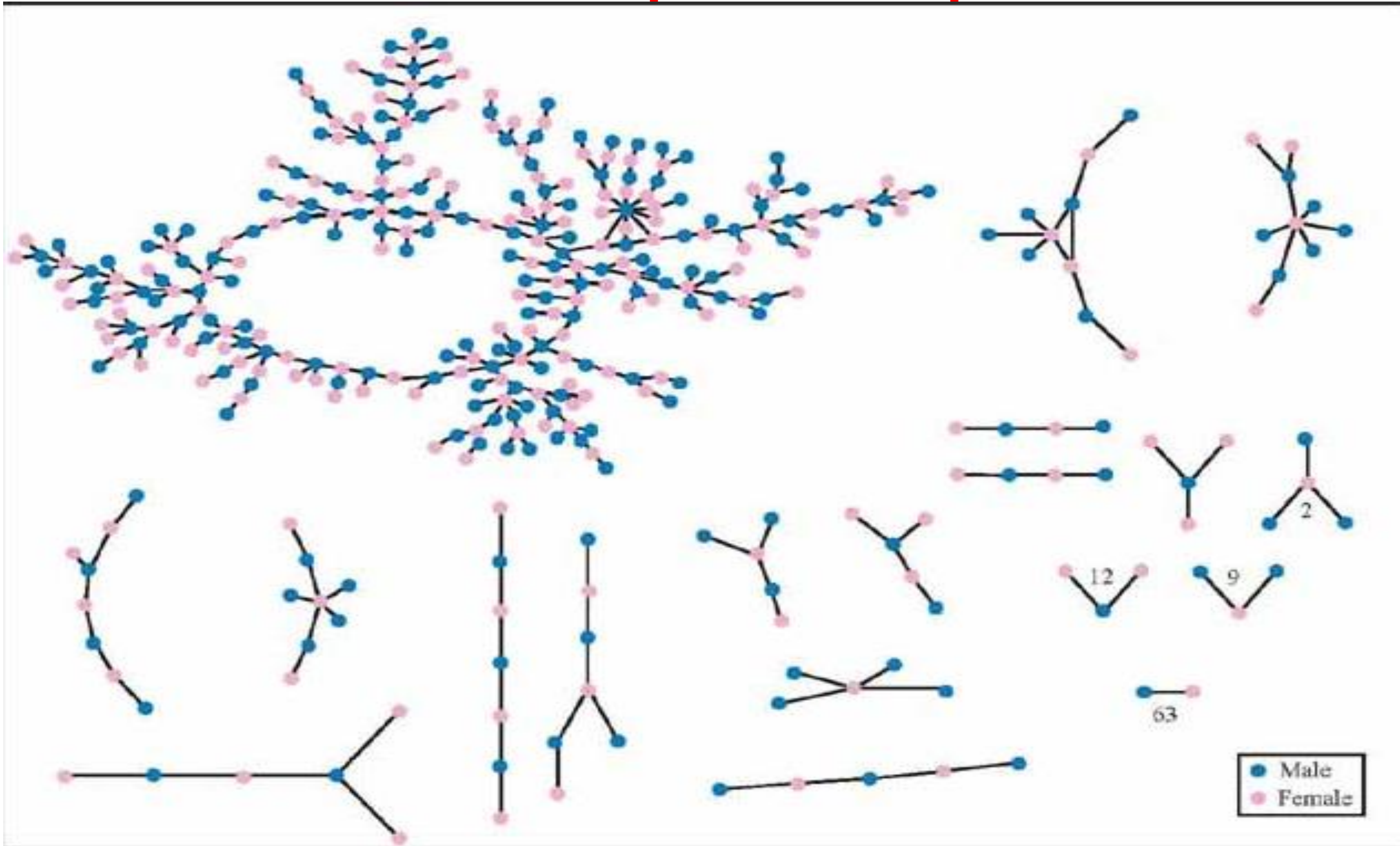
US Airport Network

Example Graph





Example Graph



"Jefferson" High School, Ohio Chains of Affection: The Structure of Adolescent Romantic and Sexual Networks, 2005,

Representing Graphs

- How to store a graph as a data structure?
-

Adjacency Matrix Representation

	A	Br	Bl	Ch	Co	E	FG	G	Pa	Pe	S	U	V
A	0	1	1	1	0	0	0	0	1	0	0	1	0
Br	1	0	1	0	1	0	1	1	1	1	1	1	1
Bl	1	1	0	1	0	0	0	0	1	1	0	0	0
Ch	1	0	1	0	0	0	0	0	0	1	0	0	0
Co	0	1	0	0	0	1	0	0	0	1	0	0	1
E	0	0	0	0	1	0	0	0	0	1	0	0	0
FG	0	1	0	0	0	0	0	0	0	0	1	0	0
G	0	1	0	0	0	0	0	0	0	0	1	0	1
Pa	1	1	1	0	0	0	0	0	0	0	0	0	0
Pe	0	1	1	1	1	1	0	0	0	0	0	0	0
S	0	1	0	0	0	0	1	1	0	0	0	0	0
U	1	1	0	0	0	0	0	0	0	0	0	0	0
V	0	1	0	0	1	0	0	1	0	0	0	0	0

Country	Code
Argentina	A
Brazil	Br
Bolivia	Bl
Chile	Ch
Colombia	Co
Ecuador	E
French Guiana	FG
Guyana	G
Paraguay	Pa
Peru	Pe
Suriname	S
Uruguay	U
Venezuela	V

Undirected Graph?

- Use a ragged 2d array to save space

NORTH ISLAND DISTANCE CHART

	AUCKLAND	CAPE REINGA	DARGAVILLE	GISBORNE	HAMILTON	HICKS BAY	KAITAIA	MASTERTON	NAPIER	NEW PLYMOUTH	PAIHIA	PALMERSTON NORTH	ROTORUA	TAUPO	TAUMARUNUI	TAURANGA	THAMES	WAIKAREMOANA	WAITOMO	WHANGANUI	WELLINGTON	WHAKAPAPA VILLAGE	WHAKATANE
CAPE REINGA	436																						
DARGAVILLE	180	285																					
GISBORNE	499	935	684																				
HAMILTON	126	562	306	394																			
HICKS BAY	503	939	683	180	398																		
KAITAIA	320	116	169	819	446	823																	
MASTERTON	646	1082	826	448	520	628	966																
NAPIER	421	857	601	215	295	395	744	233															
NEW PLYMOUTH	357	797	537	585	231	739	677	343	412														
PAIHIA	236	223	129	735	362	739	107	882	657	593													
PALMERSTON NORTH	537	973	717	393	411	573	857	109	178	234	773												
ROTORUA	234	670	414	286	108	290	554	448	223	299	470	339											
TAUPO	278	714	458	332	152	370	598	368	143	296	514	259	80										
TAUMARUNUI	286	722	466	449	160	487	606	348	260	183	522	239	172	117									
TAURANGA	205	641	385	298	106	302	525	524	299	308	441	415	86	156	235								
THAMES	114	550	294	410	108	414	434	574	349	339	350	465	164	206	268	116							
WAIKAREMOANA	390	826	570	162	264	342	710	416	183	482	626	361	156	186	303	242	320						
WAITOMO	200	636	380	445	74	449	520	450	306	173	436	341	166	163	102	151	182	322					
WHANGANUI	457	894	637	467	331	643	777	183	252	160	693	74	309	229	171	439	479	435	273				
WELLINGTON	658	1094	838	538	532	718	978	102	323	355	894	145	460	380	371	546	586	506	473	195			
WHAKAPAPA VILLAGE	345	780	525	429	219	467	665	335	240	242	581	223	177	97	59	236	303	283	159	141	344		
WHAKATANE	298	737	478	201	193	205	618	533	308	384	534	424	85	165	257	97	209	241	235	358	545	262	
WHANGAREI	165	271	58	664	291	668	155	811	586	522	71	702	400	443	451	370	279	555	365	622	823	510	463

At a Glance

Auckland to Paihia [236 km; 3 hr, 20 min]

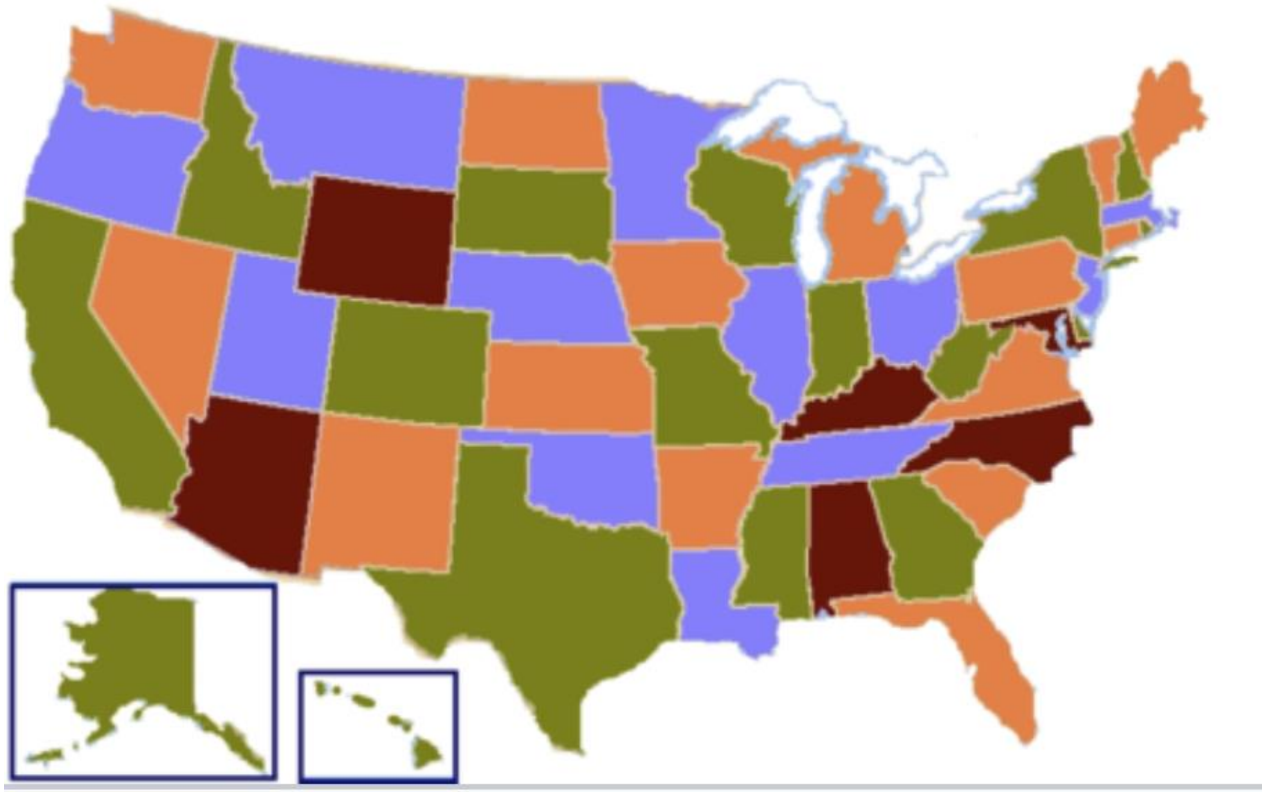
Auckland to Rotorua [234 km; 3 hr, 20 min]

Auckland to Wellington [658 km; 9 hr, 25 min]

The Map Coloring Problem

- How many colors do you need to color a map, so that no 2 countries that have a common border (not a point) are colored the same?
- How to solve using Brute Force?

Example



A four-coloring of a map of the states of the United States (ignoring lakes).

Source: https://en.wikipedia.org/wiki/Four_color_theorem

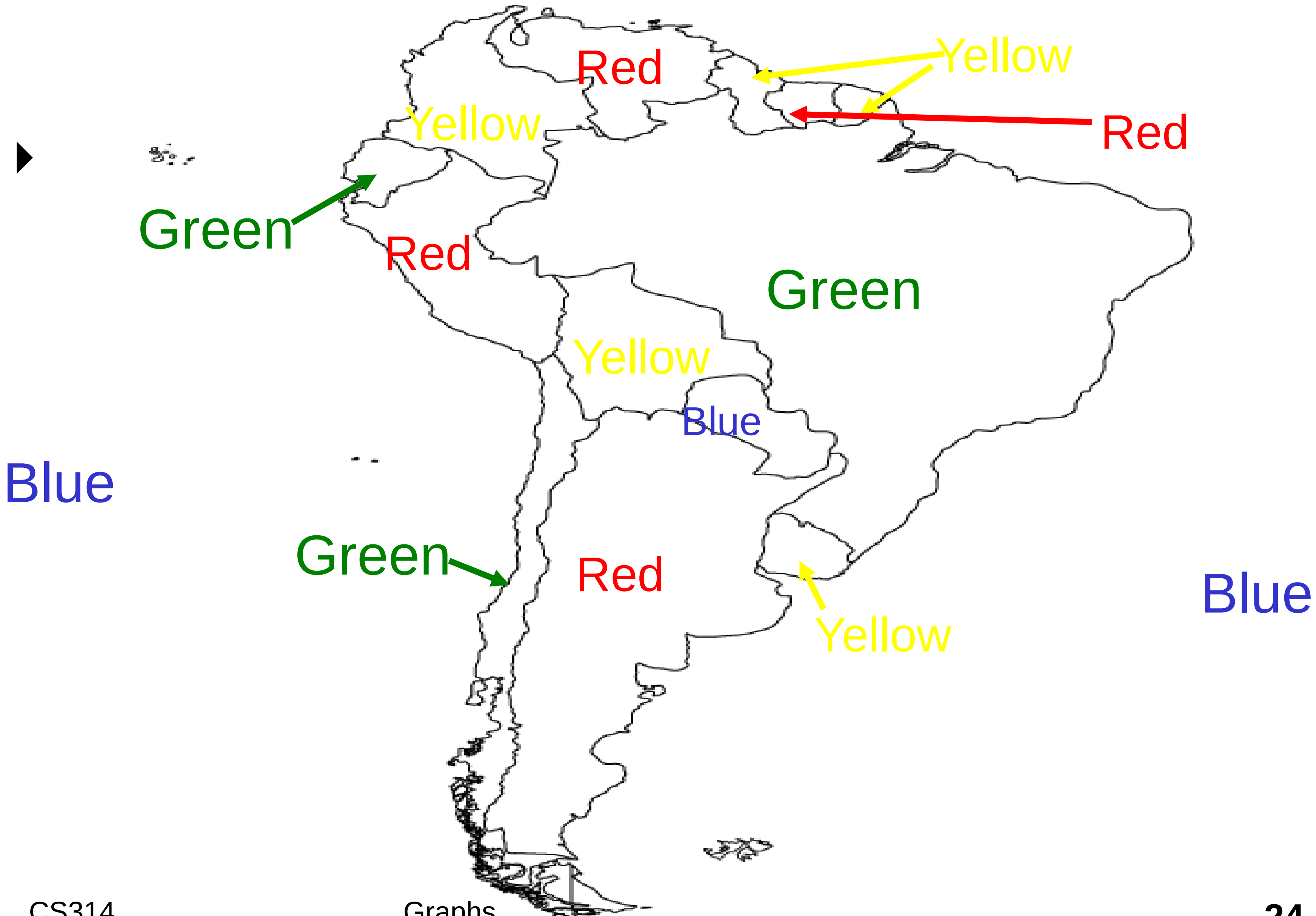
A Solution


Blue
Blue
Yellow
Yellow
Green
Blue
Green
Yellow
Red
Green
Blue
Yellow

What About the Ocean?

	A	Br	Bl	Ch	Co	E	FG	G	Pa	Pe	S	U	V	Oc
A	0	1	1	1	0	0	0	0	1	0	0	1	0	1
Br	1	0	1	0	1	0	1	1	1	1	1	1	1	1
Bl	1	1	0	1	0	0	0	0	1	1	0	0	0	0
Ch	1	0	1	0	0	0	0	0	0	1	0	0	0	1
Co	0	1	0	0	0	1	0	0	0	1	0	0	1	1
E	0	0	0	0	1	0	0	0	0	1	0	0	0	1
FG	0	1	0	0	0	0	0	0	0	0	1	0	0	1
G	0	1	0	0	0	0	0	0	0	0	1	0	1	1
Pa	1	1	1	0	0	0	0	0	0	0	0	0	0	0
Pe	0	1	1	1	1	1	0	0	0	0	0	0	0	1
S	0	1	0	0	0	0	1	1	0	0	0	0	0	1
U	1	1	0	0	0	0	0	0	0	0	0	0	0	1
V	0	1	0	0	1	0	0	1	0	0	0	0	0	1
Oc	1	1	0	1	1	1	1	1	0	1	1	1	1	0

Make the Ocean Blue



More Definitions

- A *dense* graph is one with a "large" number of edges
  - maximum number of edges?
- A "*sparse"* graph is one in which the number of edges is "much less" than the maximum possible number of edges
  - No standard cutoff for dense and sparse graphs

Graph Representation

- For dense graphs the adjacency matrix is a reasonable choice
  - For weighted graphs change booleans to double or int
  - Can the adjacency matrix handle directed graphs?
- Most graphs are sparse, not dense
- For sparse graphs an *adjacency list* is an alternative that uses less space
- Each vertex keeps a list of edges to the vertices it is connected to.

Graph Implementation

```
public class Graph
    private static final double INFINITY
                              = Double.MAX_VALUE;
    private Map<String, Vertex> vertices;

    public Graph() // create empty Graph

    public void addEdge(String source,
                 String dest, double cost)

    // find all paths from given vertex
    public void findUnweightedShortestPaths
                      (String startName)

    // called after findUnweightedShortestPath
    public void printPath(String destName)
```

Graph Class

- This Graph class stores vertices
- Each vertex has an adjacency list
  - what vertices does it connect to?
- shortest path method finds all paths from start vertex to every other vertex in graph
- after shortest path method called queries can be made for path length from start node to destination node

Vertex Class (nested in Graph)

```
private static class Vertex
     private String name;
     private List<Edge> adjacent;

     public Vertex(String n)

     // for shortest path algorithms
     private double distance;
     private Vertex prev;
     private int scratch;

     // call before finding new paths
     public void reset()
```

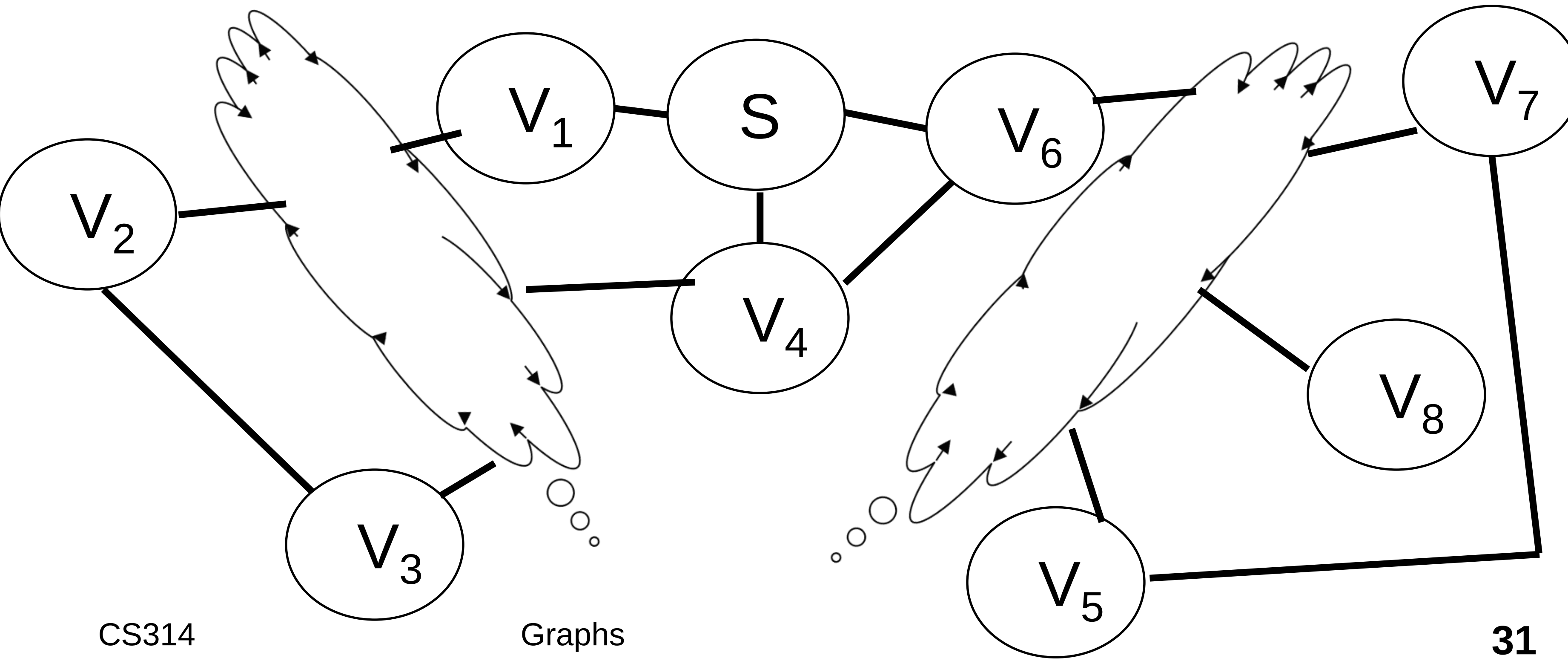
Edge Class (nested in Graph)

```
private static class Edge
    private Vertex dest;
    private double cost;

    private Edge(Vertex d, double c)
```

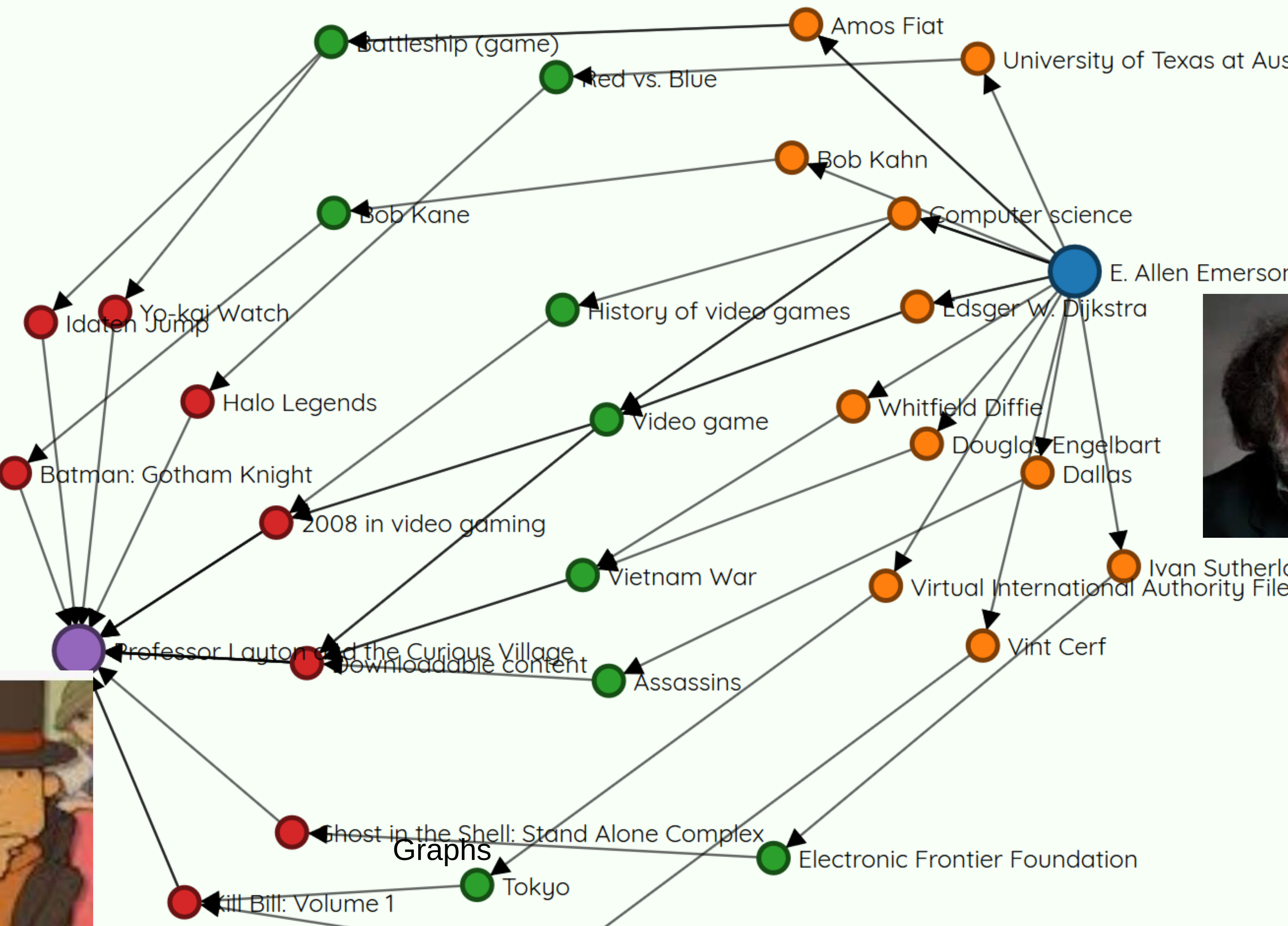
Unweighted Shortest Path

- Given a vertex, S (for start) find the shortest path from S to all other vertices in the graph
- Graph is unweighted (set all edge costs to 1)



6 Degrees of Wikipedia

- https://www.sixdegreesofwikipedia.com/



Word Ladders

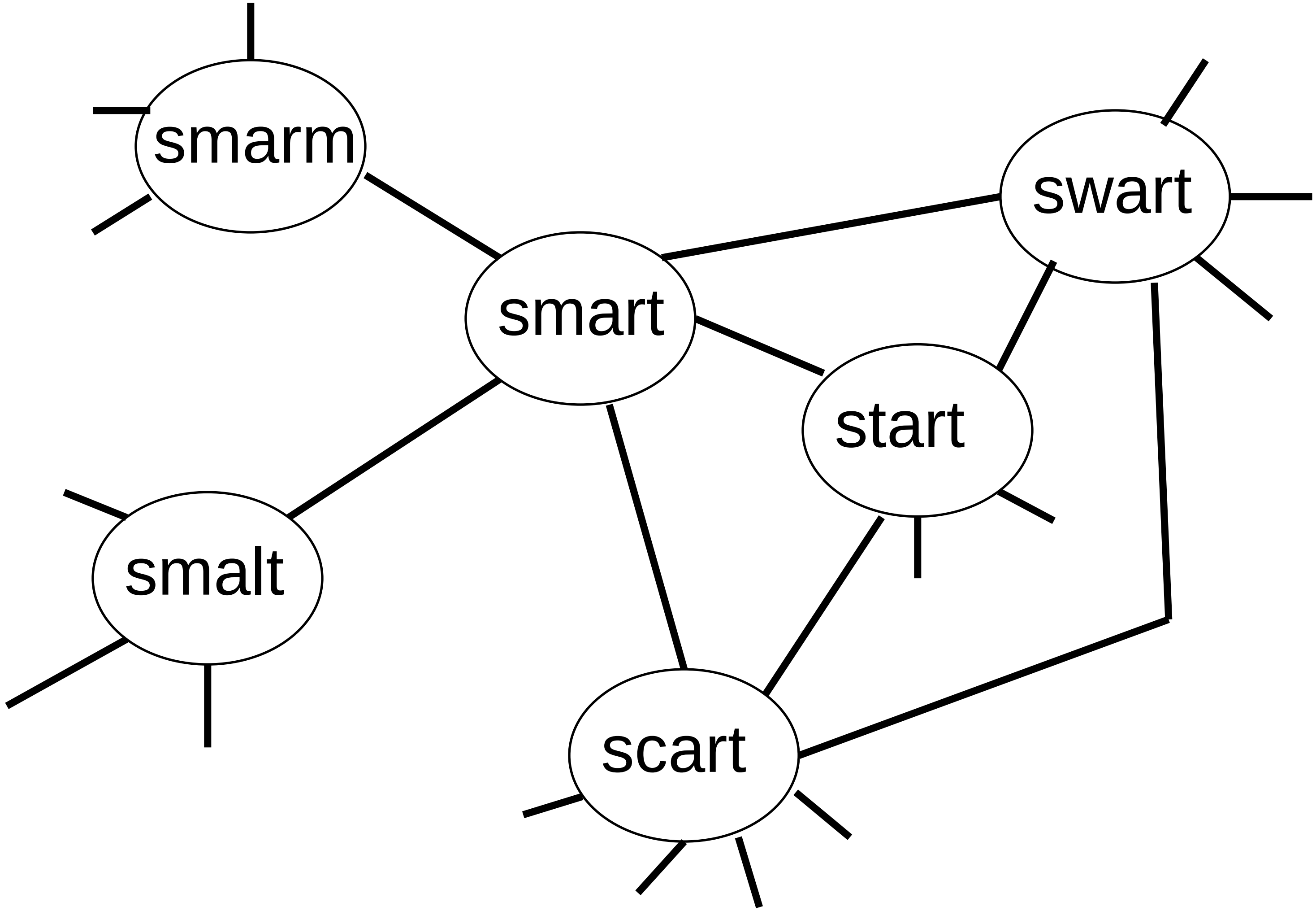
- Agree upon dictionary
- Start word and end word of same length
- Change one letter at a time to form step
- Step must also be a word
- Example: Start = silly, end = funny

```
silly
sully
sulky
hulky
hunky
funky
funny
```

Clicker 3 - Graph Representation

- What are the vertices and when does an edge exist between two vertices?

	Vertices	Edges
A.	Letters	Words
B.	Words	Words that share one or more letters
C.	Letters	Words that share one or more letters
D.	Words	Words that differ by one letter
E.	Words	Letters



Portion of Graph

Clicker 4 - Size of Graph

- Number of vertices and edges depends on dictionary
- Modified Scrabble dictionary, 5 letter words
- Words are vertices
  - 8660 words, 7915 words that are one letter different from at least one other word
- Edge exists between words if they are one letter different
  - 24,942 edges

Is this graph sparse or dense?

A. Sparse

B. Dense

Max number of edges =
N * (N - 1) / 2
37,493,470

Clicker 5 - Unweighted Shortest Path Algorithm

- Problem: Find the **shortest word ladder** between two words if one exists
- What kind of search should we use?

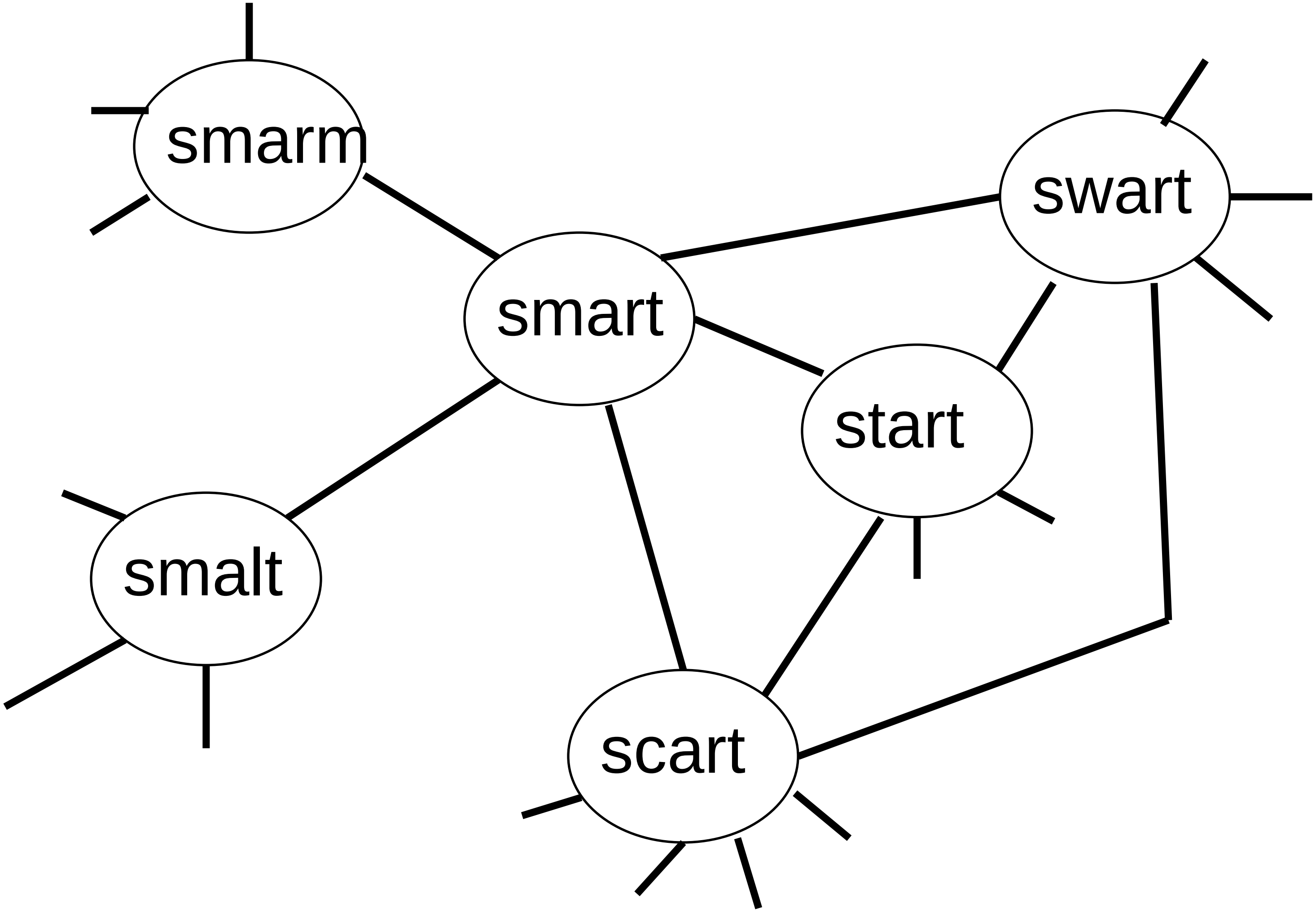
A. Breadth First Search

B. Depth First Search

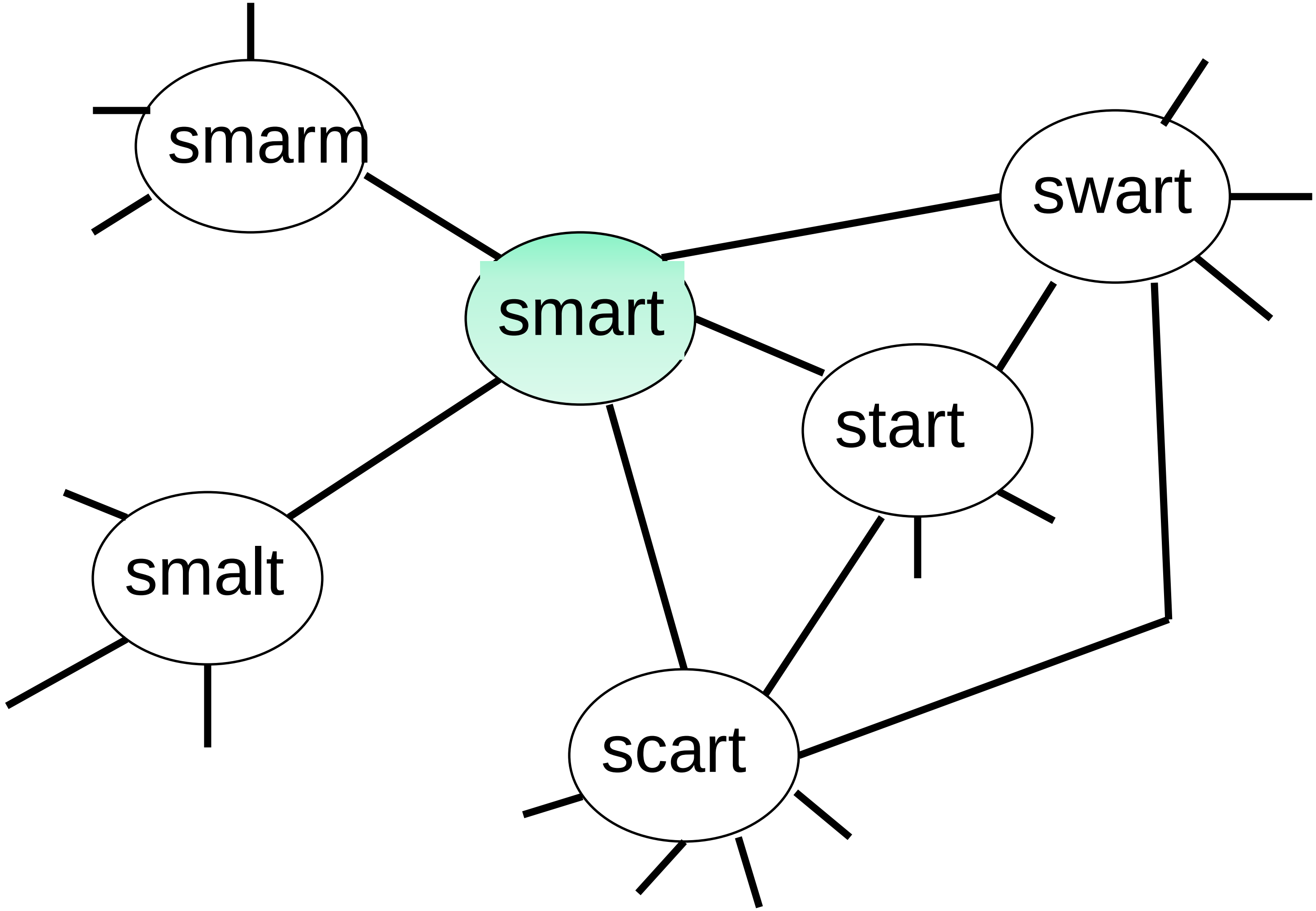
C. Either one

Unweighted Shortest Path Algorithm

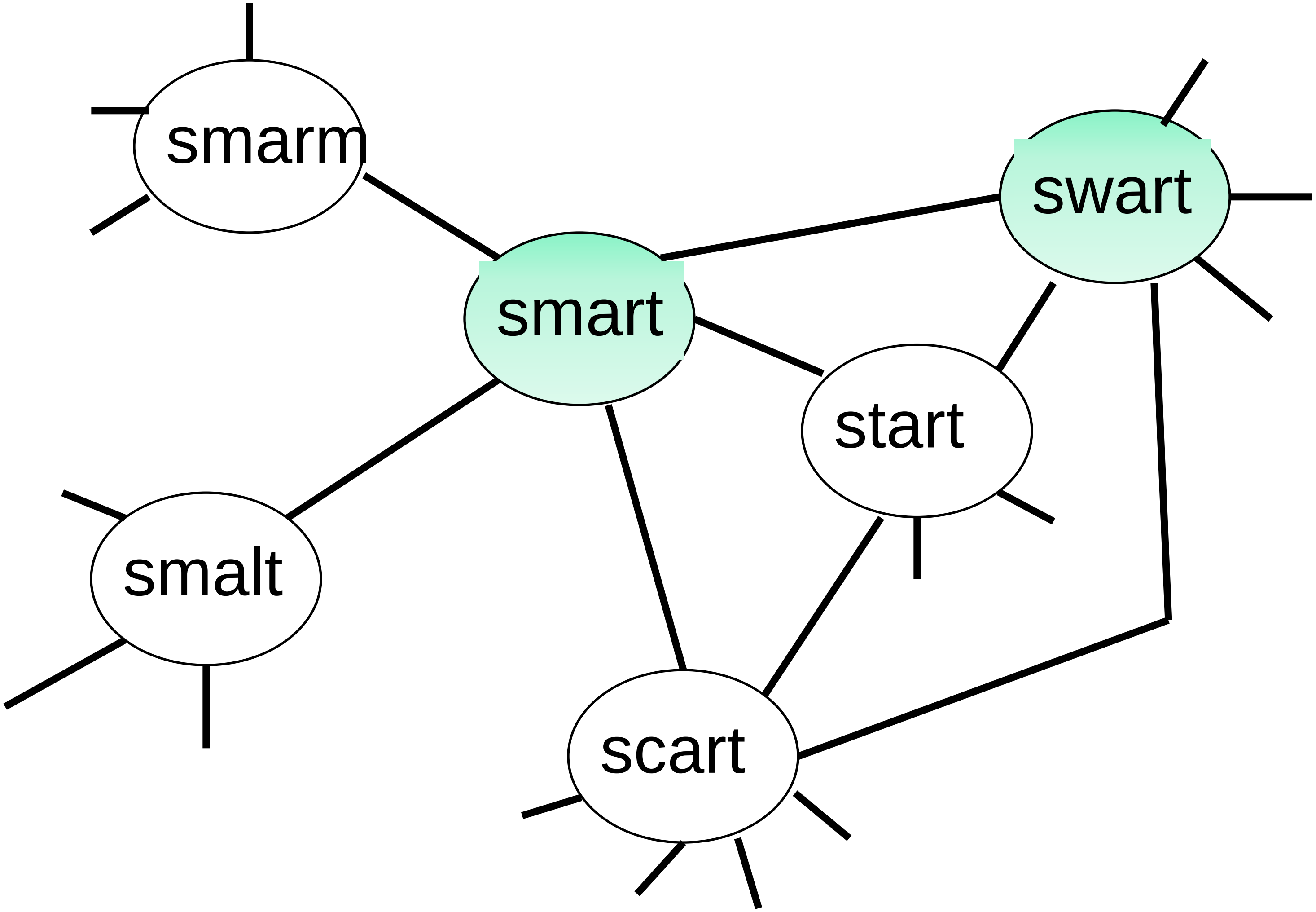
- Set distance of start to itself to 0
- Create a queue and add the start vertex
- while the queue is not empty
  - remove front
  - loop through all edges of current vertex
    - get vertex edge connects to
    - if this vertex has not been visited (have not found path to the destination of the edge)
      - sets its distance to current distance + 1
      - sets its previous vertex to current vertex
      - add new vertex to queue



Portion of Graph

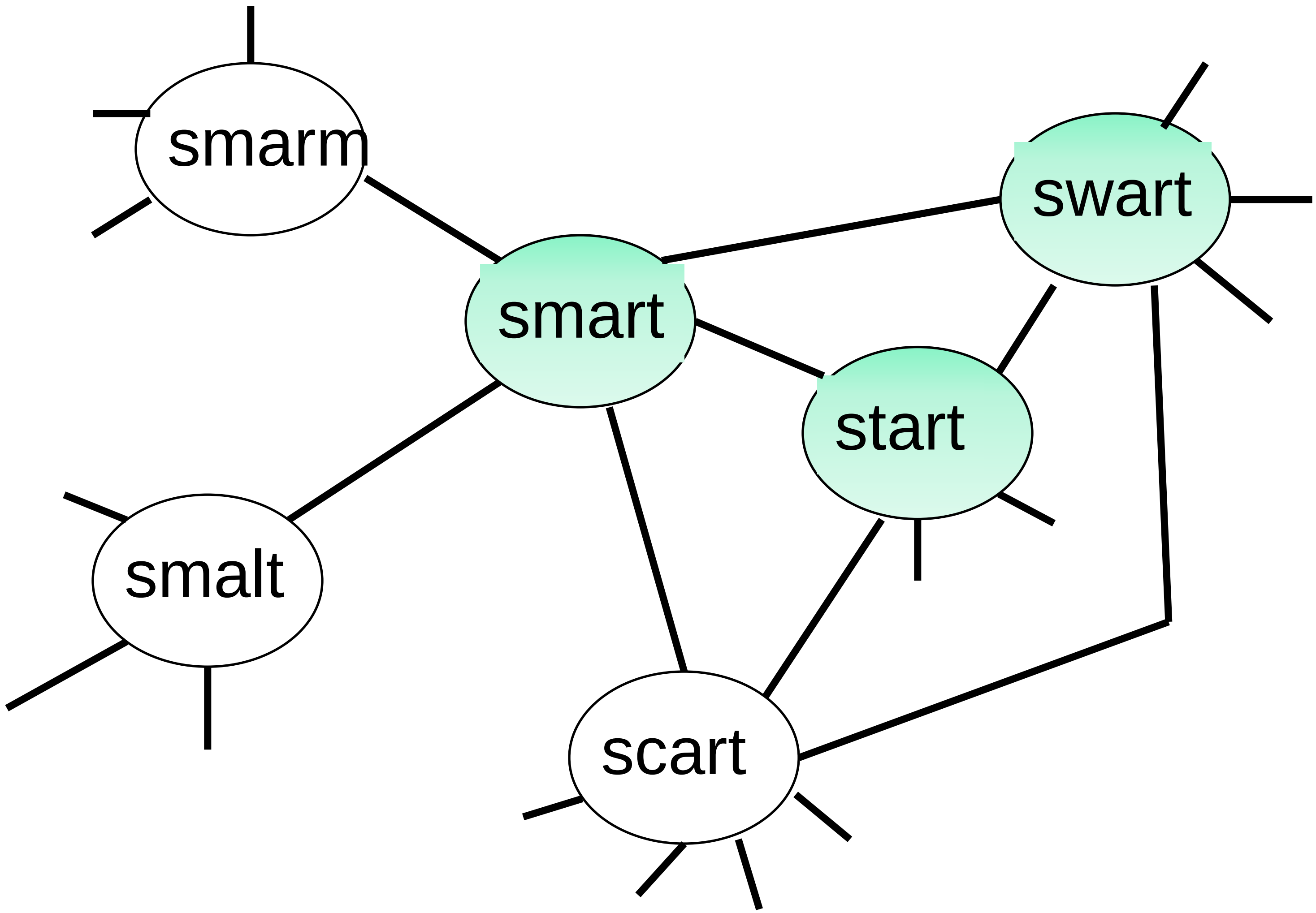


Start at "smart" and enqueue it
[smart]



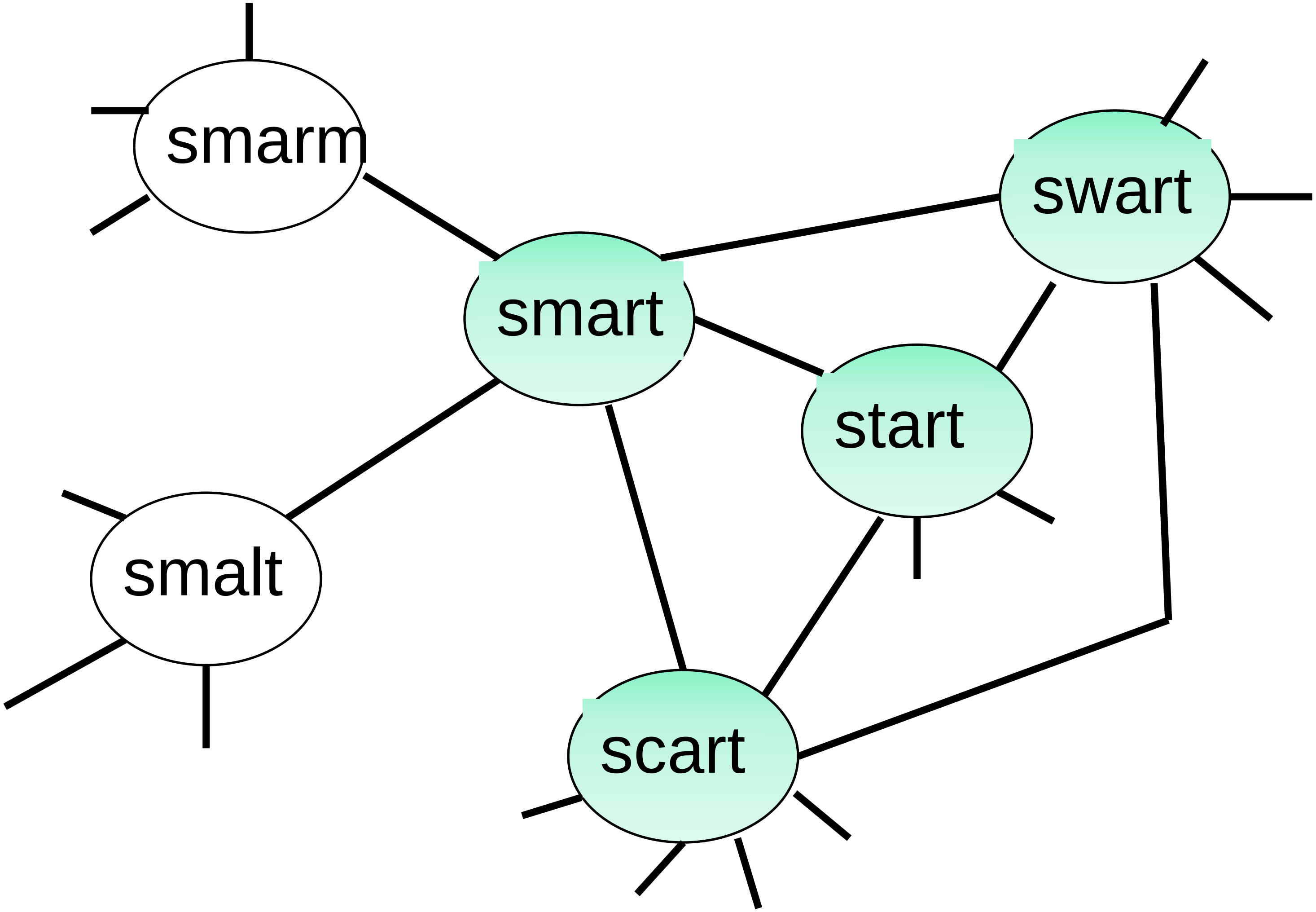
Dequeue (smart), loop through edges

[swart]



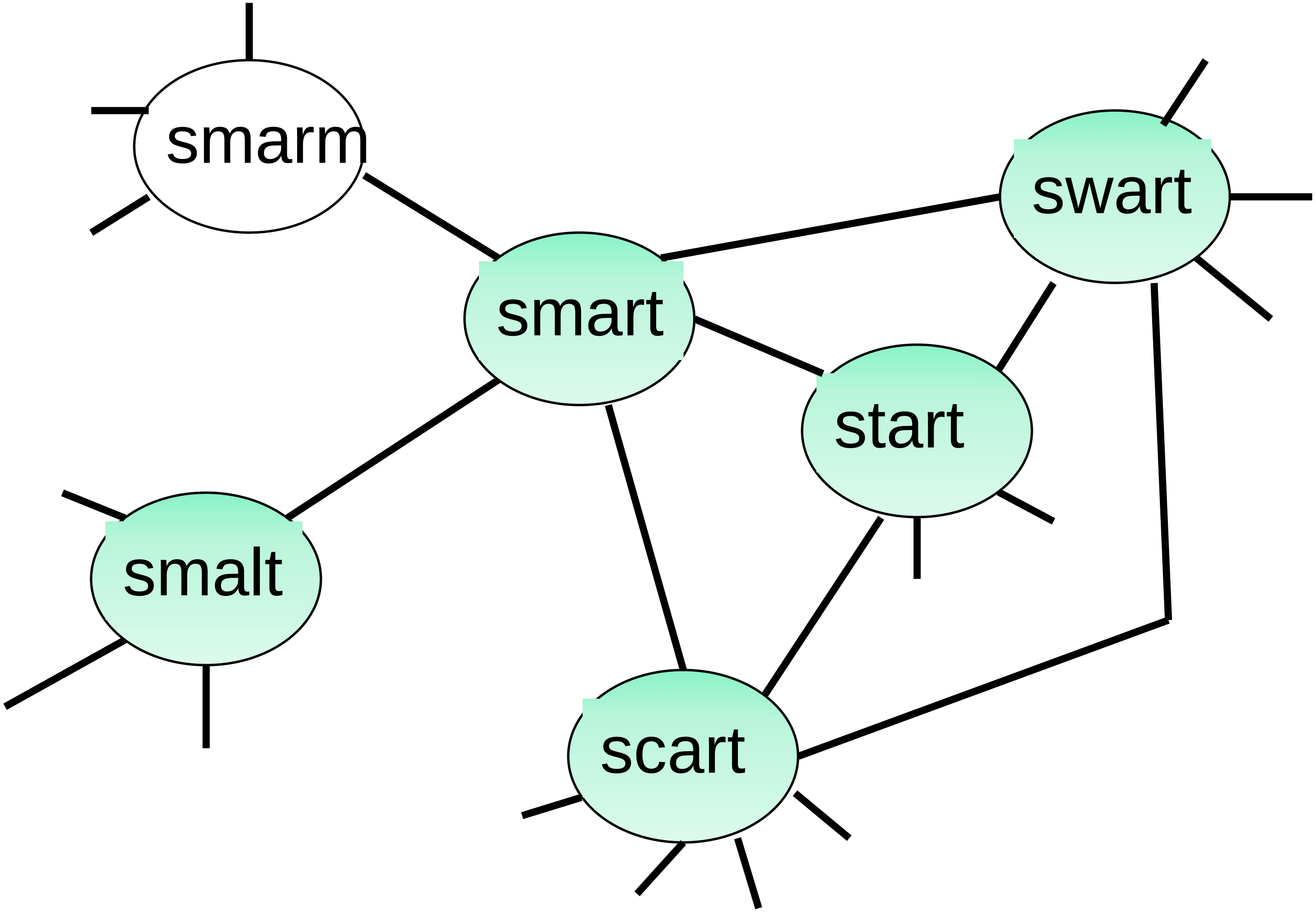
Dequeue (smart), loop through edges

[swart, start]



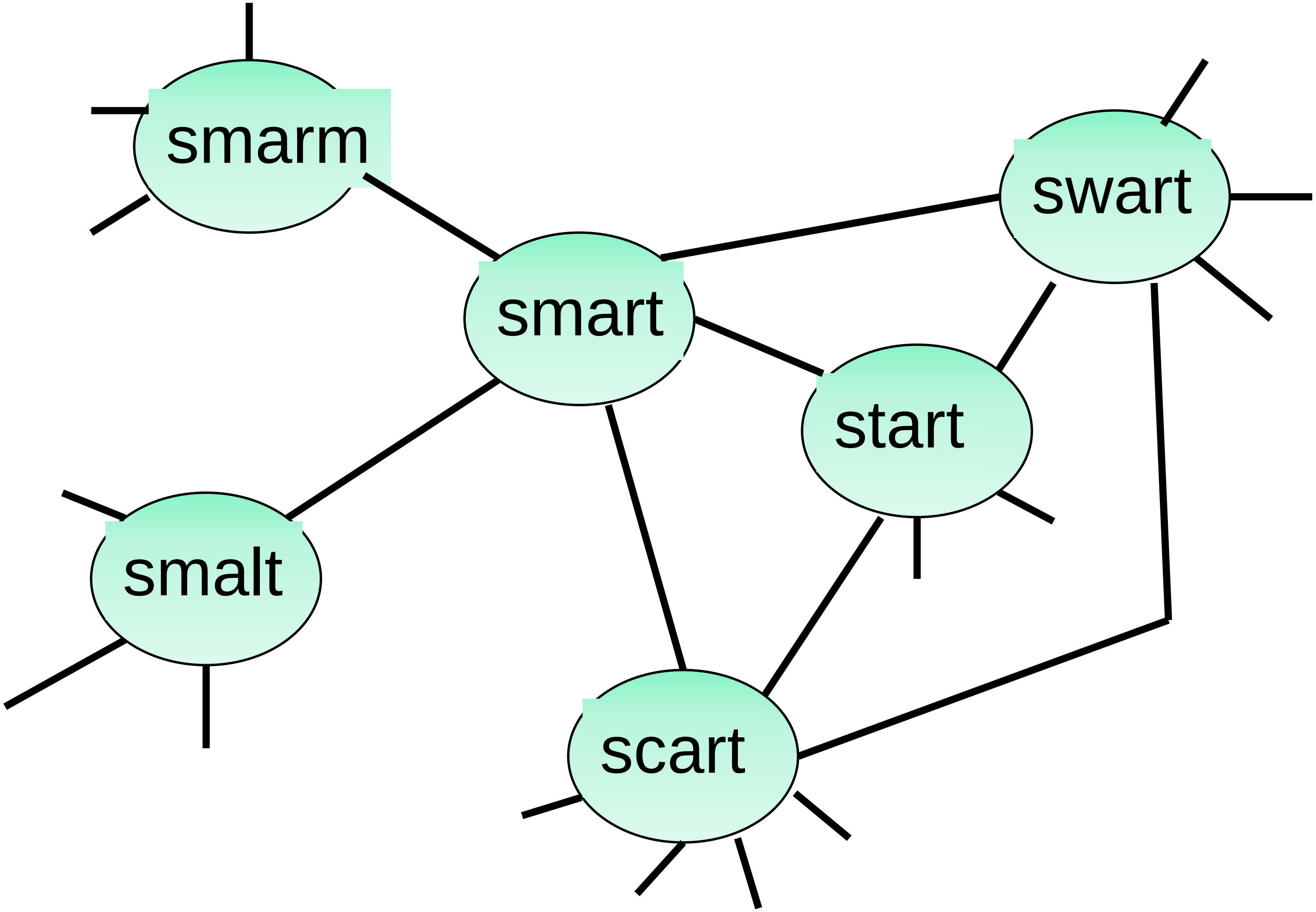
Dequeue (smart), loop through edges

[swart, start, scart]



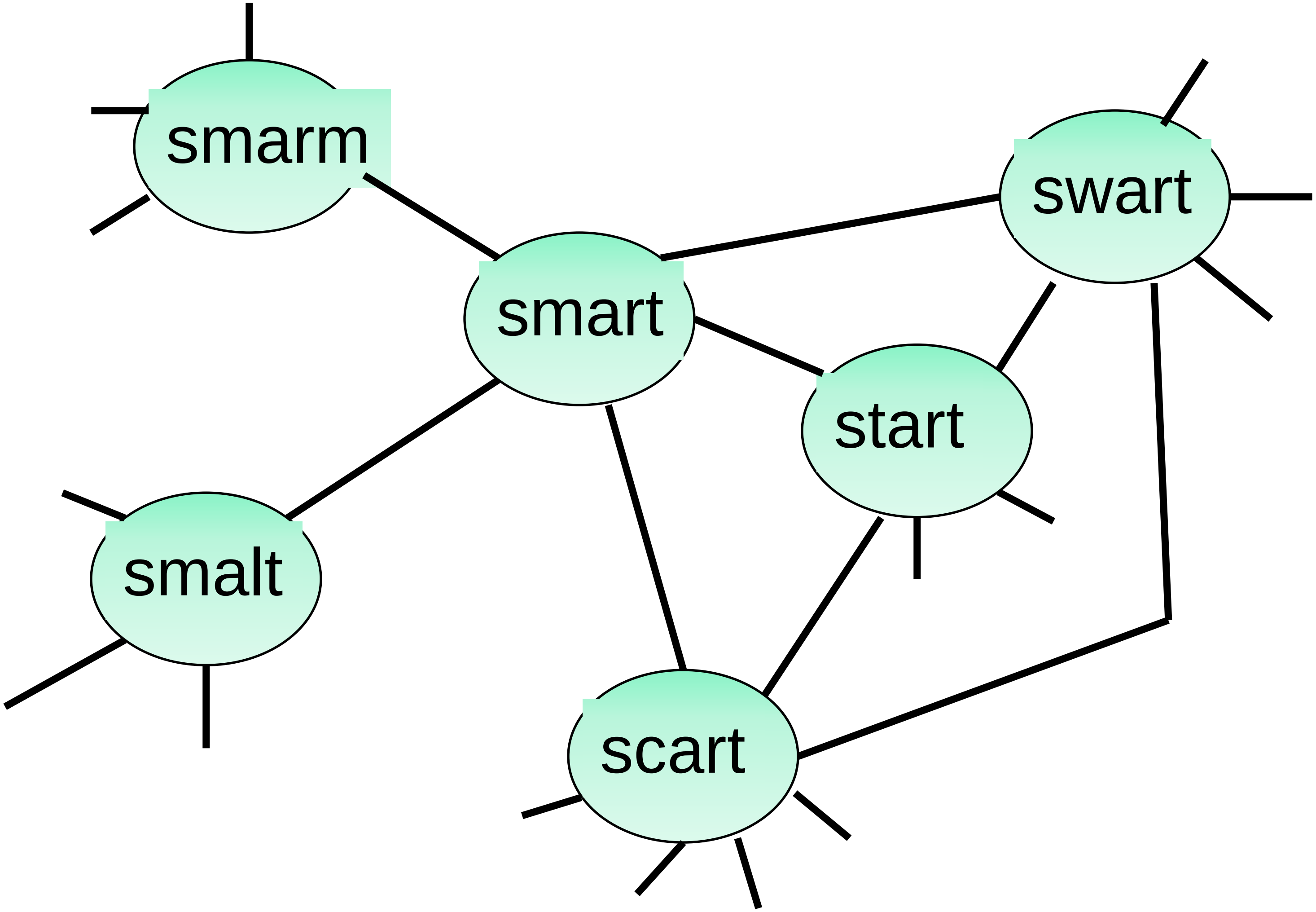
Dequeue (smart), loop through edges

[swart, start, scart, smalt]



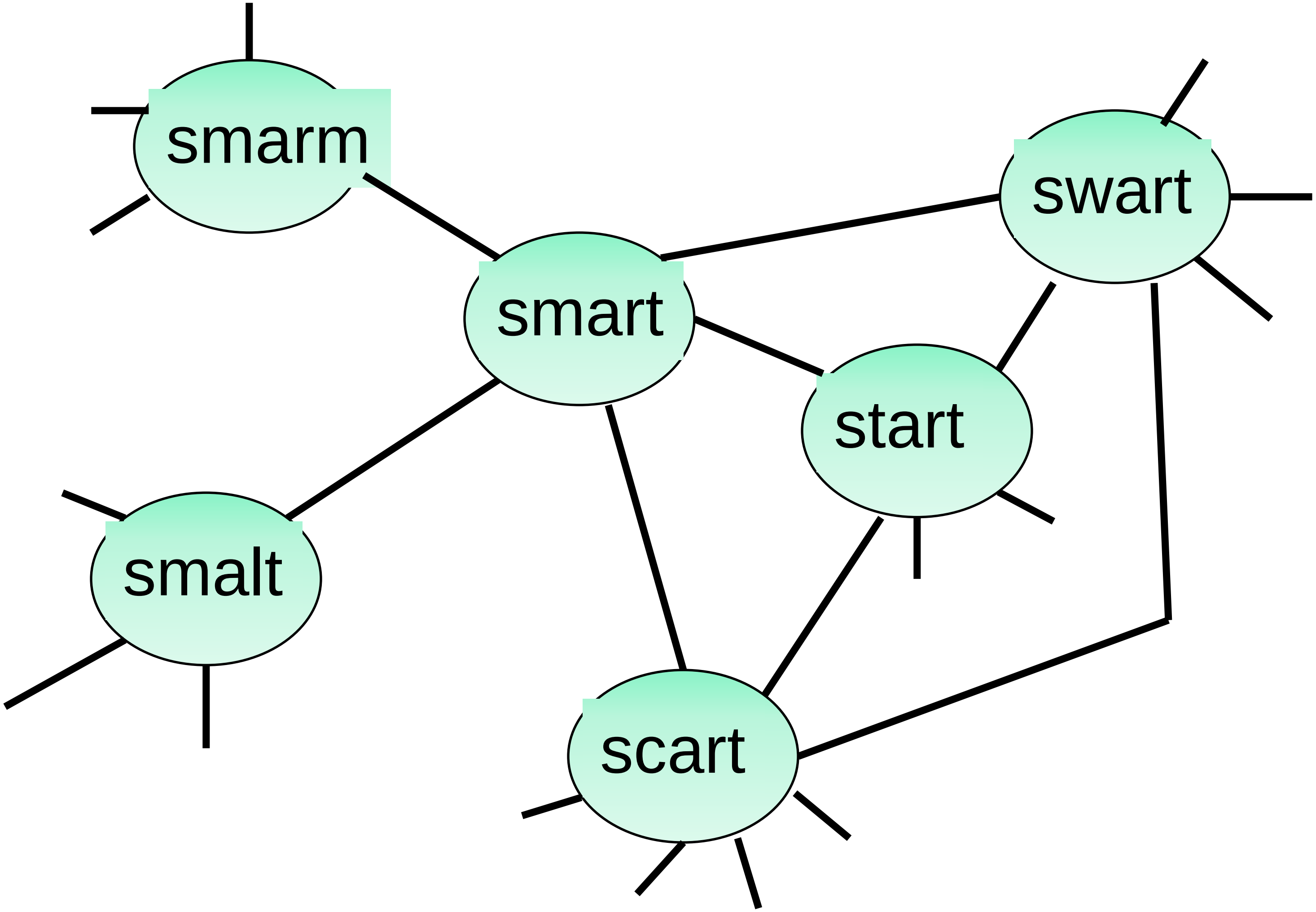
Dequeue (smart), loop through edges

[swart, start, scart, smalt, smarm]



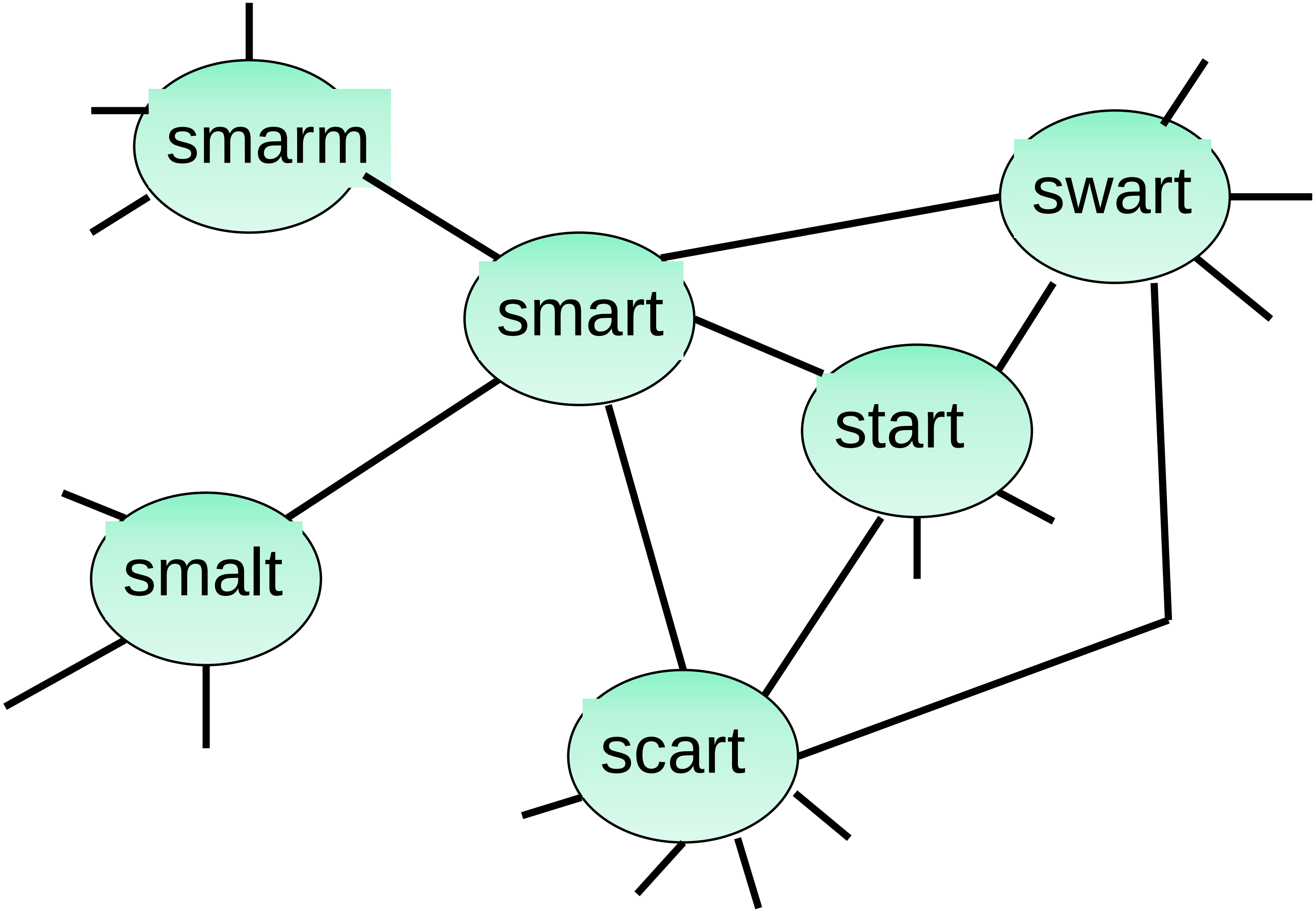
Done with smart, dequeue (swart)

[start, scart, smalt, smarm]



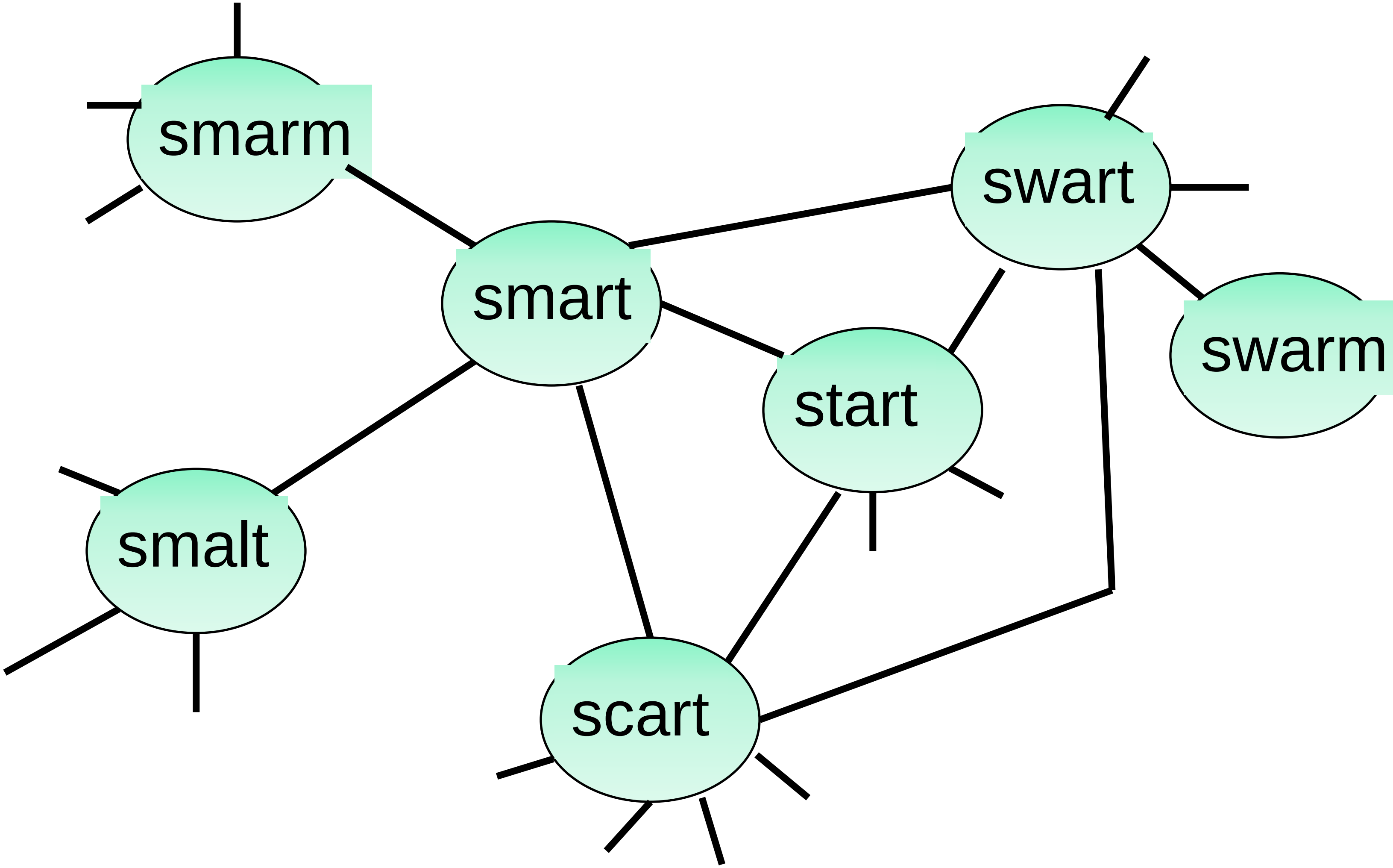
loop through edges of swart (start already present)

[start, scart, smalt, smarm]



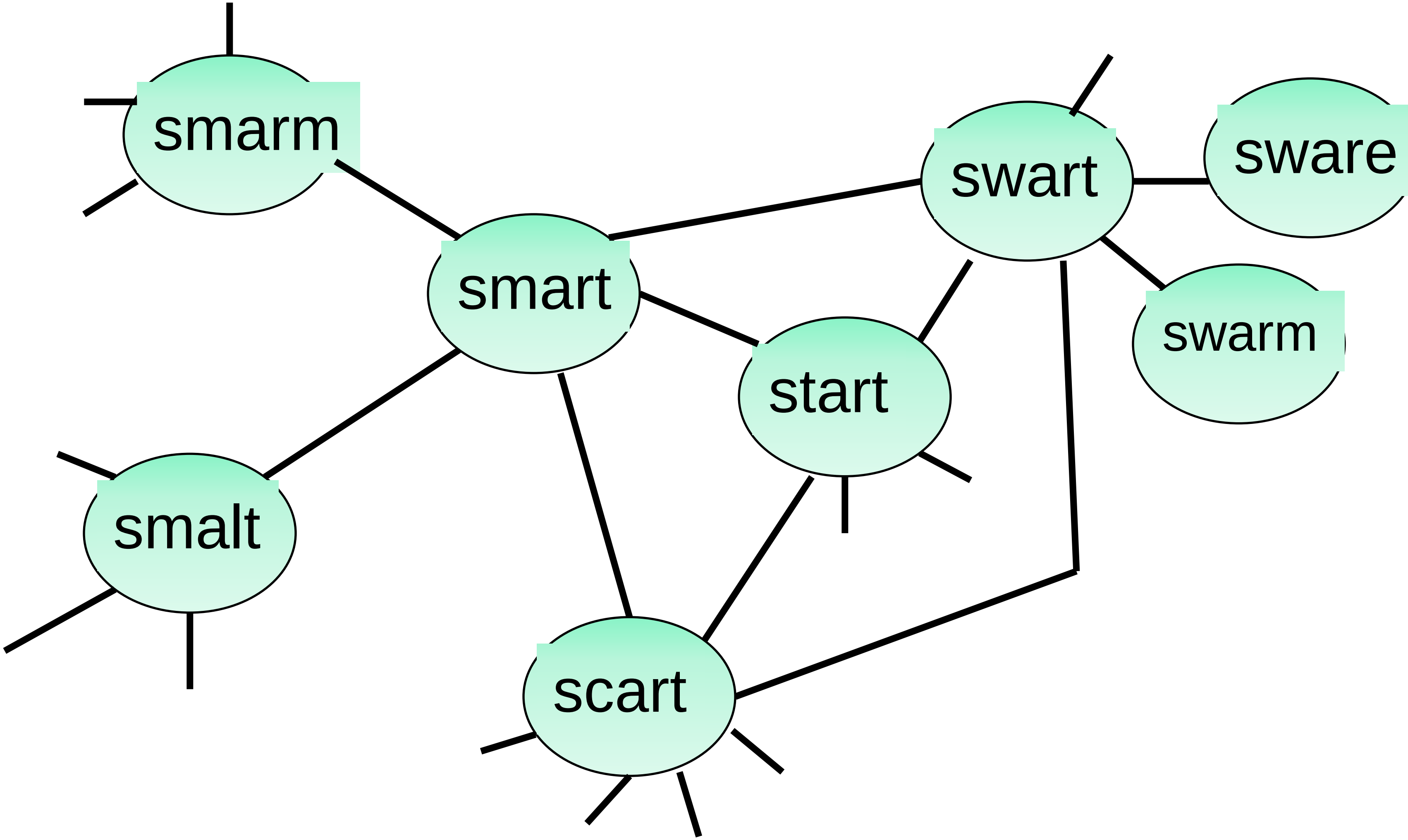
loop through edges of swart (scart already present)

[start, scart, smalt, smarm]



loop through edges of swart

[start, scart, smalt, smarm, swarm]



loop through edges of swart

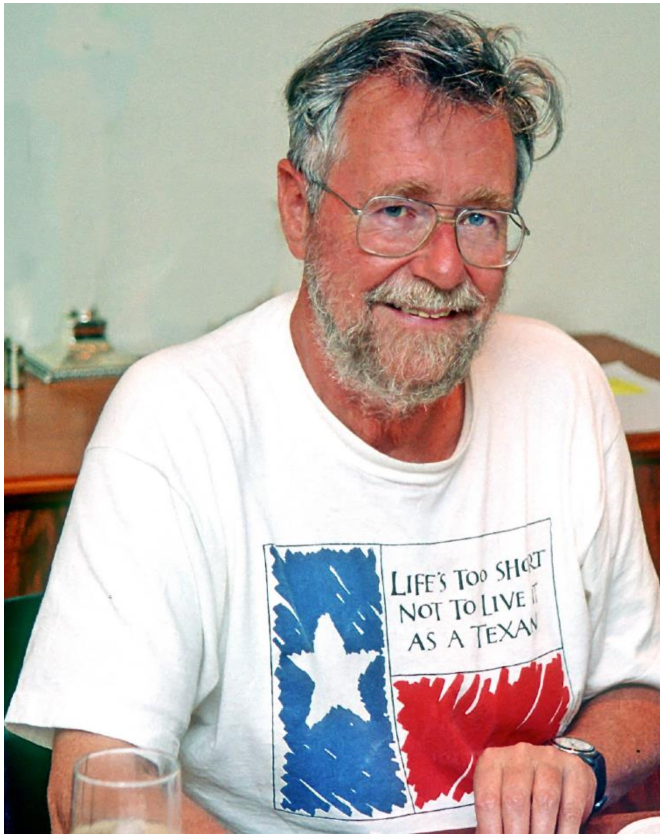
[start, scart, smalt, smarm, swarm, sware]

Unweighted Shortest Path

- Implement method
- demo
- how is path printed?
- The *diameter* of a graph is the longest shortest past in the graph
- How to find?
- How to find *center* of graph?
  – many measures of centrality
  – ours: vertex connected to the largest number of other vertices with the shortest average path length

Positive Weighted Shortest Path

- Edges in graph are weighted and all weights are positive
- Similar solution to unweighted shortest path
- Dijkstra's algorithm
- Edsger W. Dijkstra, 1930–2002
- UT Professor 1984 - 2000
- Algorithm developed in 1956 and published in 1959.



Dijkstra on Creating the Algorithm

- What is the shortest way to travel from Rotterdam to Groningen, in general: from given city to given city. It is the algorithm for the shortest path, which **I designed in about twenty minutes**. One morning I was shopping in Amsterdam with my young fiancée, and tired, we sat down on the café terrace to drink a cup of coffee and I was just thinking about whether I could do this, and I then designed the algorithm for the shortest path. As I said, it was a twenty-minute invention. In fact, it was published in ’59, three years later. The publication is still readable, it is, in fact, quite nice. **One of the reasons that it is so nice was that I designed it without pencil and paper**. I learned later that one of the advantages of designing without pencil and paper is that you are almost forced to avoid all avoidable complexities. **Eventually that algorithm became, to my great amazement, one of the cornerstones of my fame.**
- — Edsger Dijkstra, in an interview with Philip L. Frana, Communications of the ACM, 2001 (wiki page on the algorithm)

Vertex Class (nested in Graph)

```
private static class Vertex
    private String name;
    private List<Edge> adjacent;

    public Vertex(String n)

    // for shortest path algorithms
    private double distance;
    private Vertex prev;
    private int scratch;

    // call before finding new paths
    public void reset()
```

Dijkstra's Algorithm

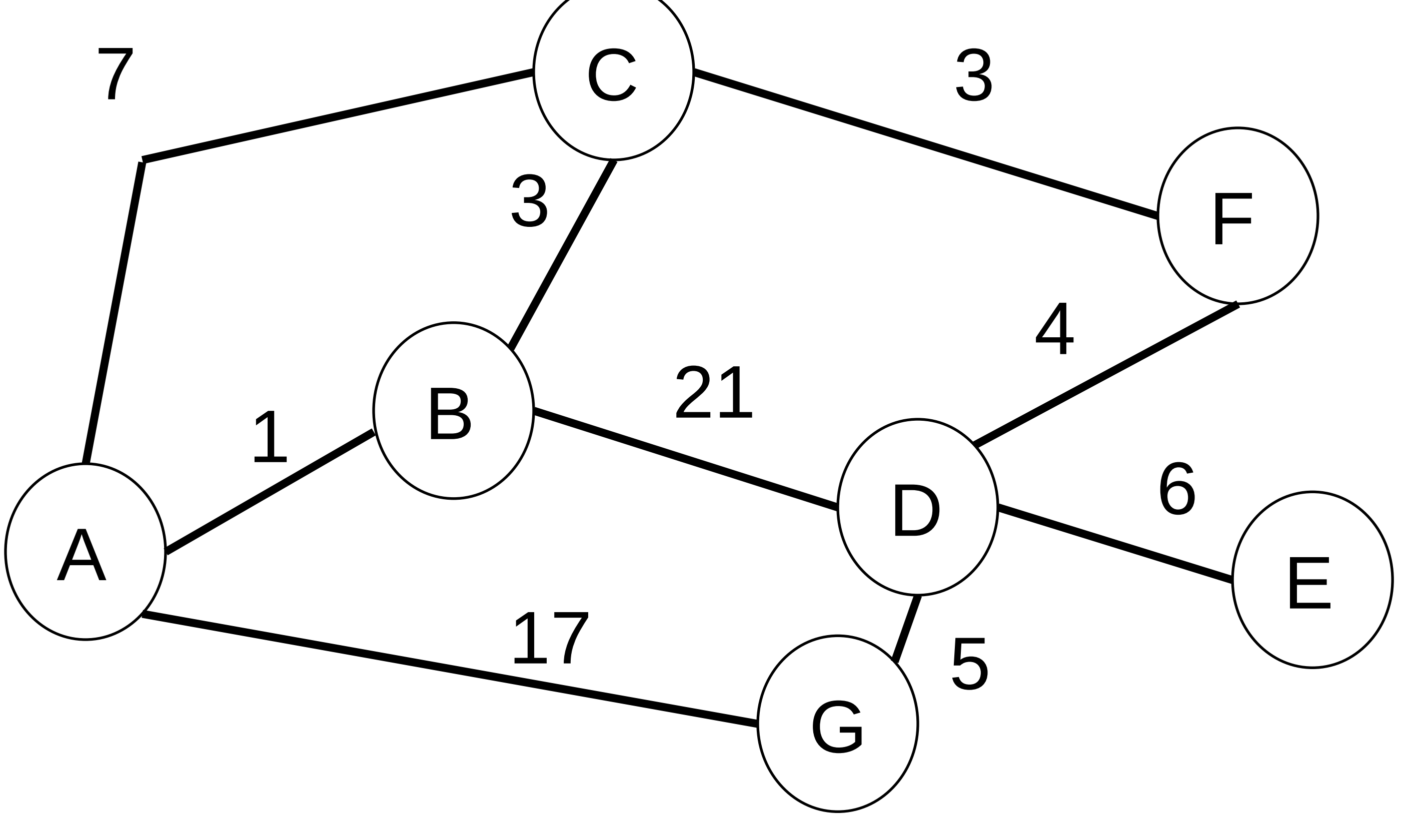
- Pick the start vertex
- Set the distance of the start vertex to 0 and all other vertices to INFINITY
- While there are unvisited vertices:
  - Let the current vertex be the vertex with the lowest cost path from start to it that has **not yet been visited**
  - mark current vertex as visited
  - for each edge from the current vertex
    - if the sum of the cost of the current vertex and the cost of the edge is less than the cost of the destination vertex
      - update the cost of the destination vertex
      - set the previous of the destination vertex to the current vertex
      - enqueue this path (not vertex) to the priority queue
      - **THIS IS NOT VISITING THE NEIGHBORING VERTEX**

Dijkstra's Algorithm

- Example of a *Greedy Algorithm*
  - A Greedy Algorithm does what appears to be the best thing at each stage of solving a problem
- Gives best solution in Dijkstra's Algorithm
- Does NOT always lead to best answer
- Fair teams:
  - (10, 10, 8, 8, 8), 2 teams
- Making change with fewest coins

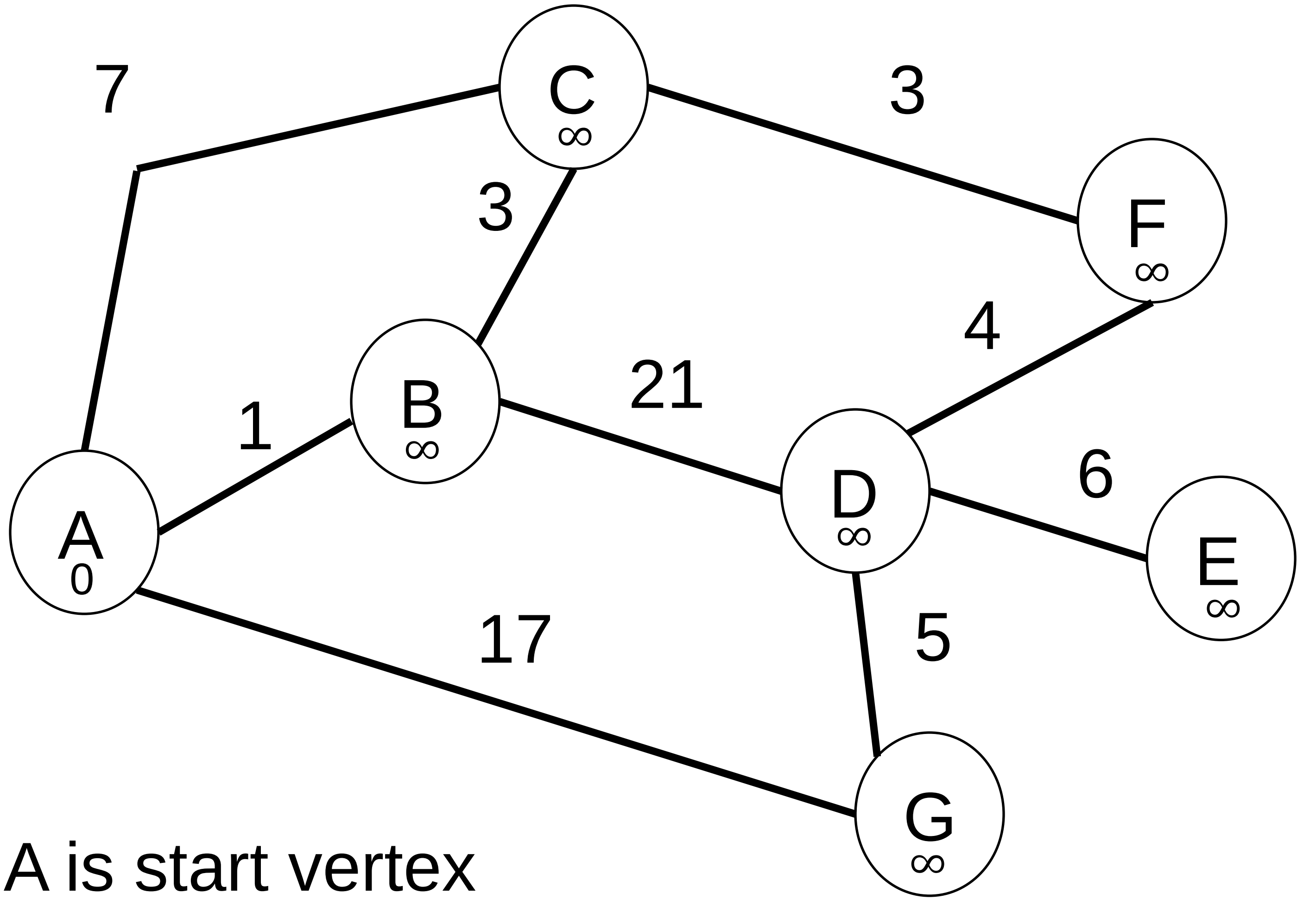
  (1, 5, 10) 15 cents

  (1, 5, 12) 15 cents



Clicker 6 - What is the cost of the lowest cost path from A to E?

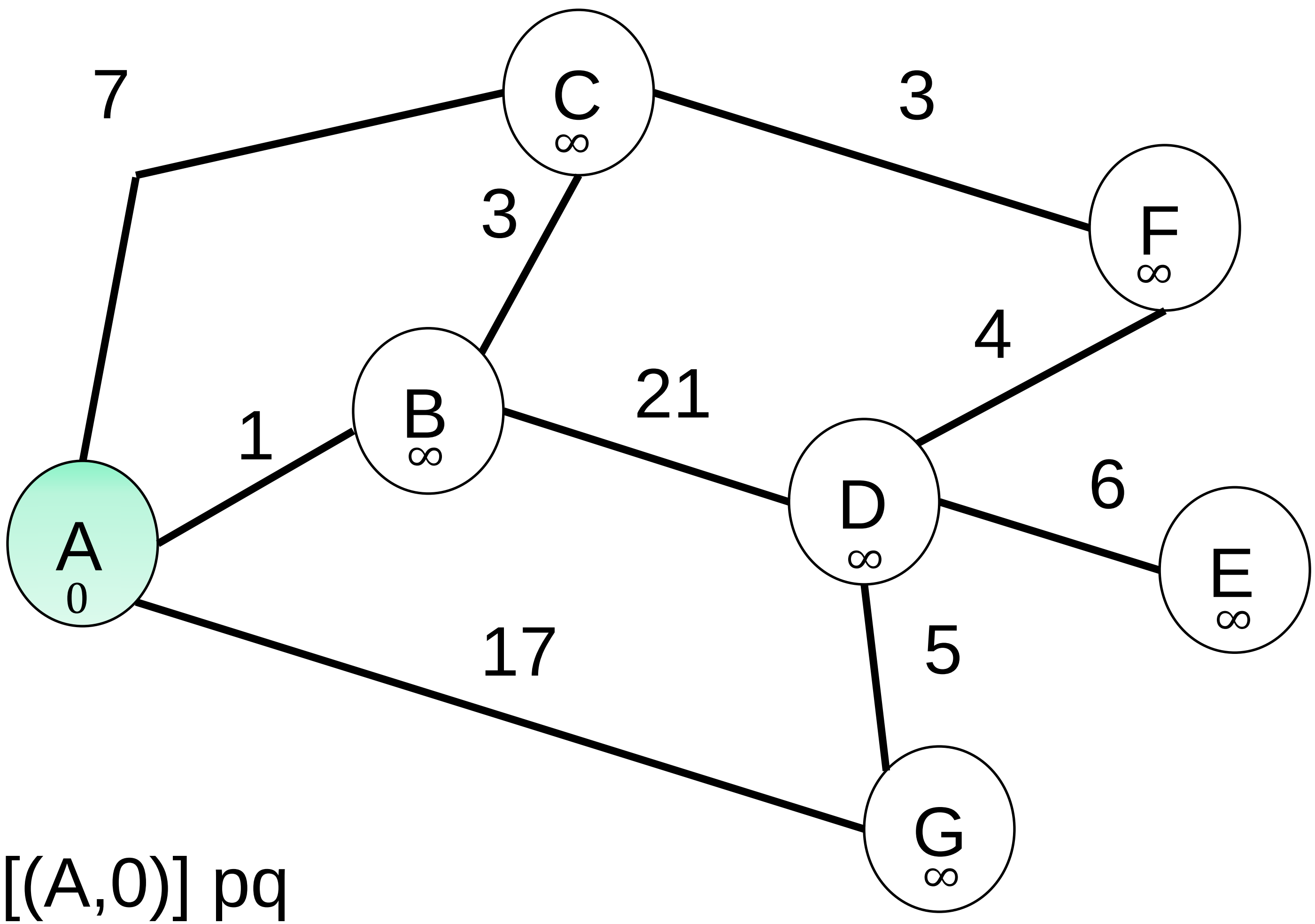
A. 5
B. 17
C. 20
D. 28
E. 37



A is start vertex

Set cost of A to 0, all others to INFINITY

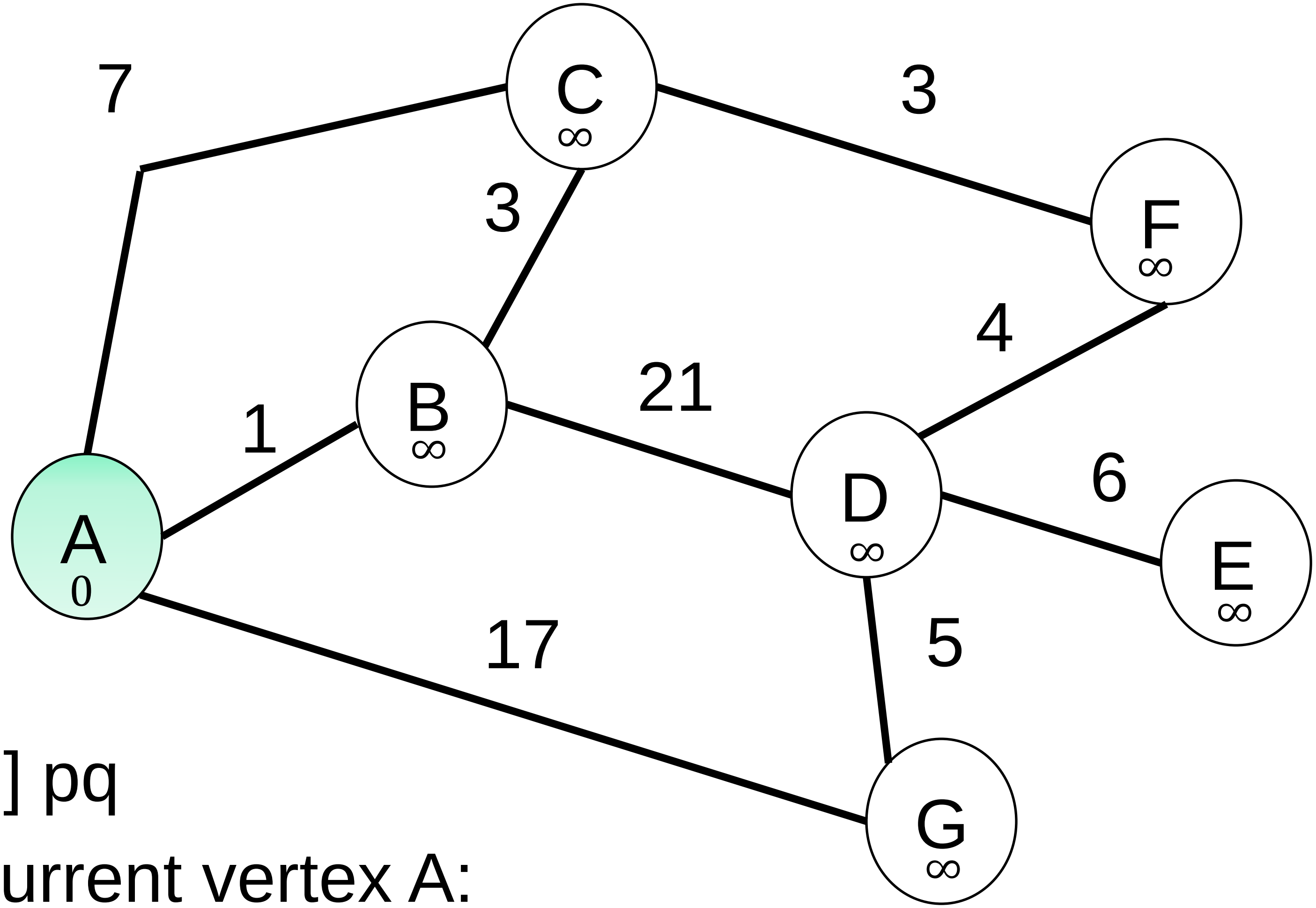
Place A in a priority queue



[(A,0)] pq

dequeue (A,0)

Mark A as visited



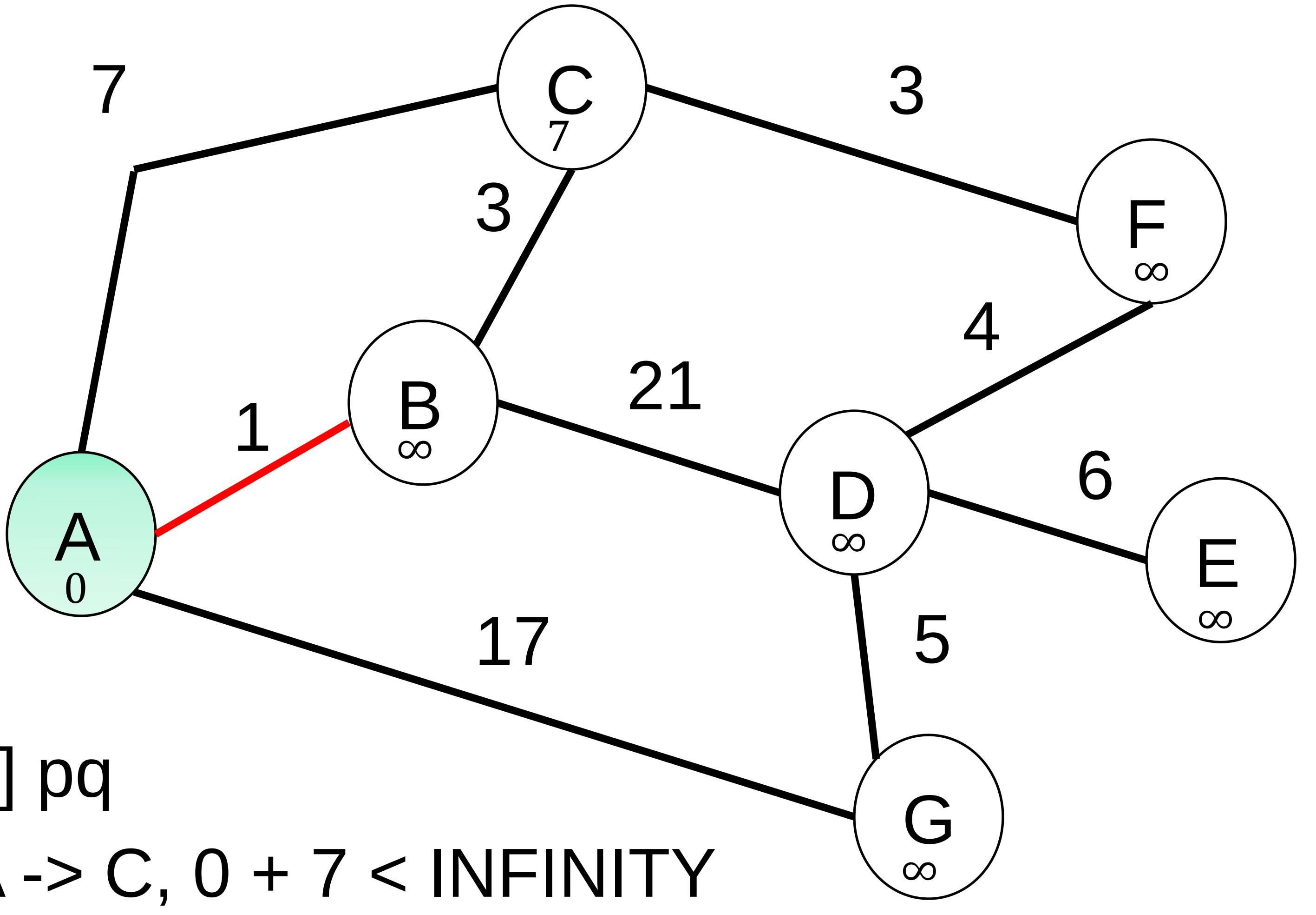
[ ] pq

current vertex A:

loop through A's edges

if sum of cost from A to dest is less than current cost

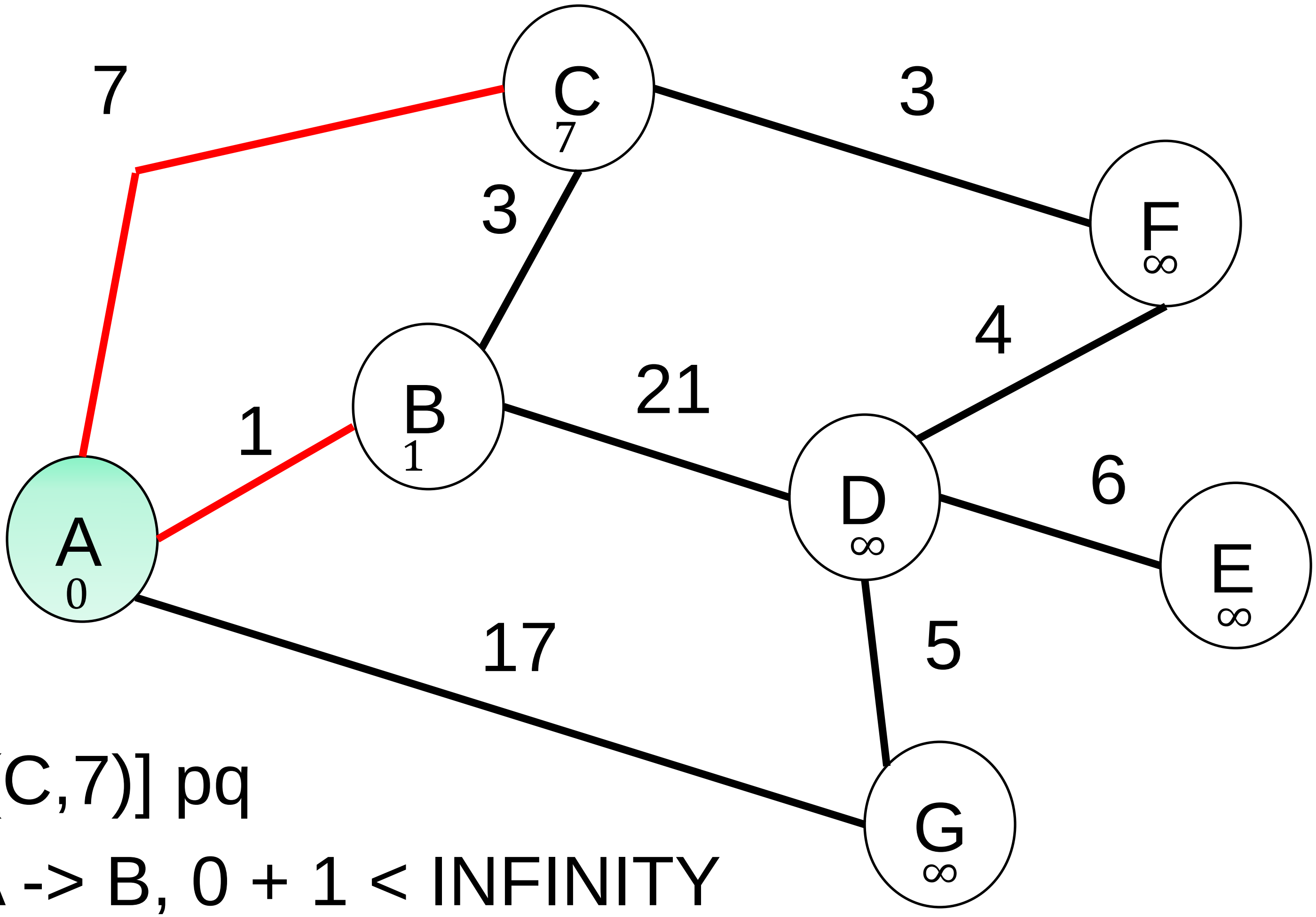
update cost and prev



[ ] pq

A -> C, 0 + 7 < INFINITY

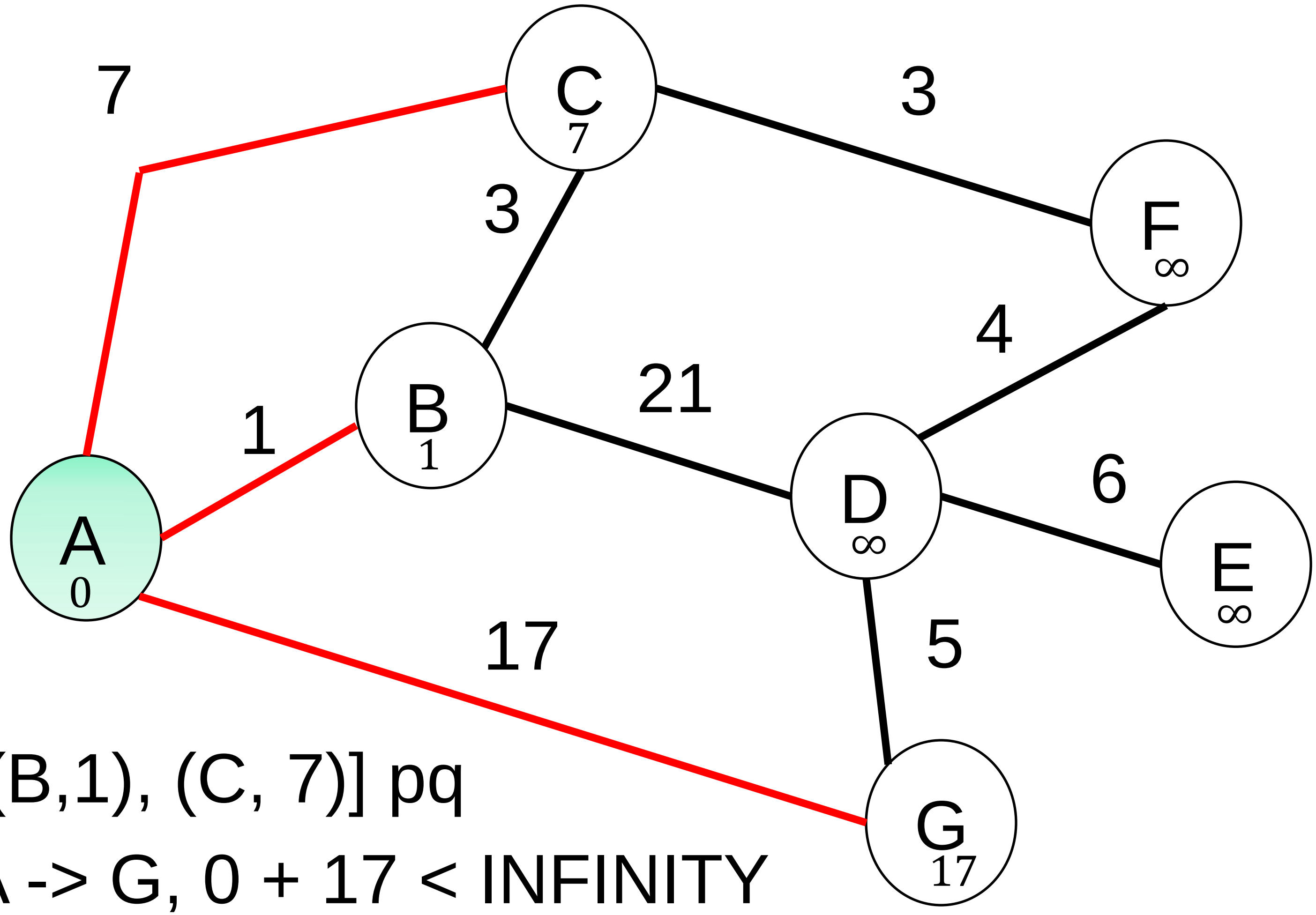
[(C,7)] pq



[(C,7)] pq

A -> B, 0 + 1 < INFINITY

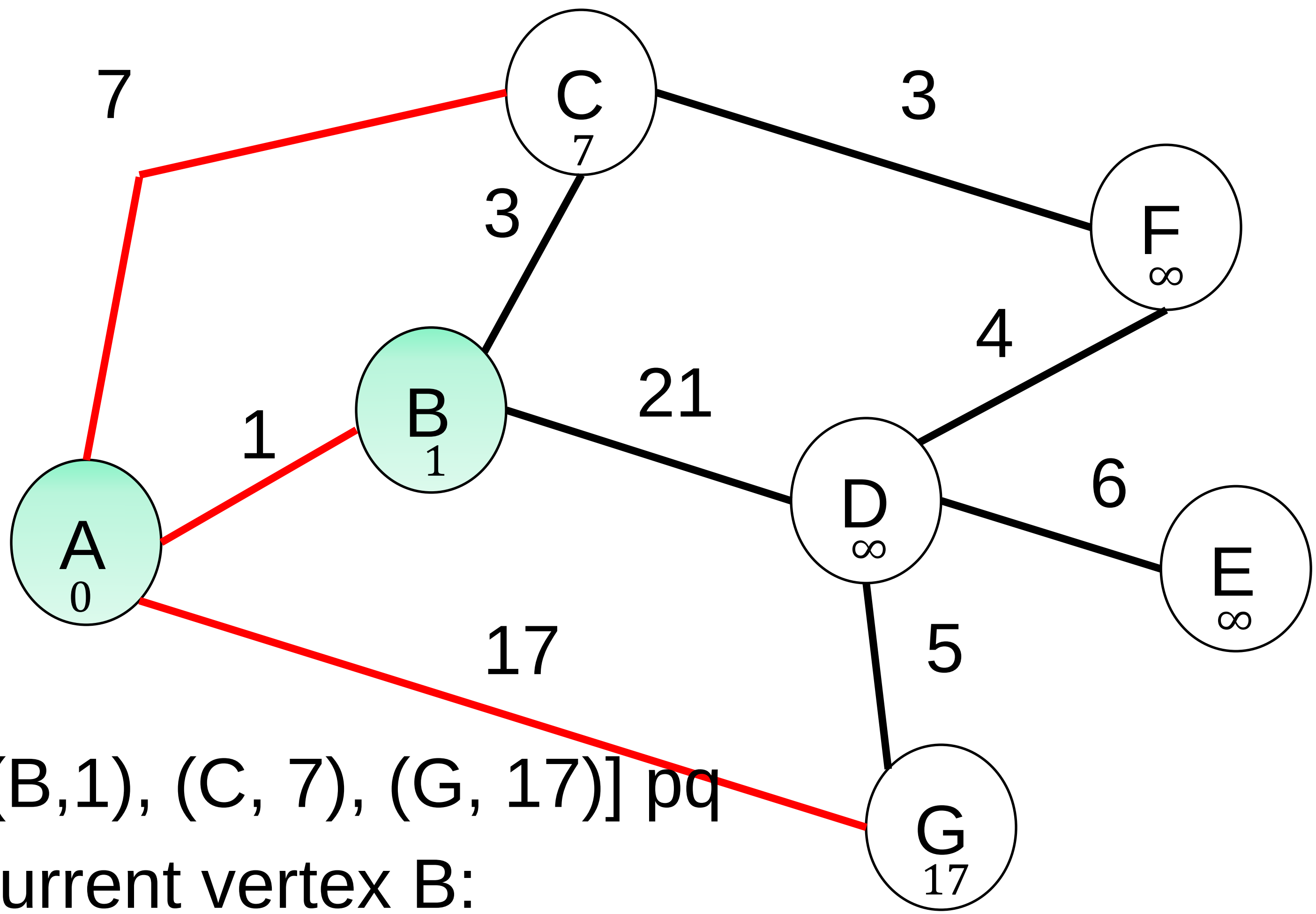
[(B,1), (C, 7)] pq (Note, the (B,1) jumps in front of (C,7)



[(B,1), (C, 7)] pq

A -> G, 0 + 17 < INFINITY

[(B,1), (C, 7), (G, 17)] pq



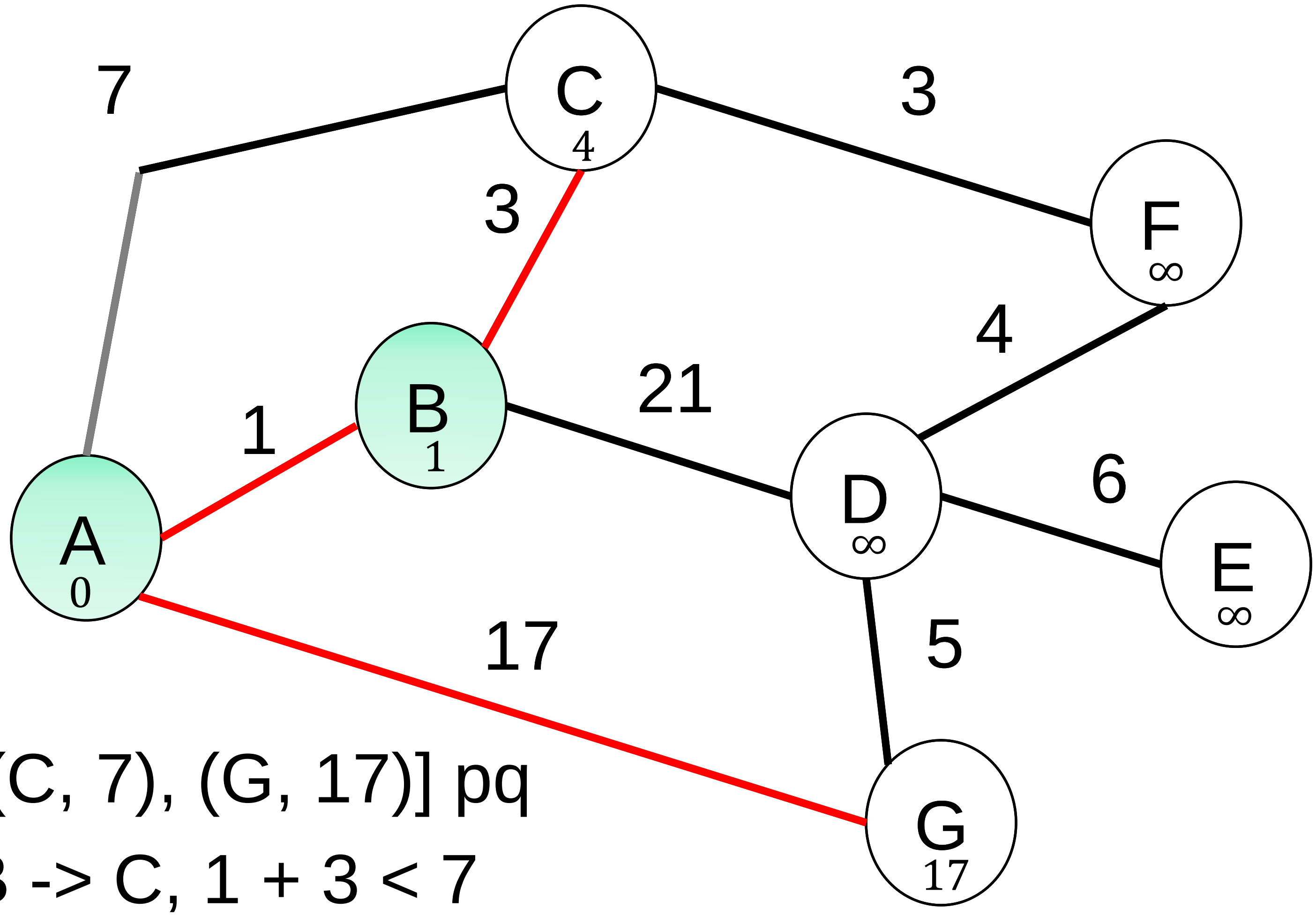
[(B,1), (C, 7), (G, 17)] pq

current vertex B:

loop through B's edges

if sum of cost from B to edge is less than current cost

update cost and prev

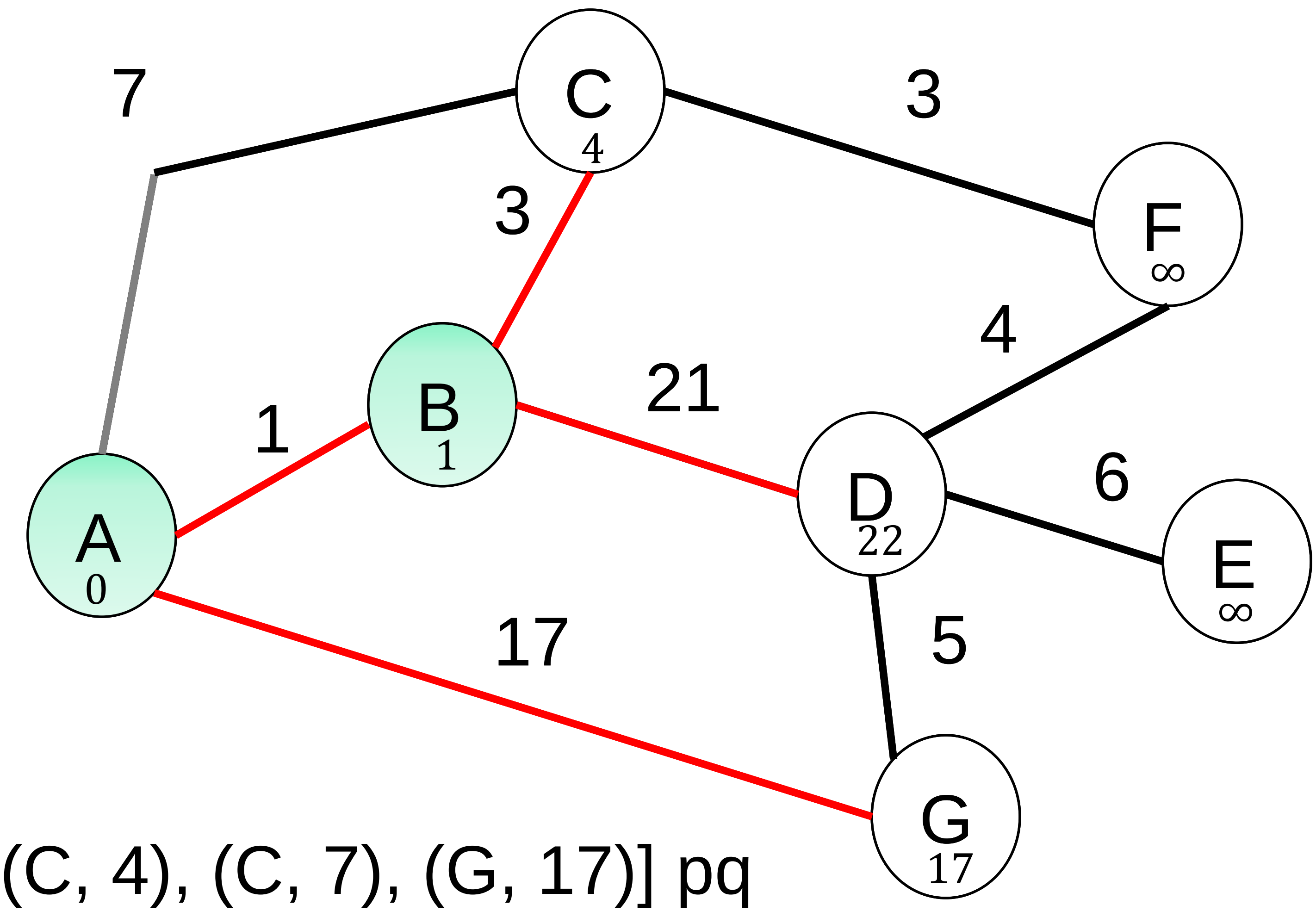


[(C, 7), (G, 17)] pq

B -> C, 1 + 3 < 7

update C's cost and previous

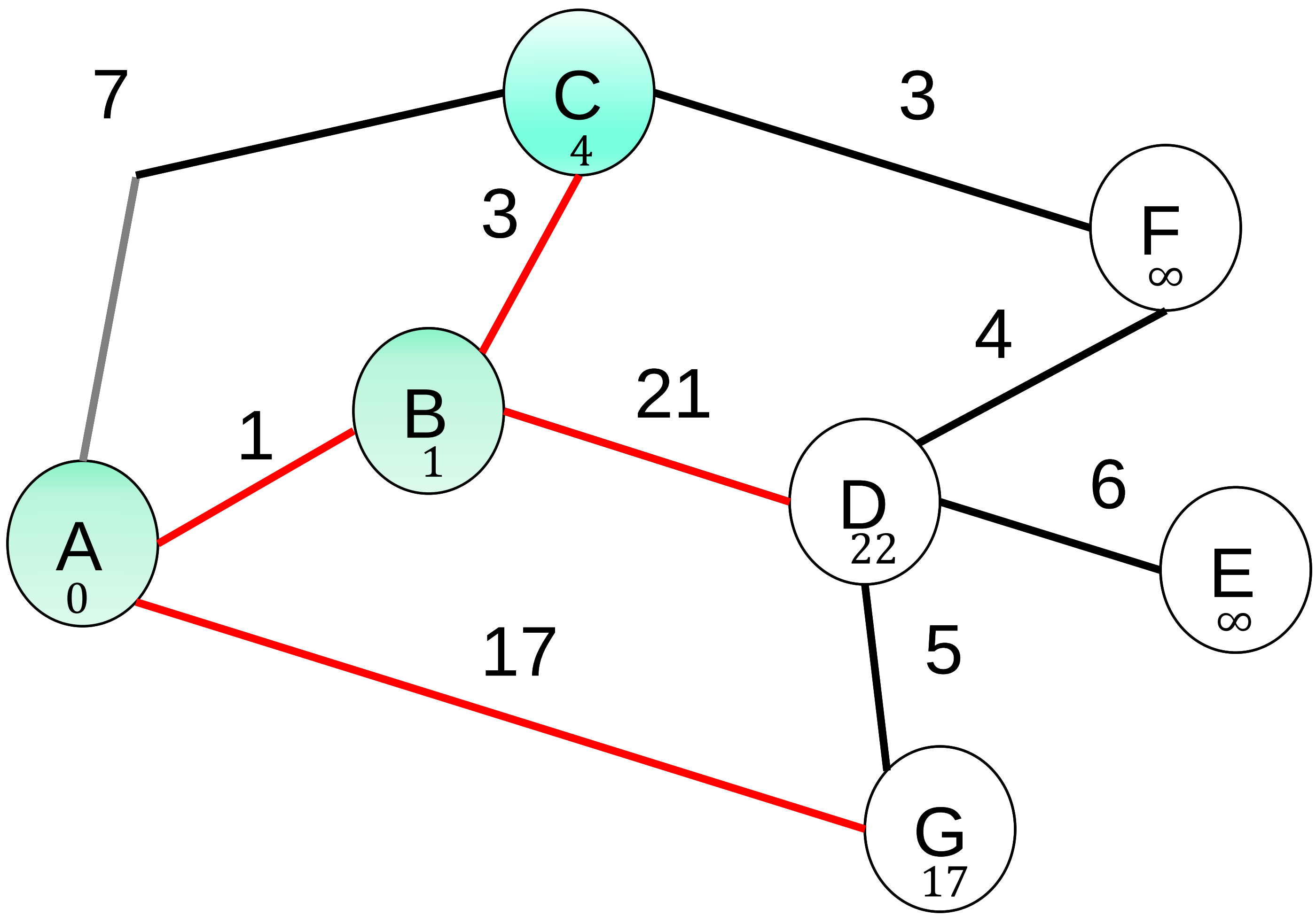
[(C, 4), (C, 7), (G, 17)] pq



[(C, 4), (C, 7), (G, 17)] pq

B -> D, 1 + 21 < INFINITY

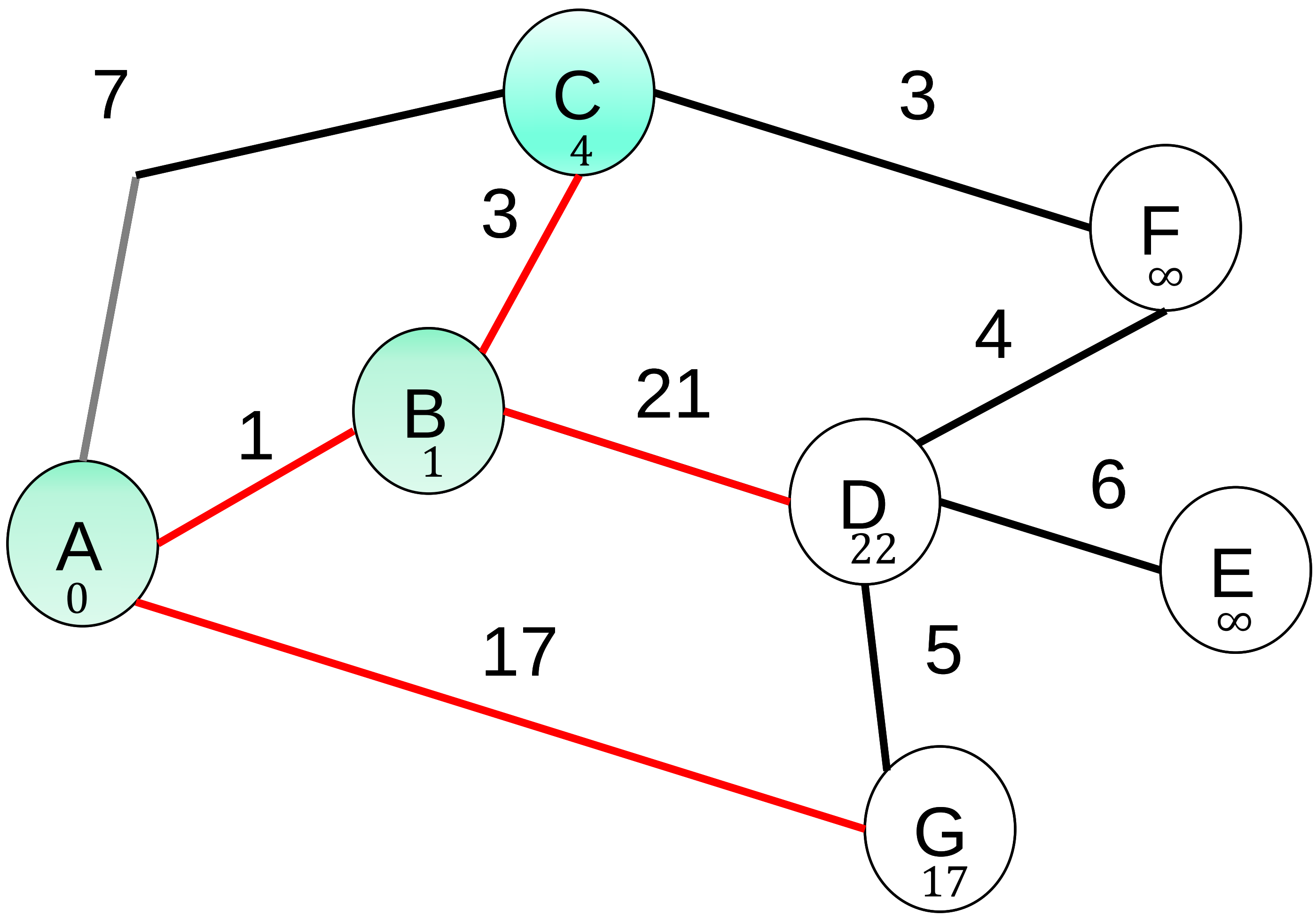
[(C, 4), (C, 7), (G, 17), (D, 22)] pq



[(C, 4), (C, 7), (G, 17), (D, 22)] pq

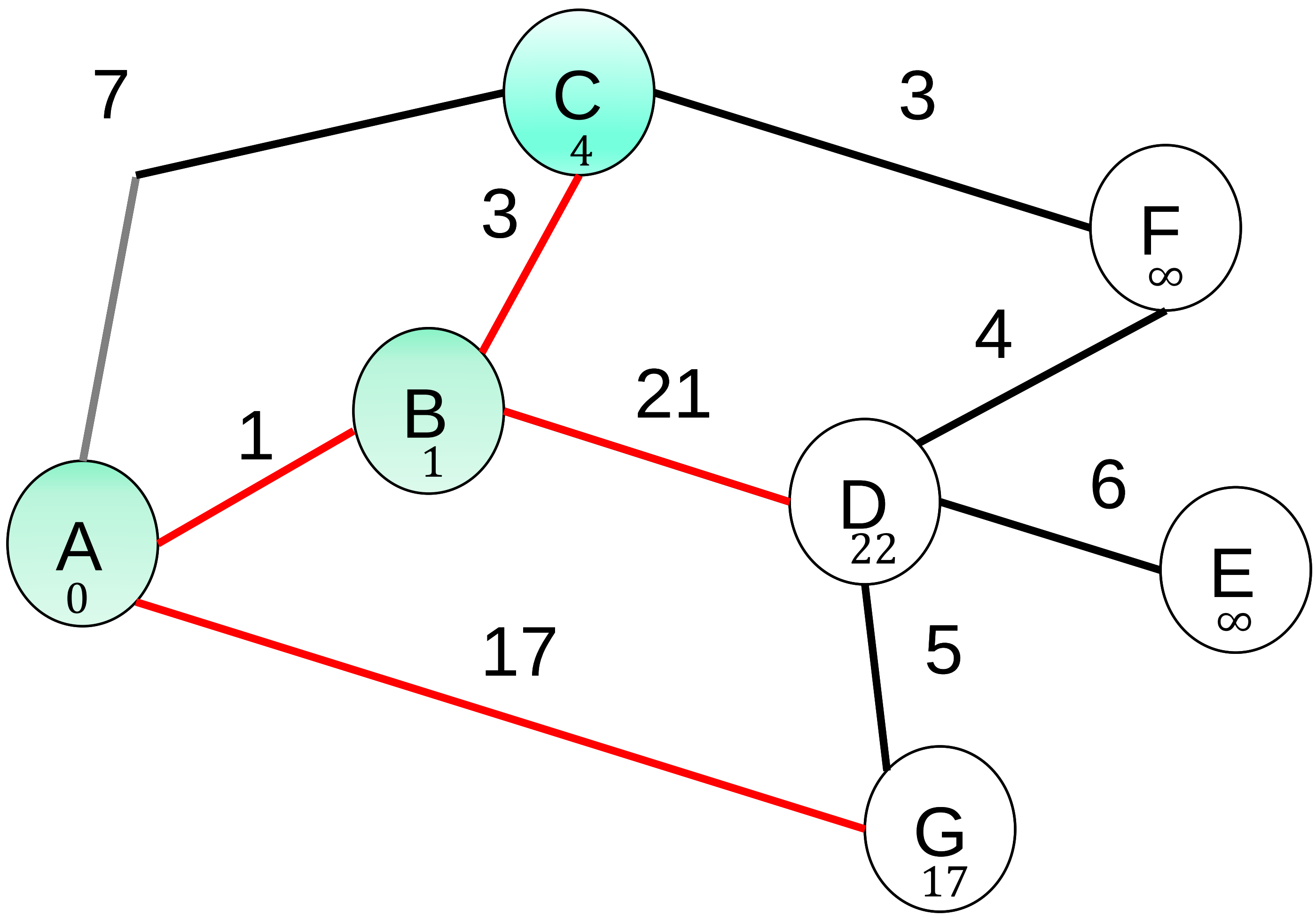
current vertex is C, cost 4

loop through C's edges



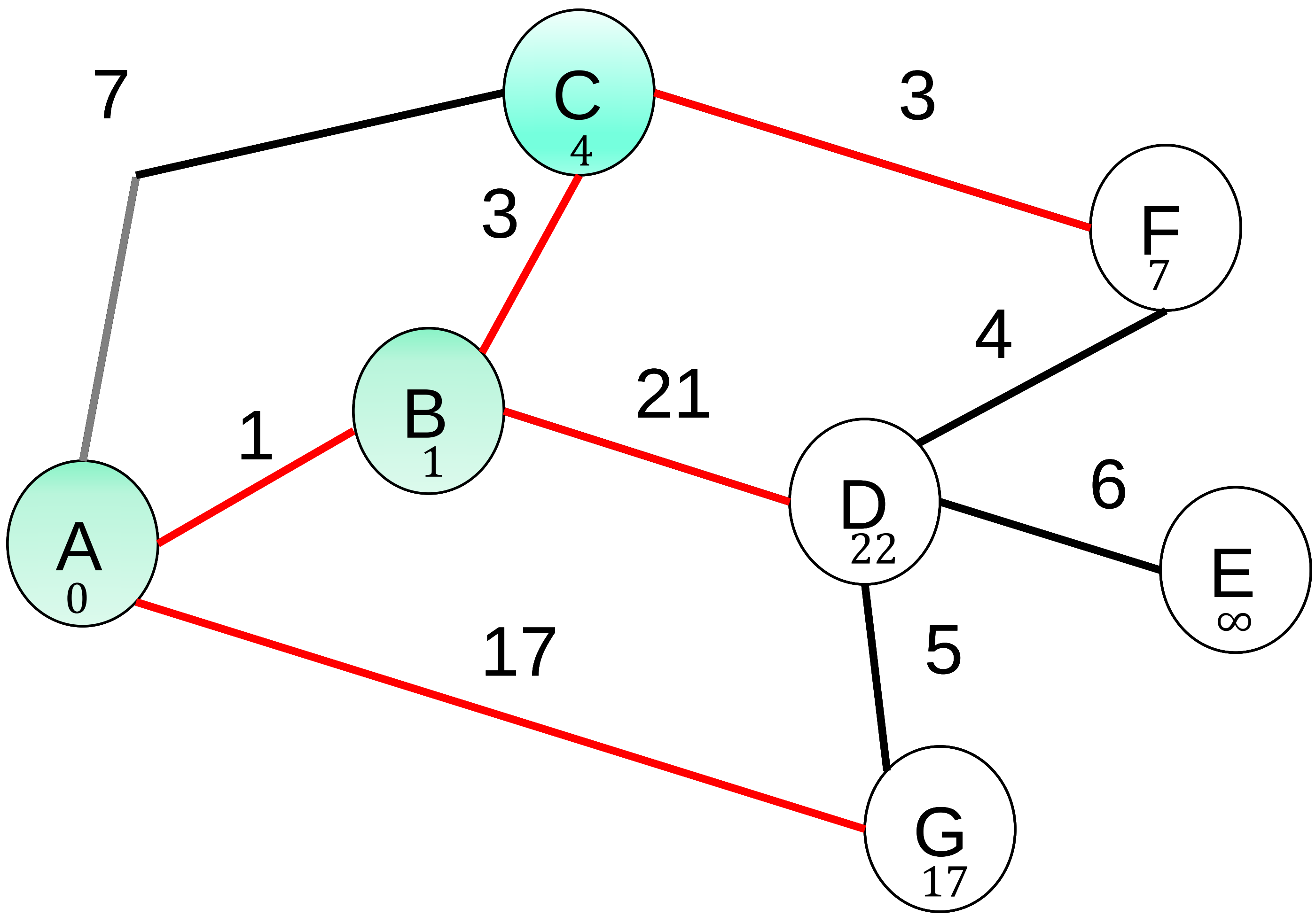
[(C, 7), (G, 17), (D, 22)] pq

C -> A, 7 + 4 !< 0, skip



[(C, 7), (G, 17), (D, 22)] pq

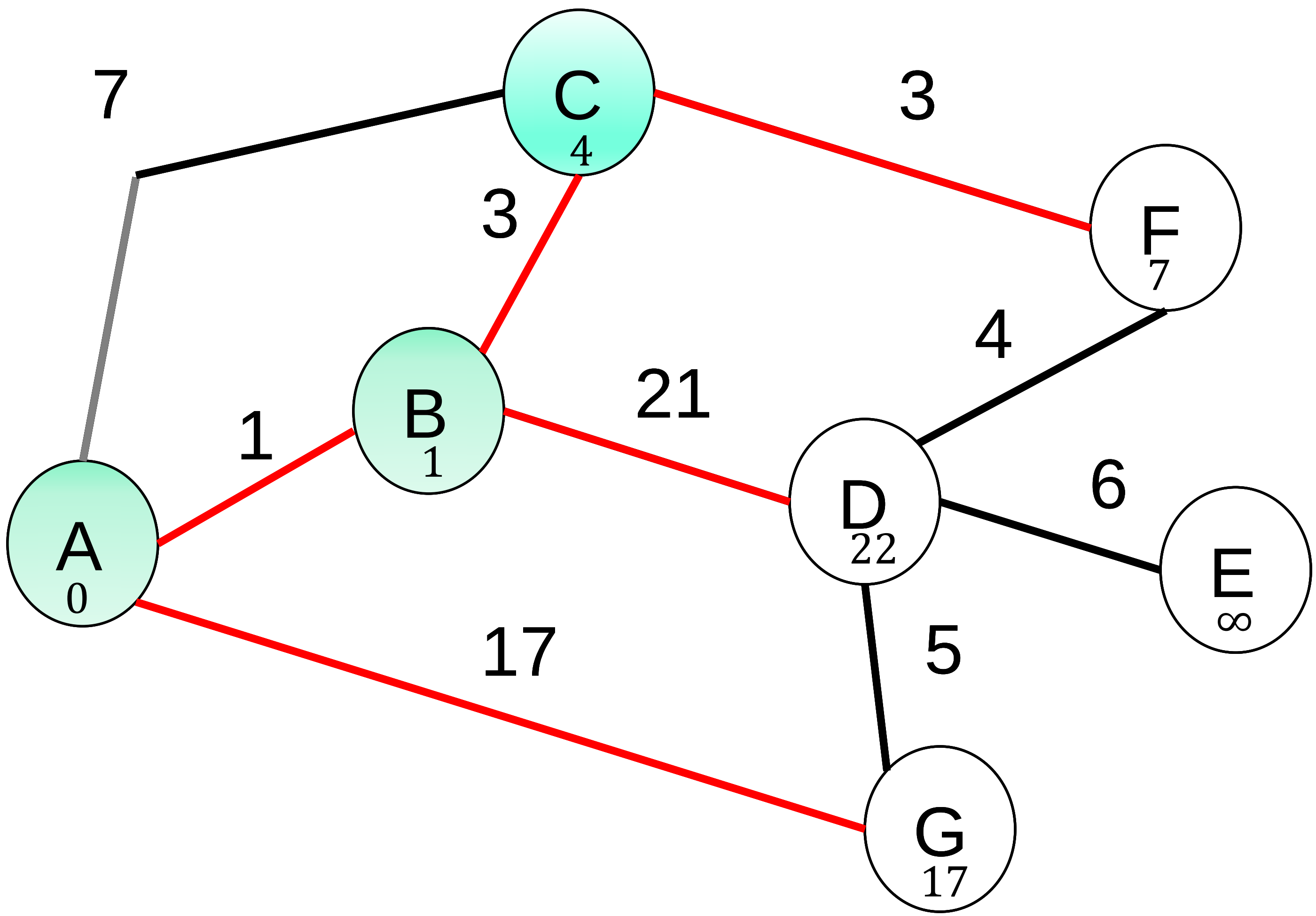
C -> B, 4 + 3 !< 1, skip



[(C, 7), (G, 17), (D, 22)] pq

C -> F, 4 + 3 < INFINITY

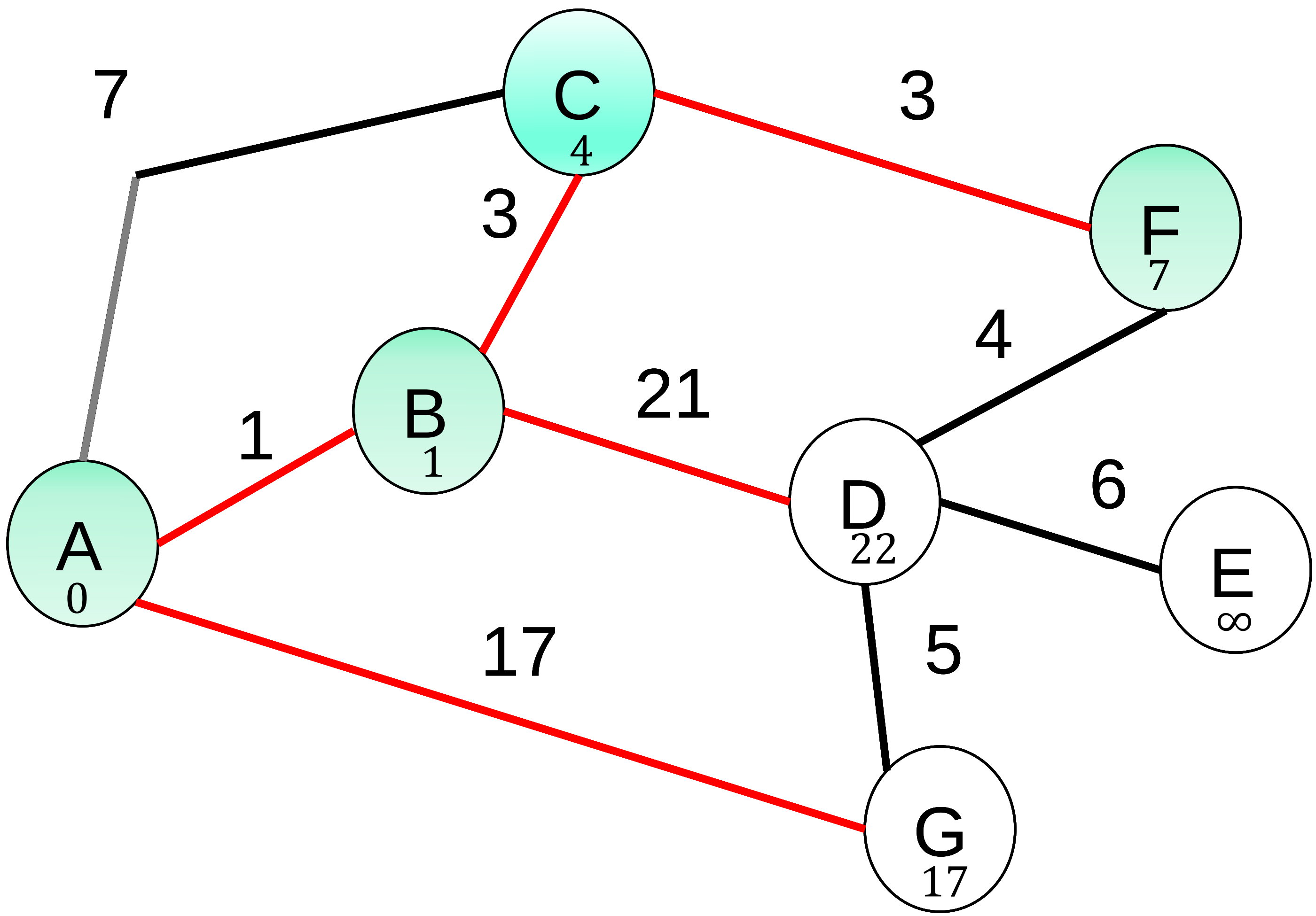
[(C, 7), (F, 7), (G, 17), (D, 22)] pq



[(C, 7), (F, 7), (G, 17), (D, 22)] pq

current vertex is C

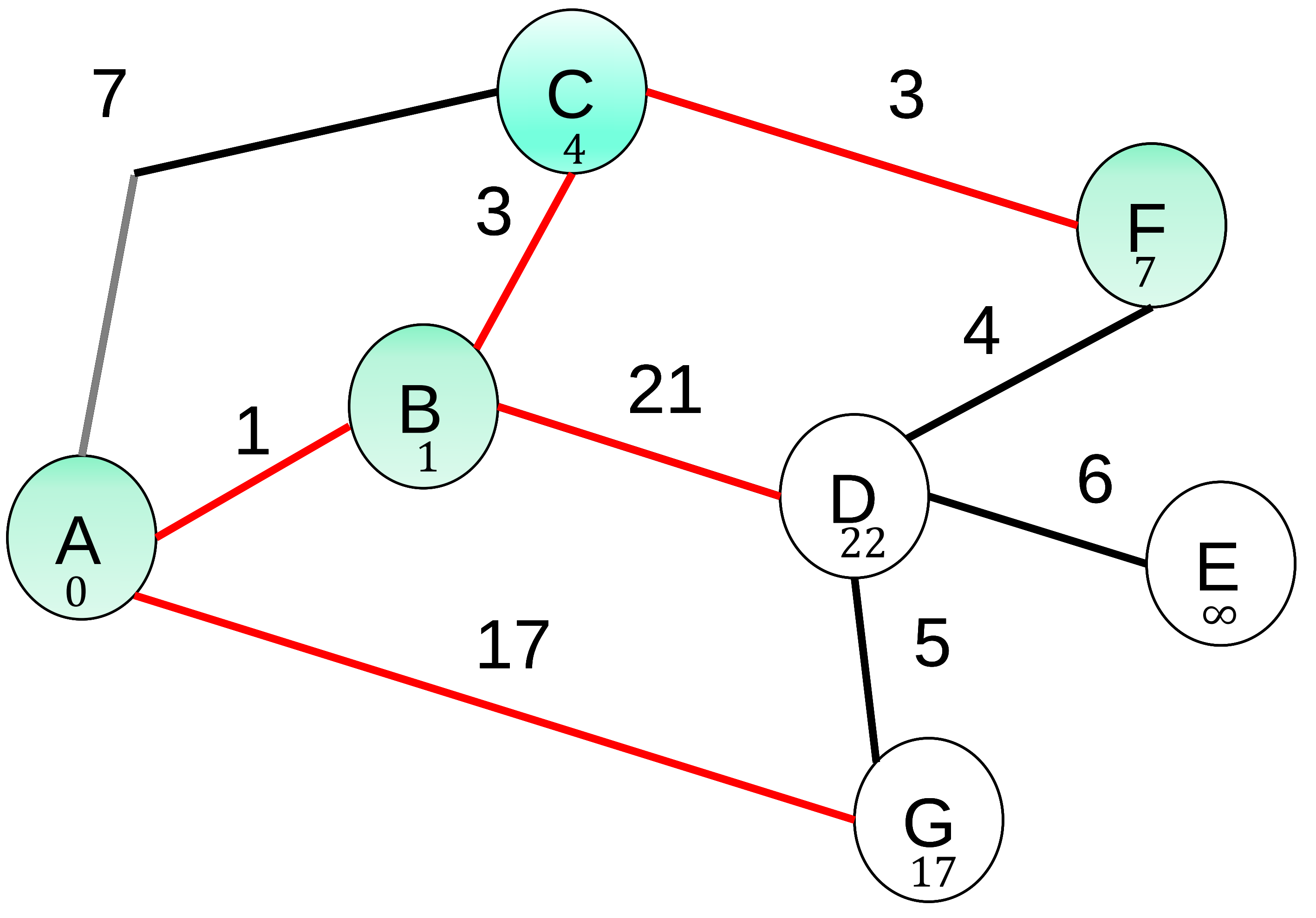
Already visited so skip



[(F, 7), (G, 17), (D, 22)] pq

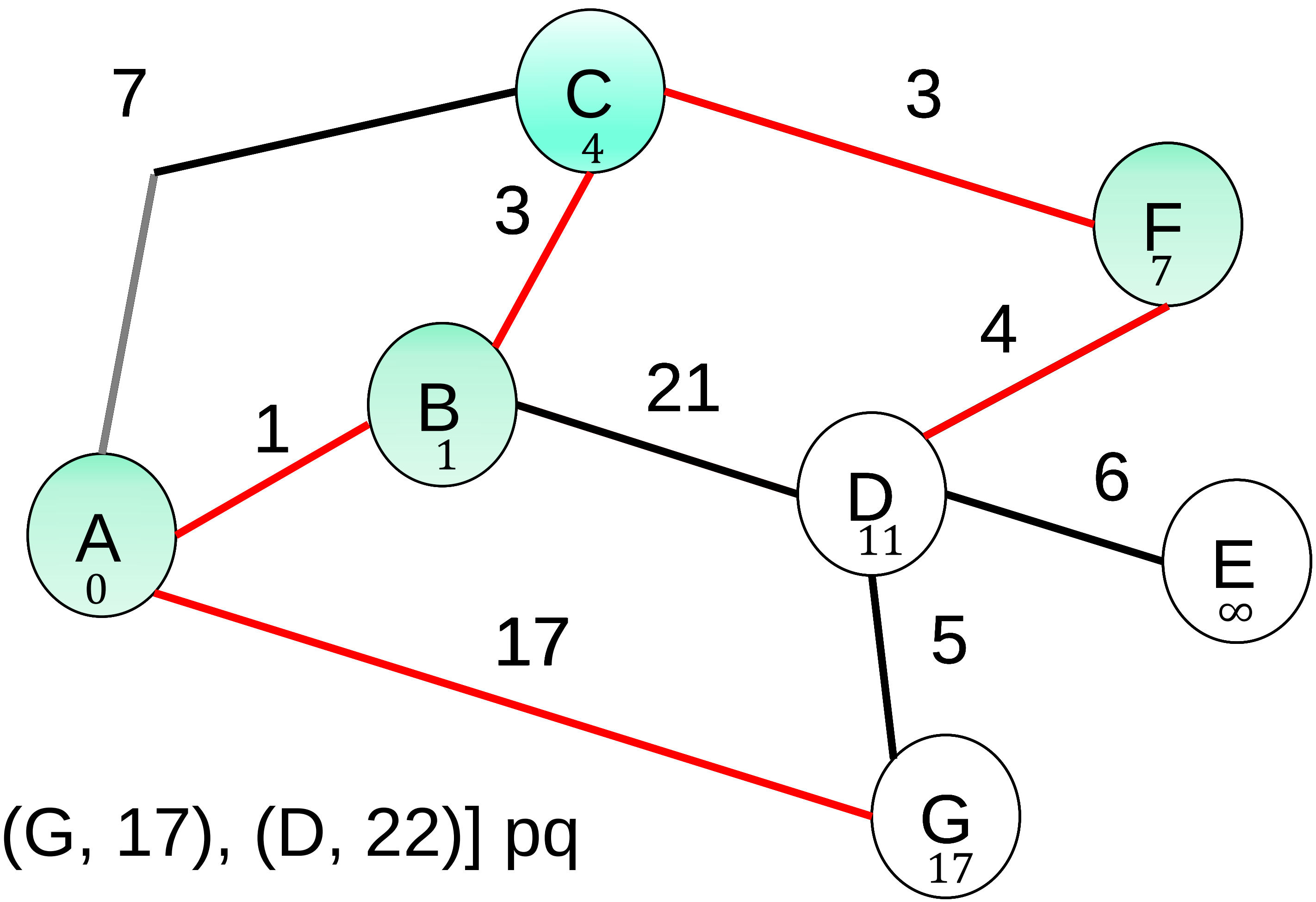
current vertex is F

loop through F's edges



[(G, 17), (D, 22)] pq

F -> C, 7 + 3 !< 4, so skip



[(G, 17), (D, 22)] pq

F -> D, 7 + 4 < 22

update D's cost and previous

[(D, 11), (G, 17), (D, 22)] pq

Aside - Implementing Dijkstra's

- Create a Path class to allow for multiple paths and distances (costs) to a given vertex

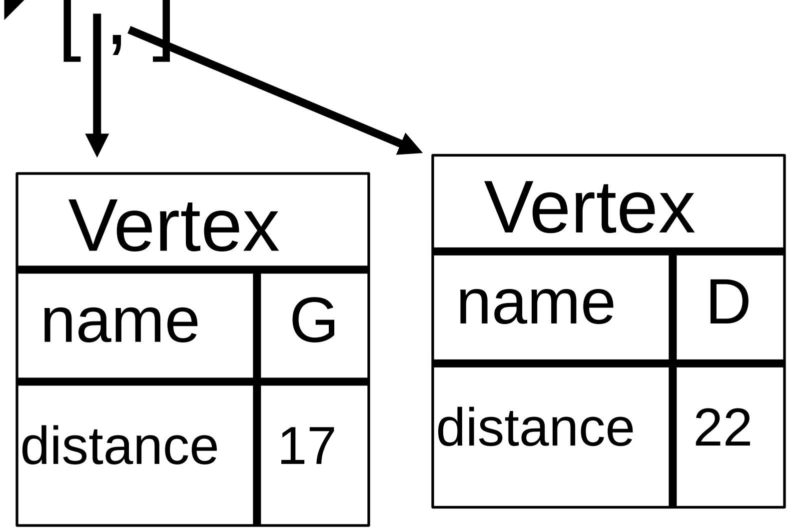
```
private static class Path
    implements Comparable<Path> {


    private Vertex dest;
    private double cost;
```

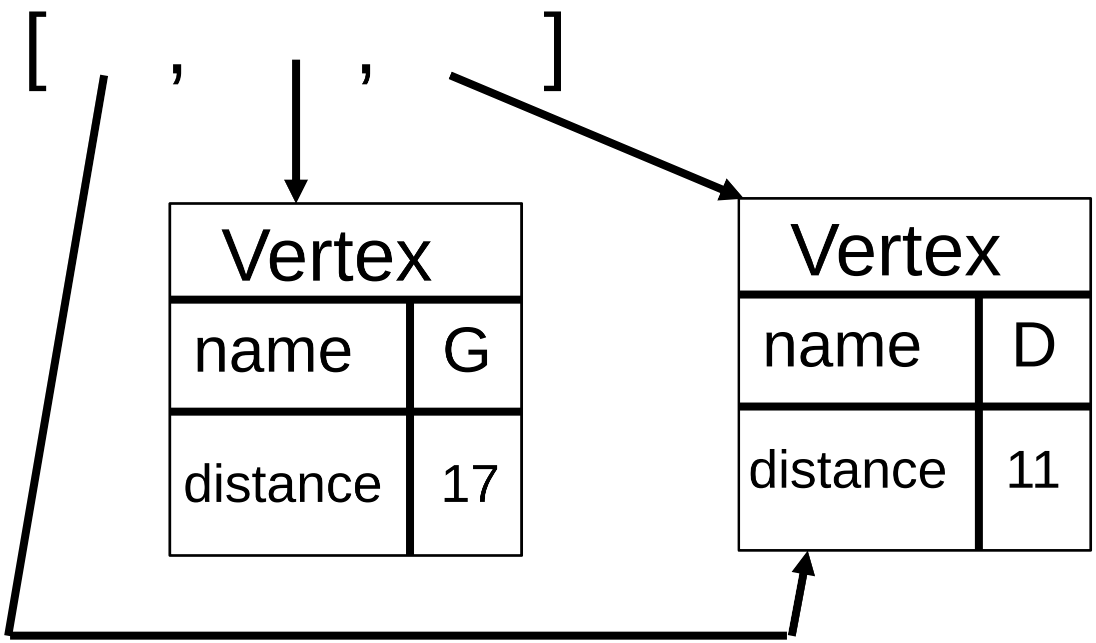
- Use a priority queue of Paths to store the vertices and distances

Why? References!!!

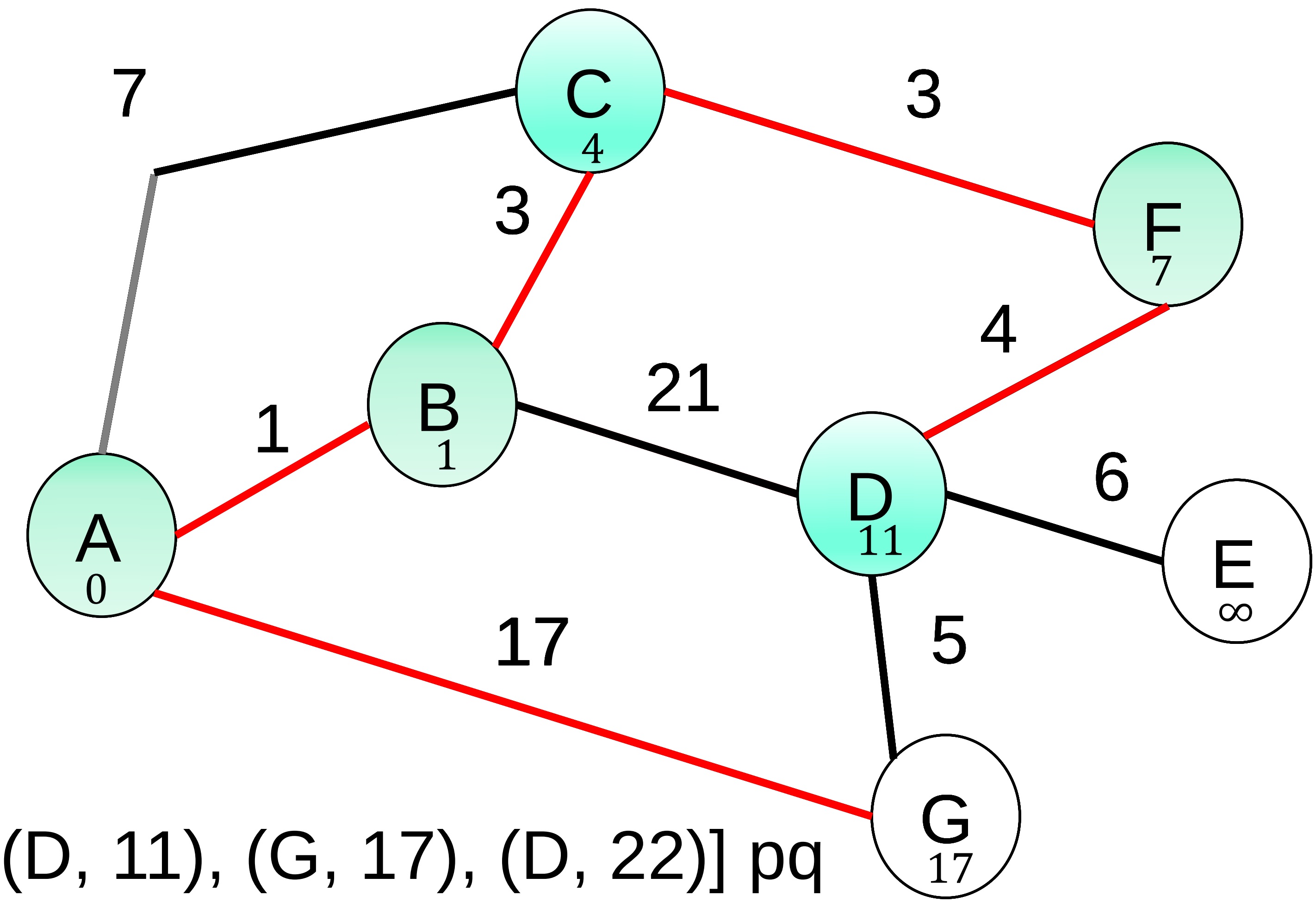
- Slide 74 and 78, adding new, lower cost path to Vertex D
- Abstractly: [(G, 17), (D, 22)] becomes
  [(D, 11) (G, 17), (D, 22)]
- What does priority queue store? ***References to Vertex Objects***
- [,]



Lower Cost Path to D

- New, lower cost path to D. Alter Vertex D's distance to 11 and add to priority queue
- 

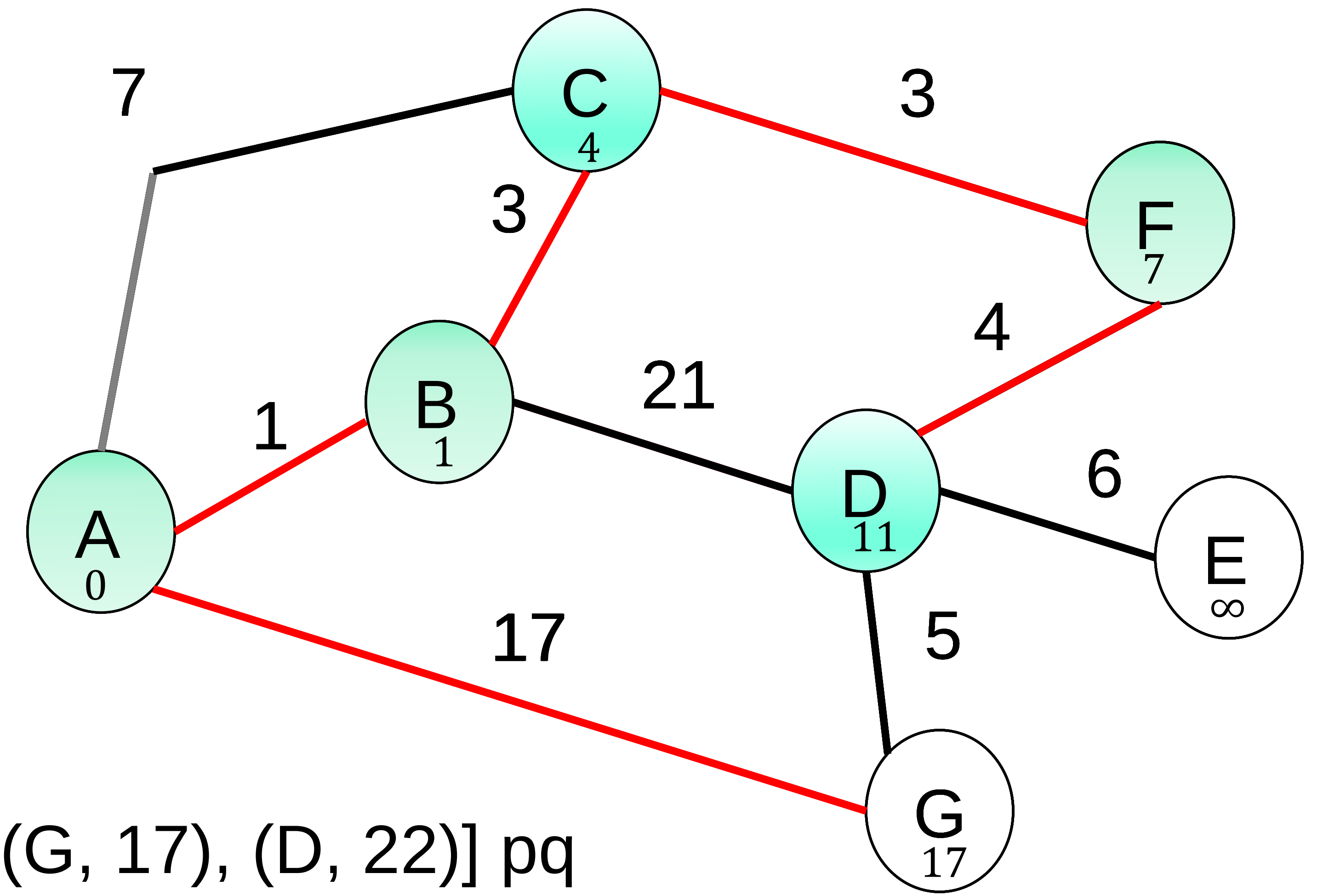
- PROBLEMS?????
- Abstractly [(D, 11), (G, 17), (D, 11)]



[(D, 11), (G, 17), (D, 22)] pq

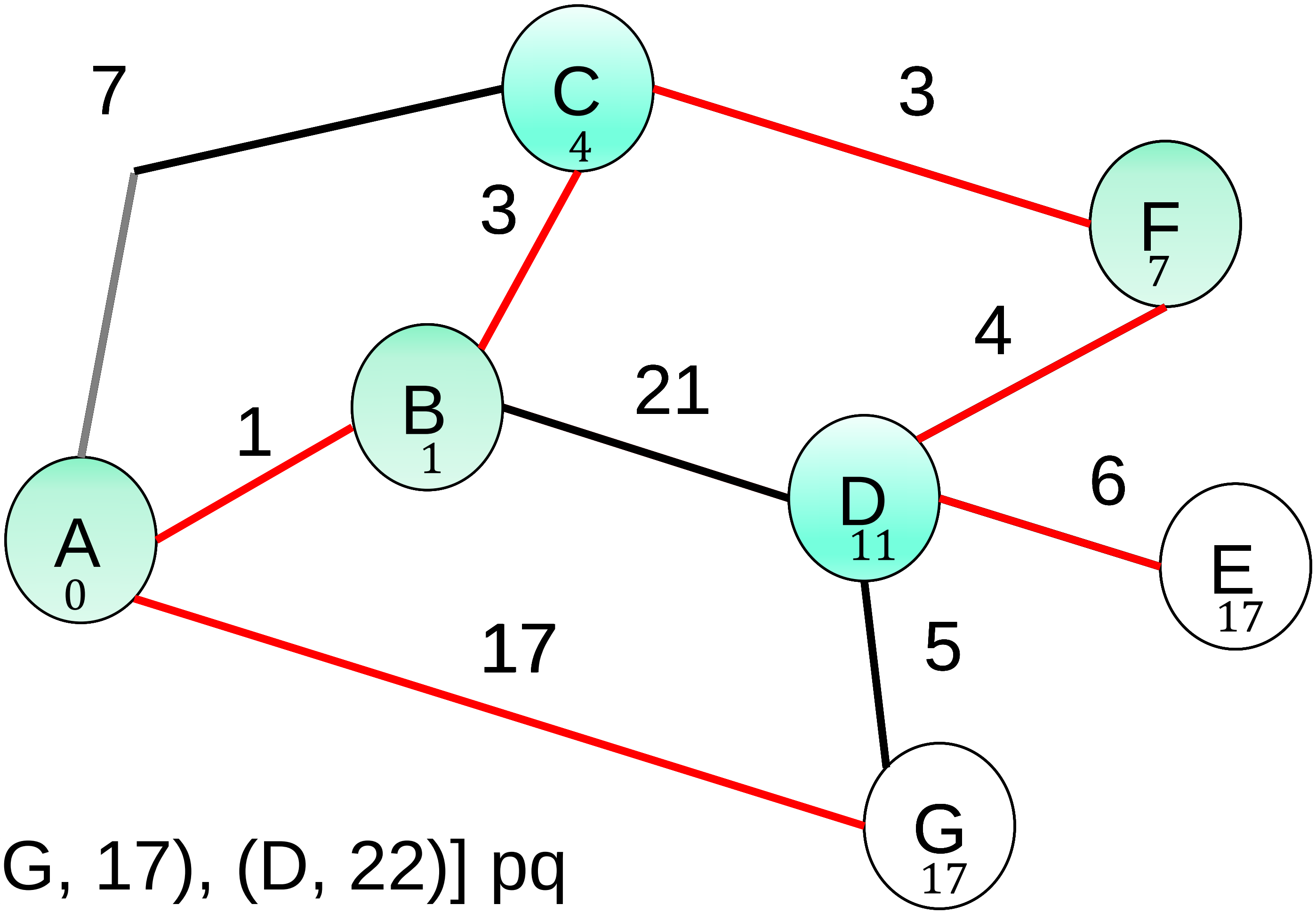
current vertex is D

loop through D's edges



[(G, 17), (D, 22)] pq

D -> B, 11 + 21 !< 1, so skip

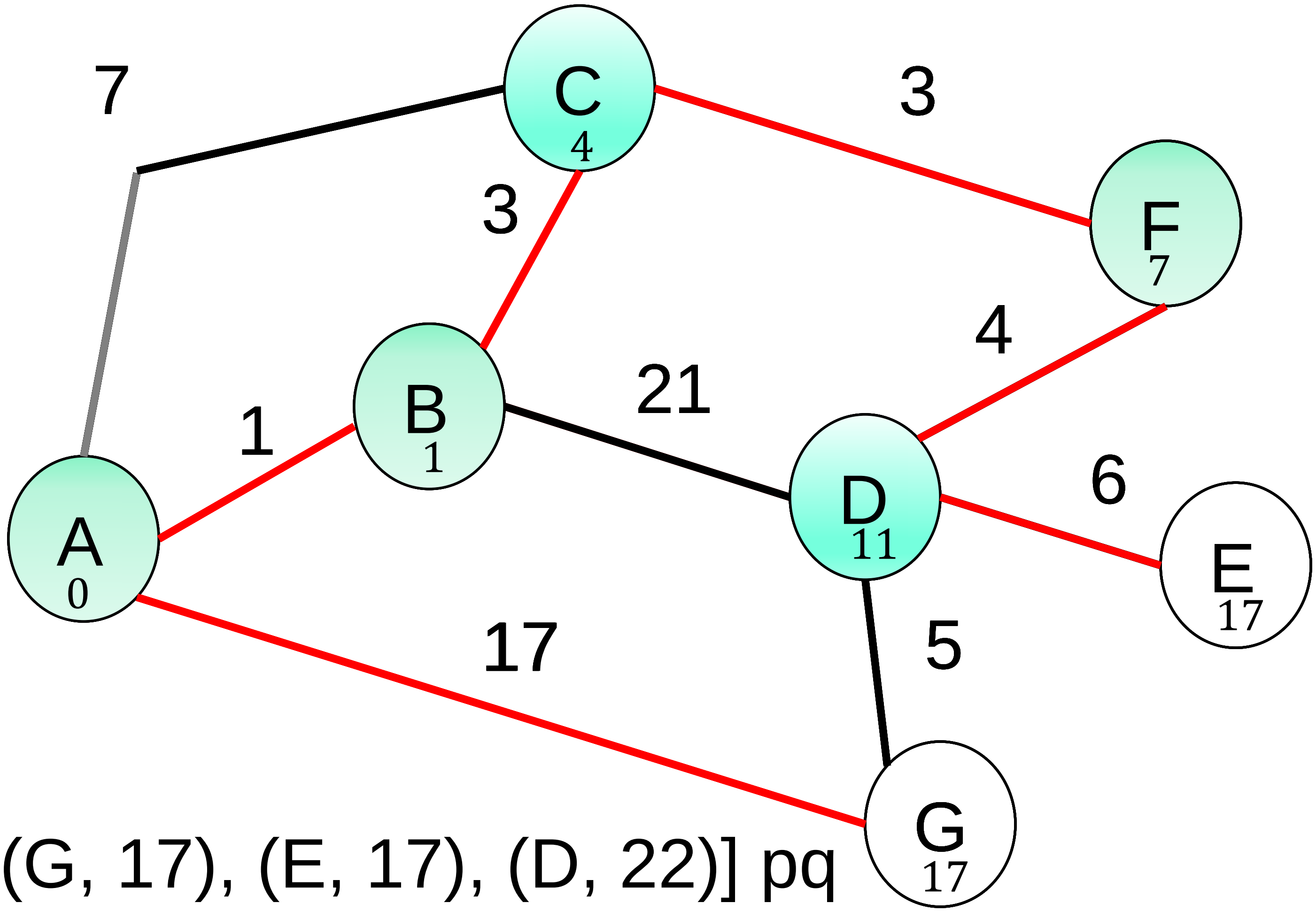


[(G, 17), (D, 22)] pq

D -> E, 11 + 6 < INFINITY

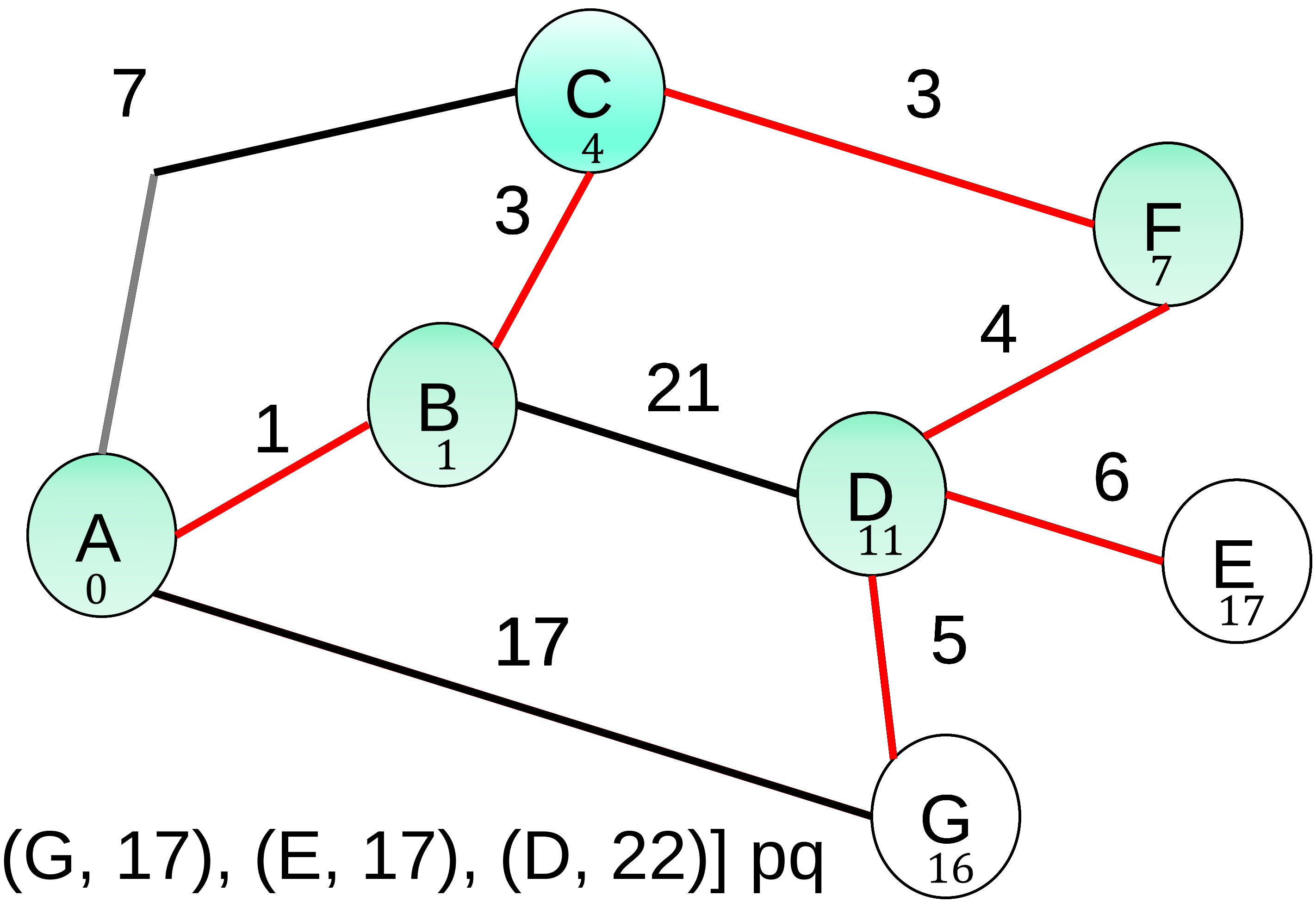
update E's cost and previous

[(G, 17), (E, 17), (D, 22)] pq



[(G, 17), (E, 17), (D, 22)] pq

D -> F, 4 + 11 !< 7, so skip

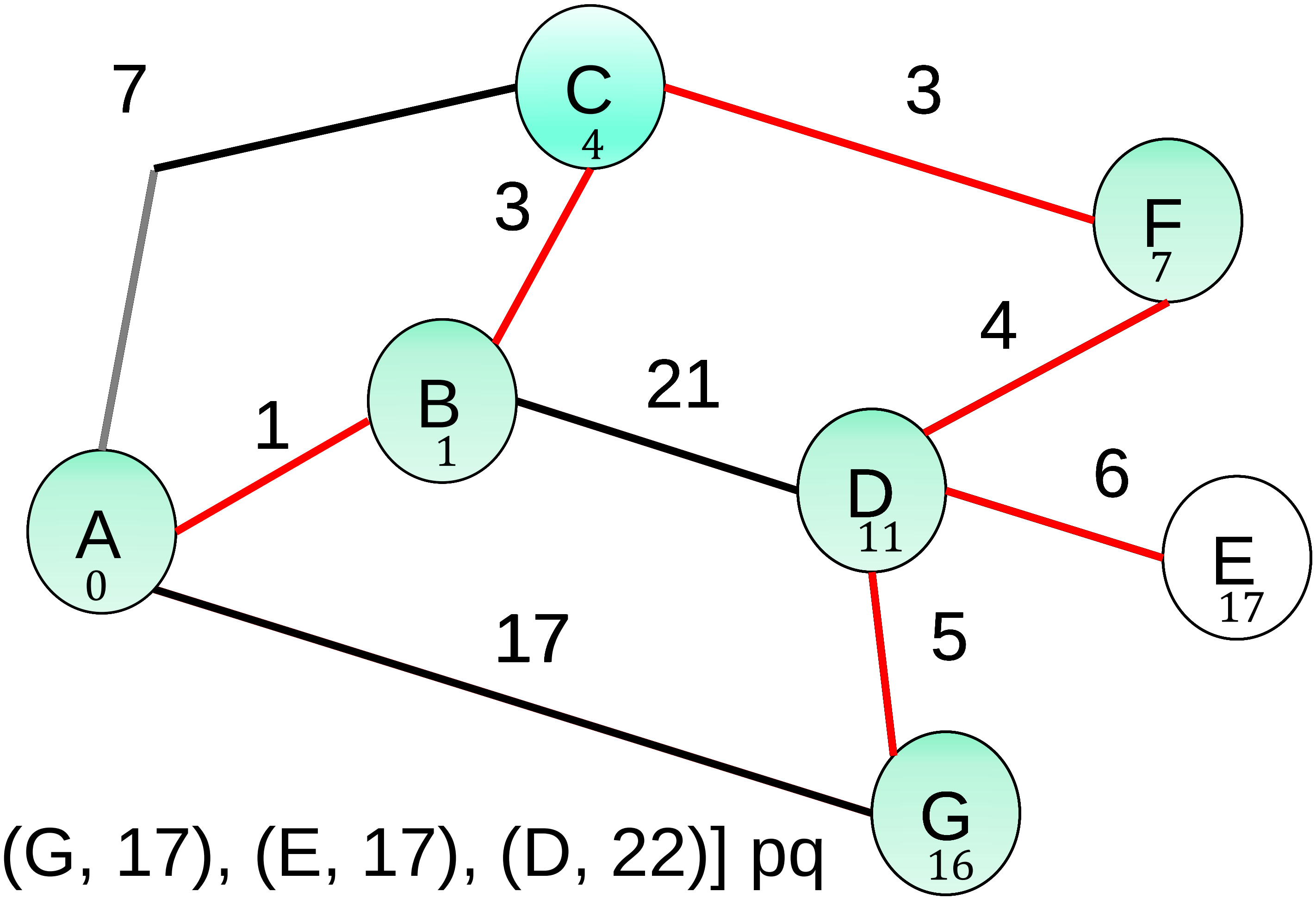


[(G, 17), (E, 17), (D, 22)] pq

D -> G, 11 + 5 < 17

update G's cost and previous

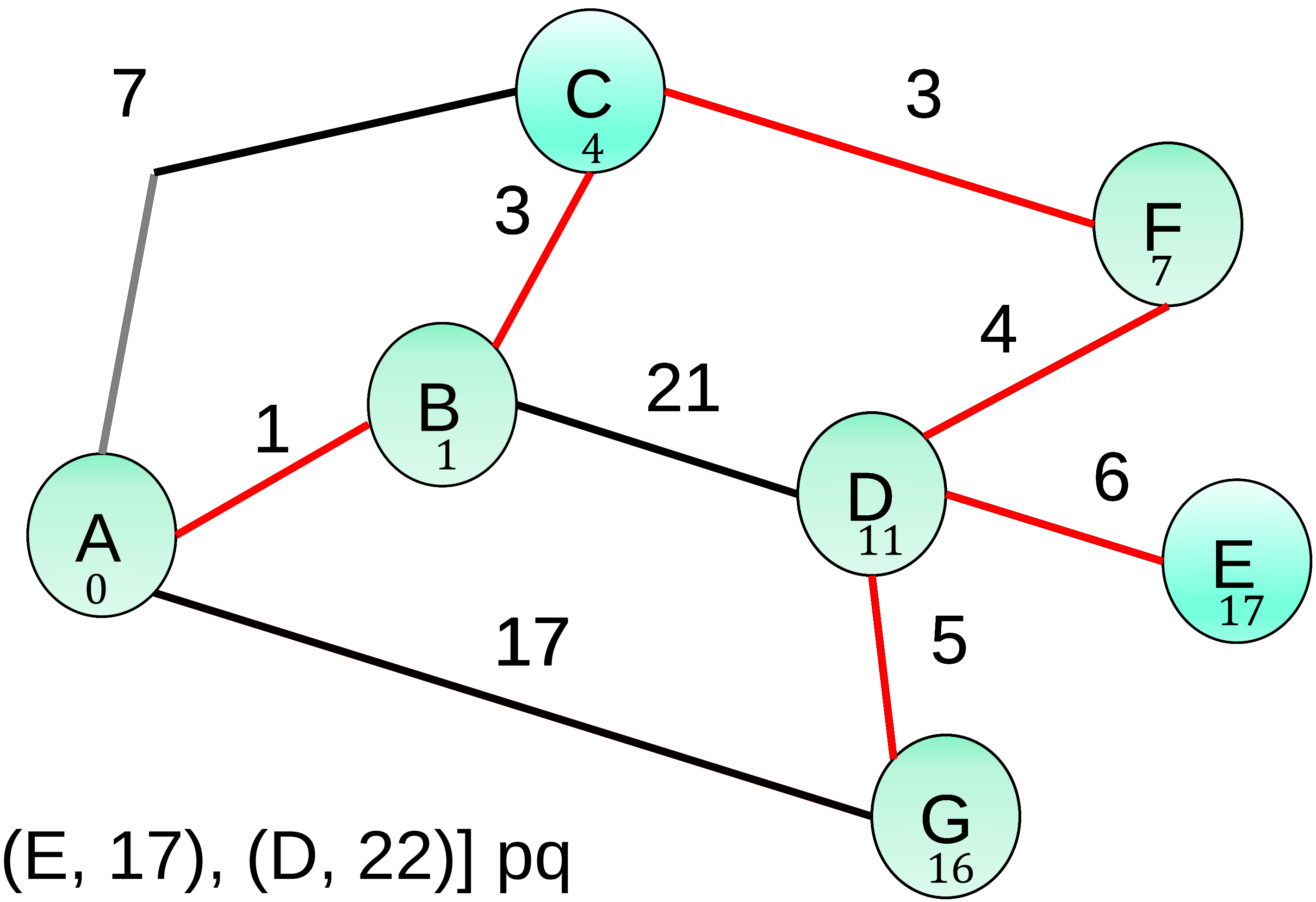
[(G, 16), (G, 17), (E, 17), (D, 22)] pq



[(G, 17), (E, 17), (D, 22)] pq

current vertex is G

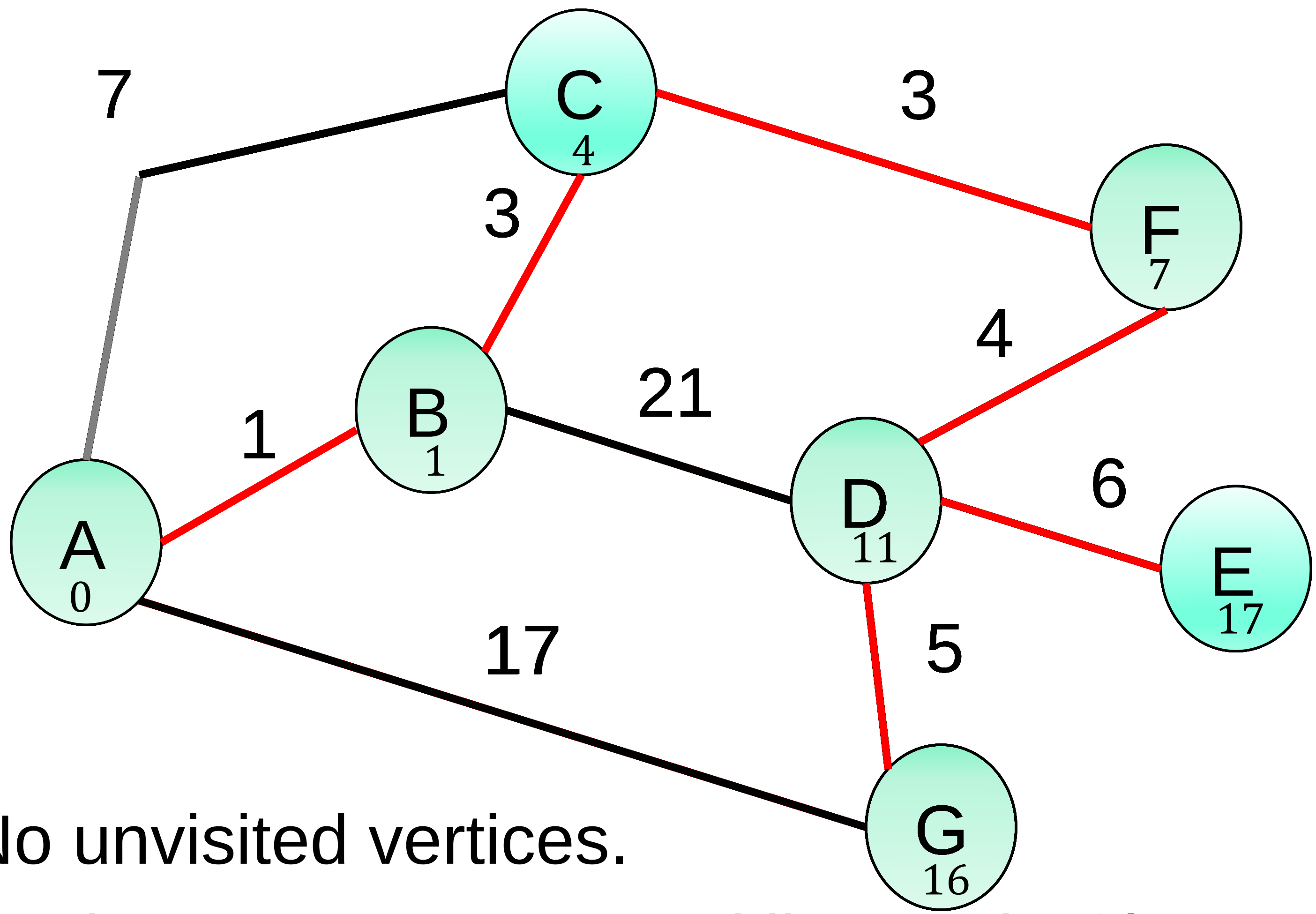
loop though edges, already visited all neighbors



[(E, 17), (D, 22)] pq

current vertex is E

loop though edges, already visited all neighbors



No unvisited vertices.

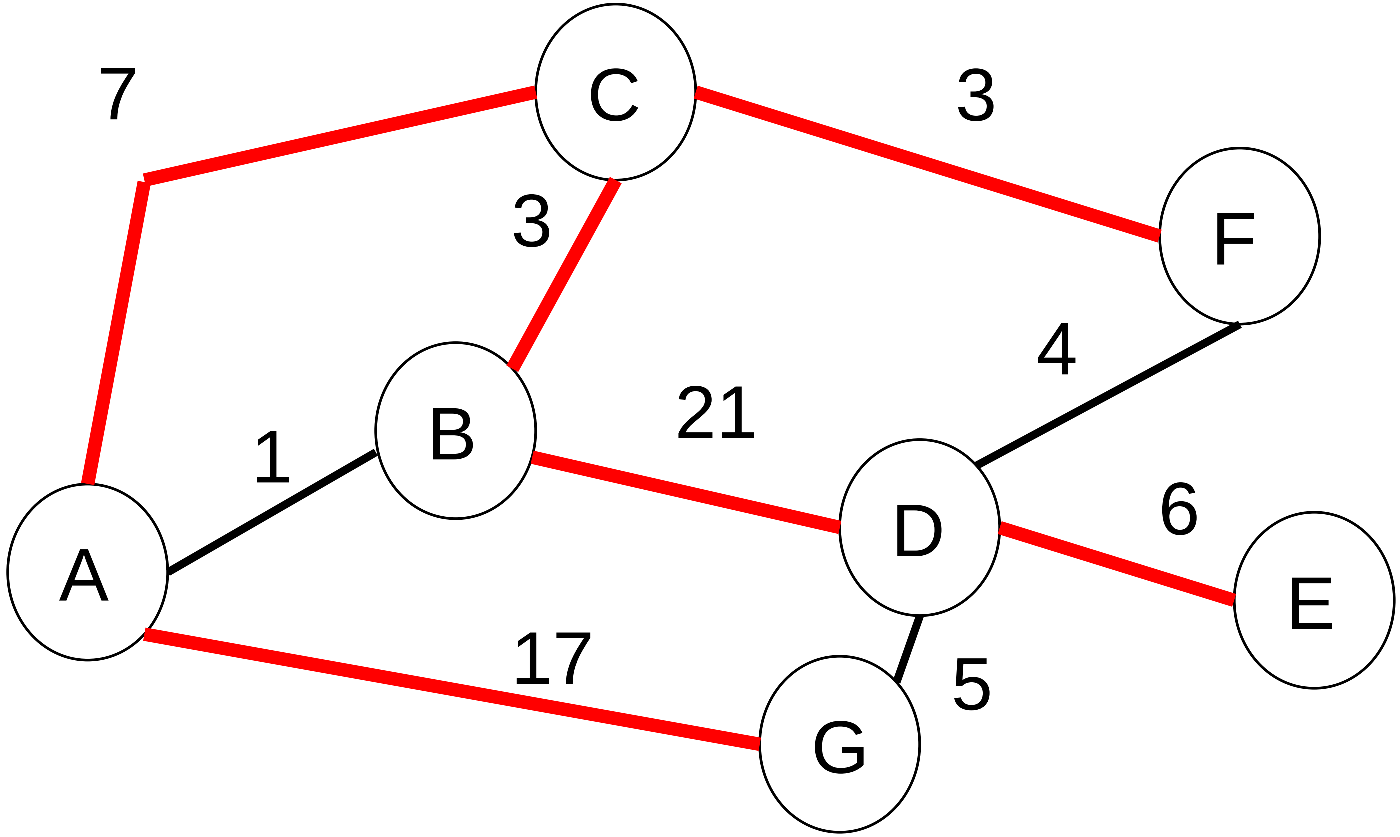
Each Vertex stores cost (distance) of lowest cost path from start Vertex to itself and previous vertex in path from start vertex to itself.

Alternatives to Dijkstra's Algorithm

- A*, pronounced "A Star"
- A heuristic, goal of finding shortest weighted path from single start vertex to goal vertex
- Uses actual distance like Dijkstra's but also estimates *remaining cost or distance*
  - *distance is set to current distance from start PLUS the* ***estimated distance*** *to the goal*
- For example when finding a path between towns, estimate the remaining distance as the straight-line (*as the crow flies)* distance between current location and goal.

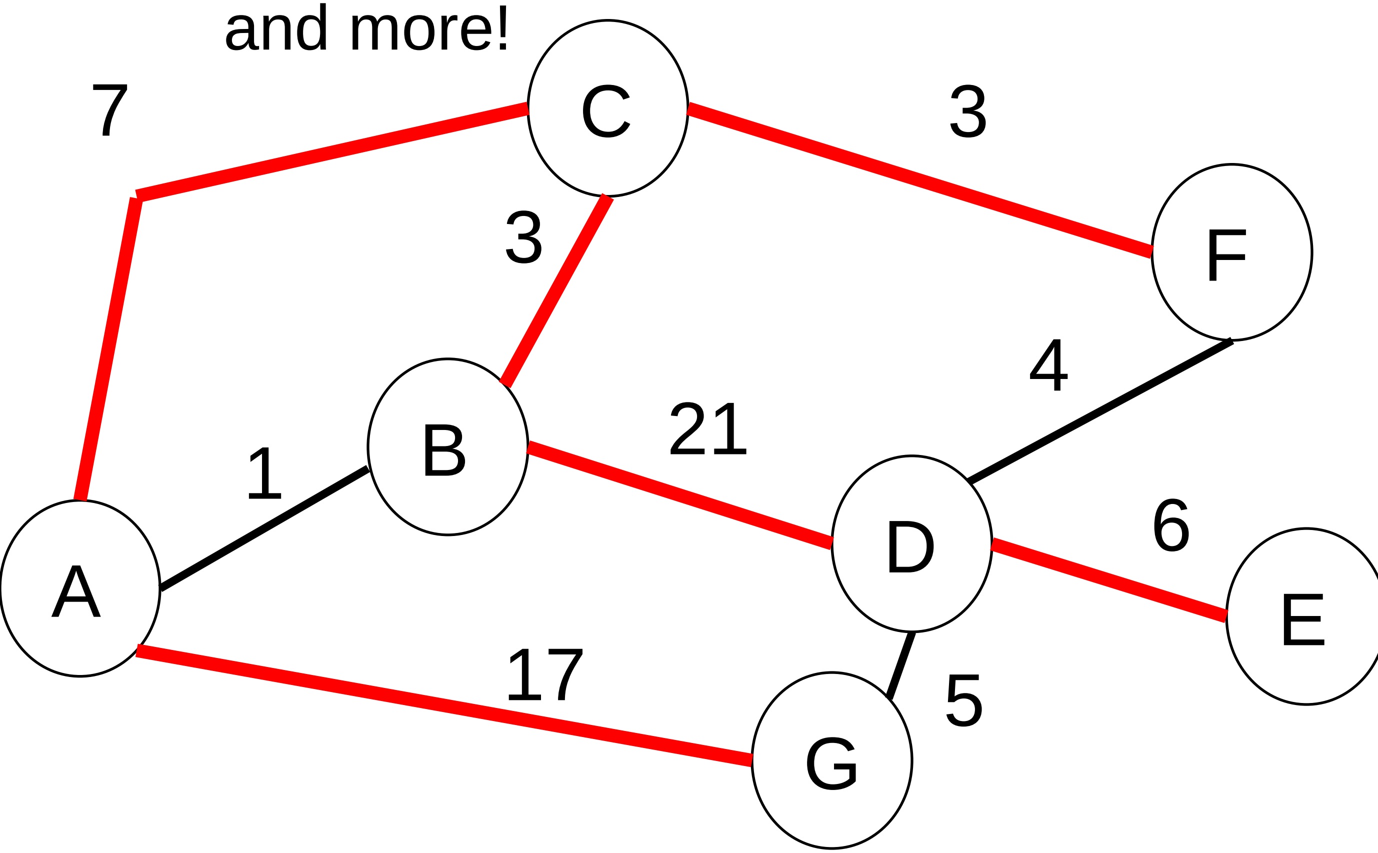
Spanning Tree

- *Spanning Tree:* A tree of edges that connects all the vertices in a graph



Clicker 7 - Minimum Spanning Tree

- *Minimum Spanning Tree*: A spanning tree in a weighted graph with the lowest total cost
  - used in network design, taxonomy, Image registration, and more!



Cost of spanning tree shown?

A. 6

B. 7

C. 29

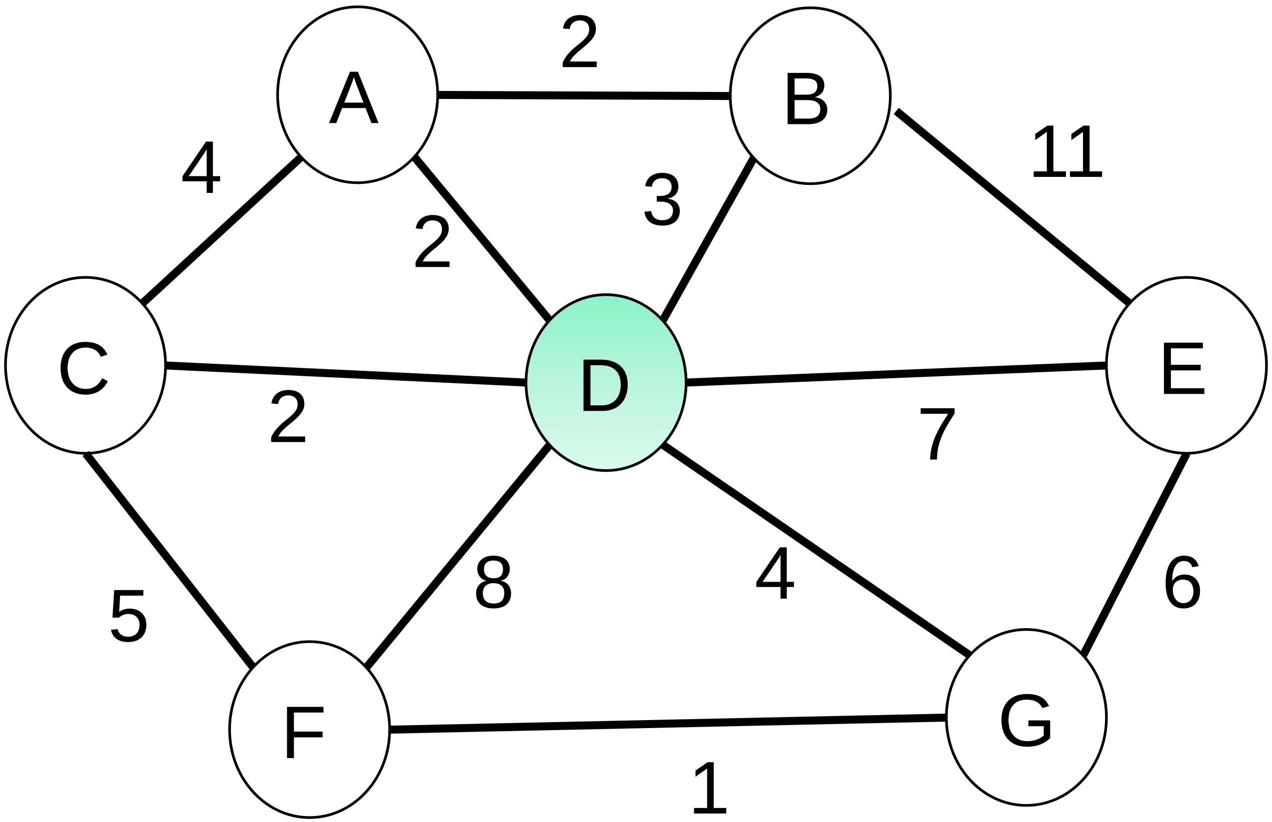
D. 61

E. None of These

Prim's Algorithm

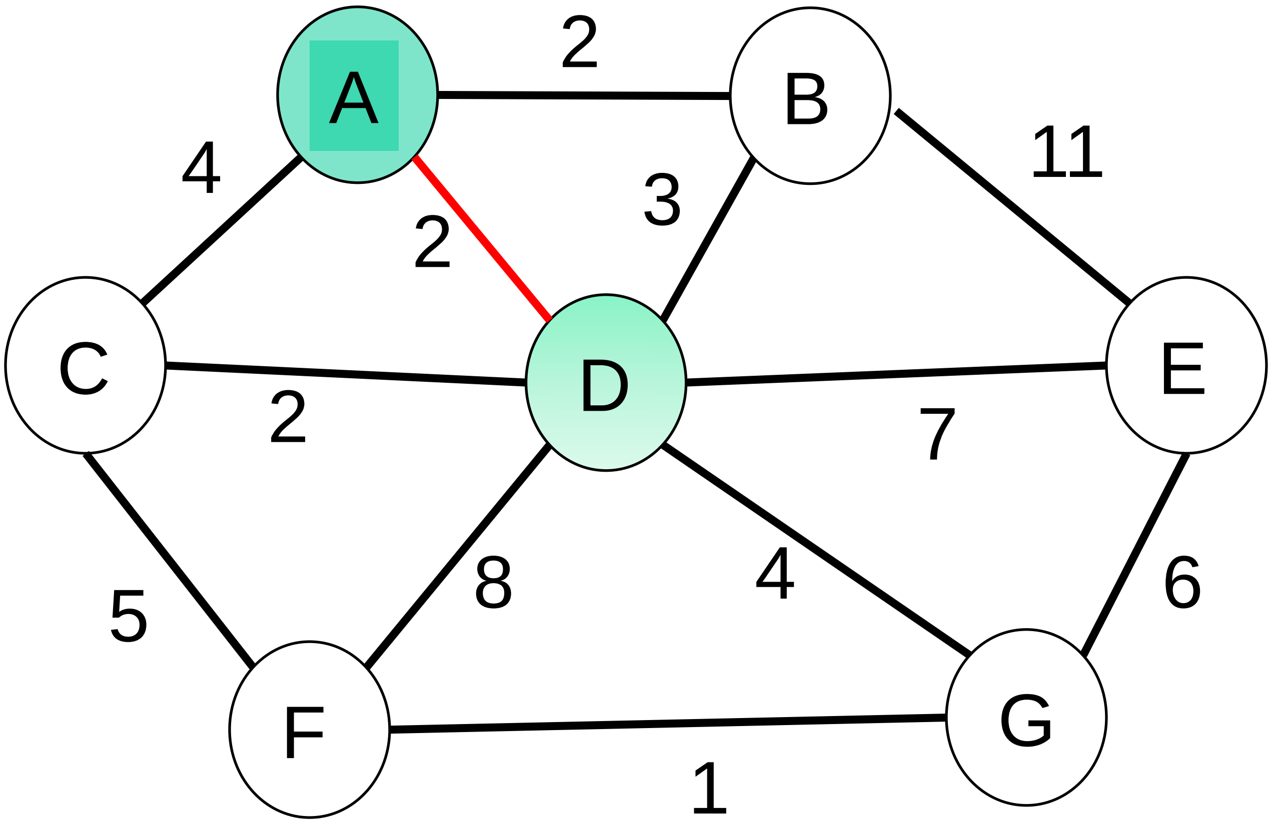
- Initially created by Vojtěch Jarník
- Rediscovered by Prim (of Sweetwater, TX) and Dijkstra
- Pick a vertex arbitrarily from graph
  - In other words, it doesn't matter which one
- Add lowest cost edge between the tree and a vertex that is not part of the tree UNTIL every vertex is part of the tree
- Greedy Algorithm, very similar to Dijkstra's

Prim's Algorithm



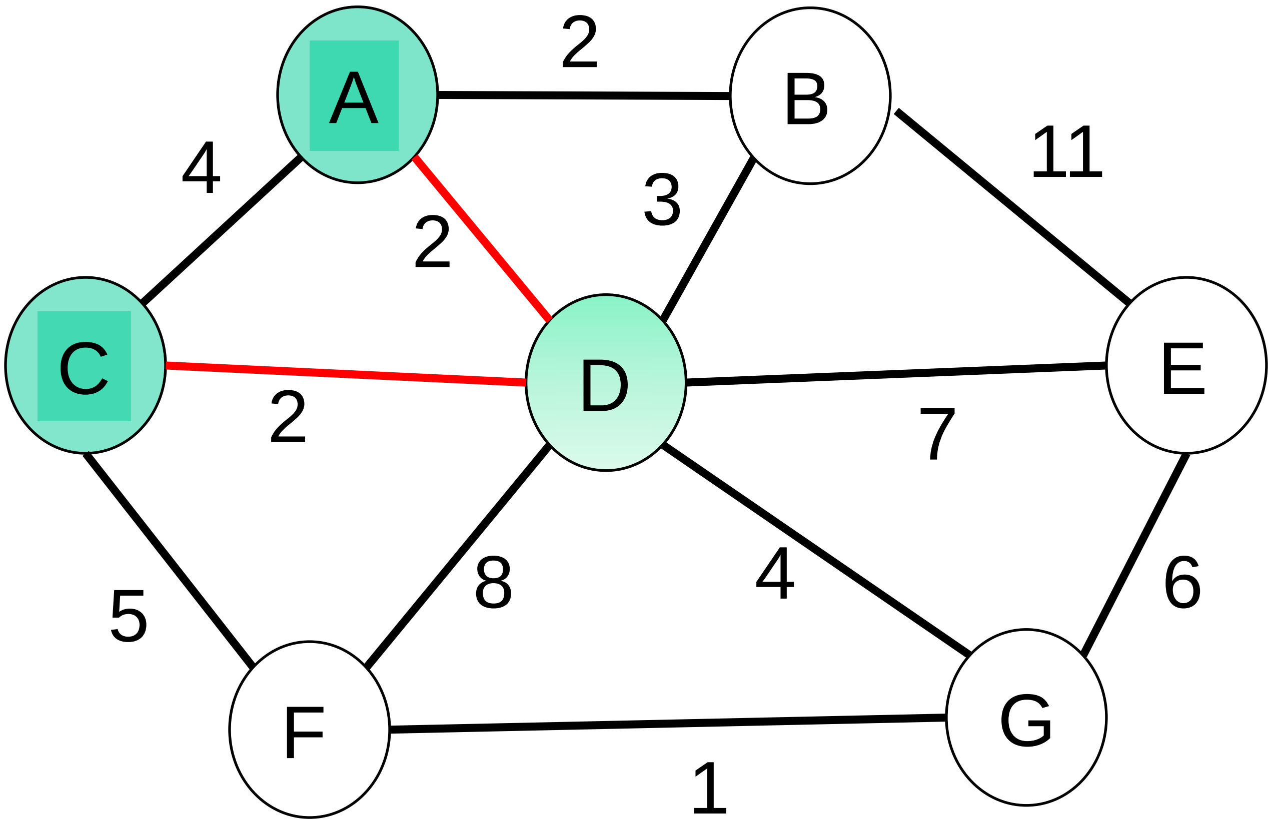
Pick D as root

Prim's Algorithm



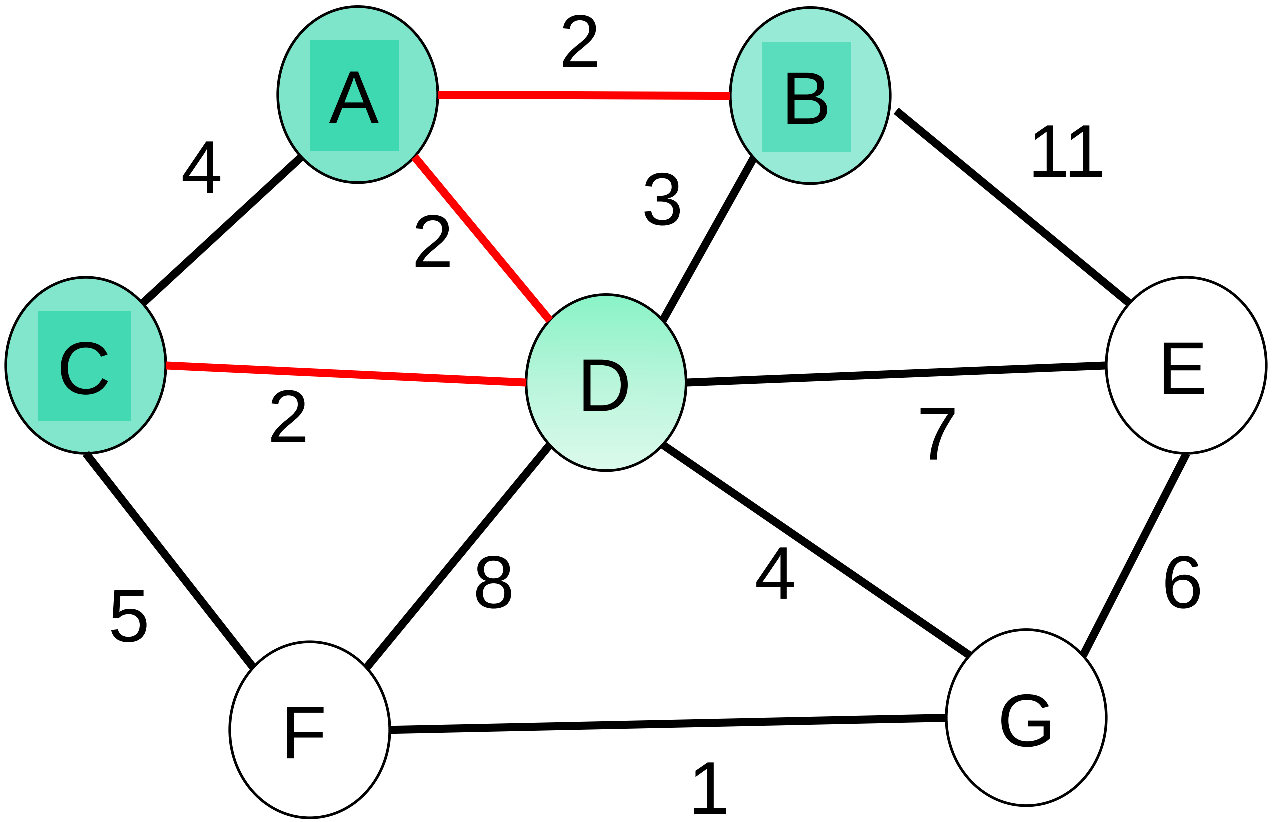
Lowest cost edge from tree to vertex not in Tree?
2 from D to A (or C)

Prim's Algorithm



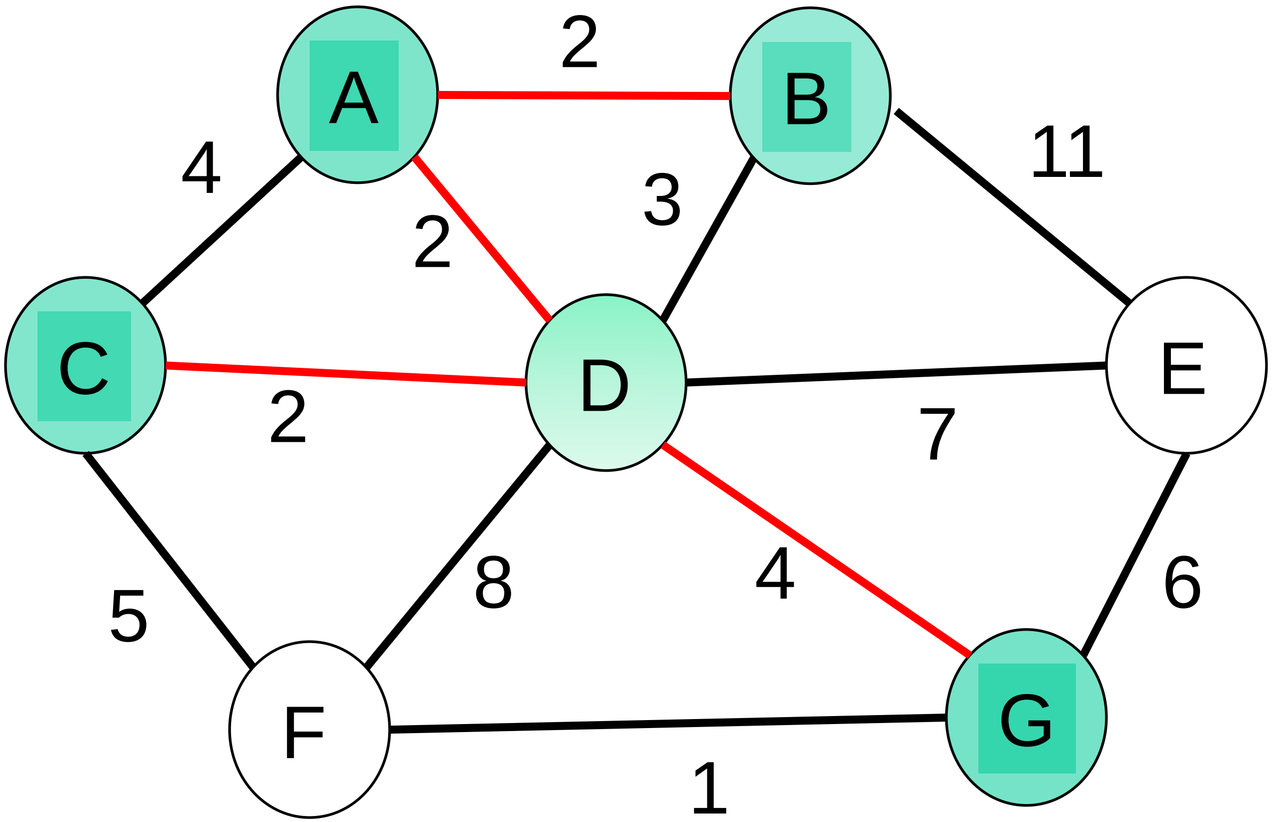
Lowest cost edge from tree to vertex not in Tree?
2 from D to C (OR from A to B)

Prim's Algorithm



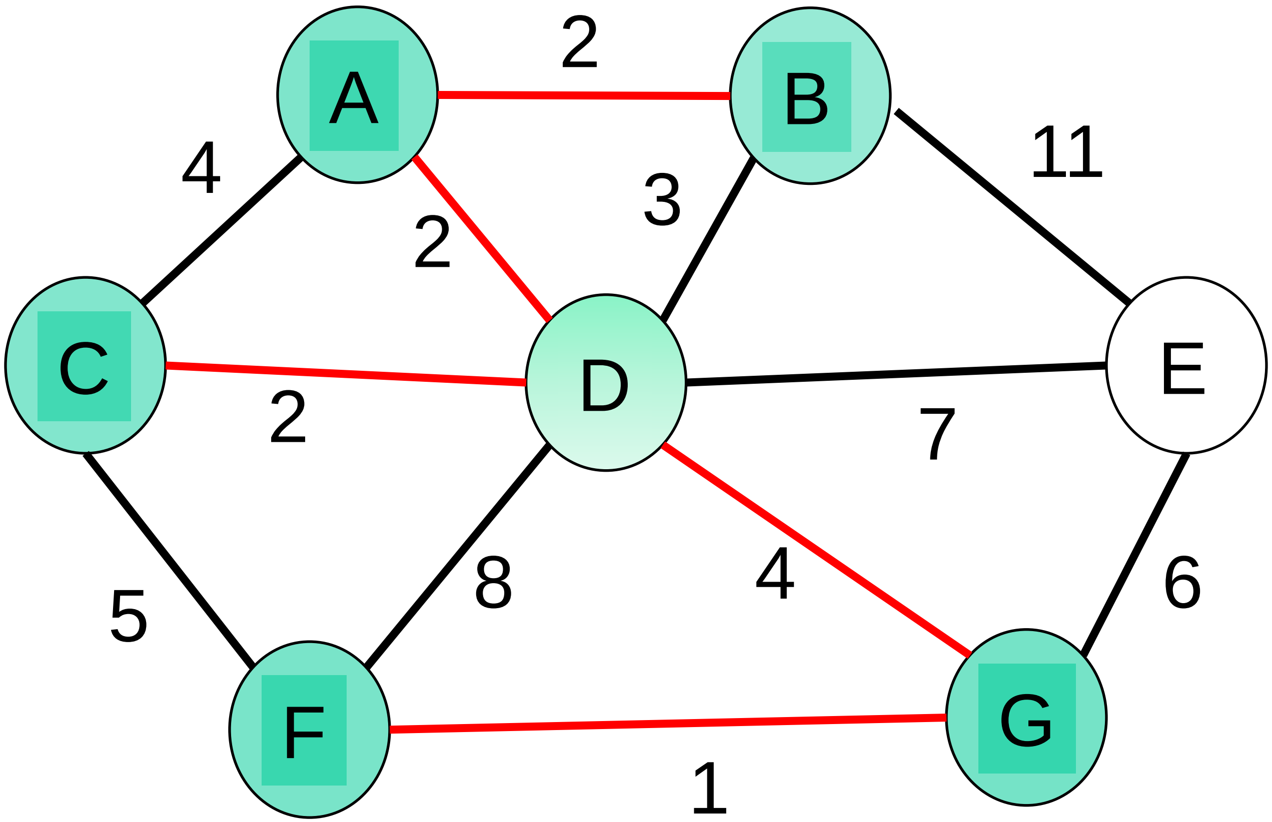
Lowest cost edge from tree to vertex not in Tree?
2 from A to B

Prim's Algorithm



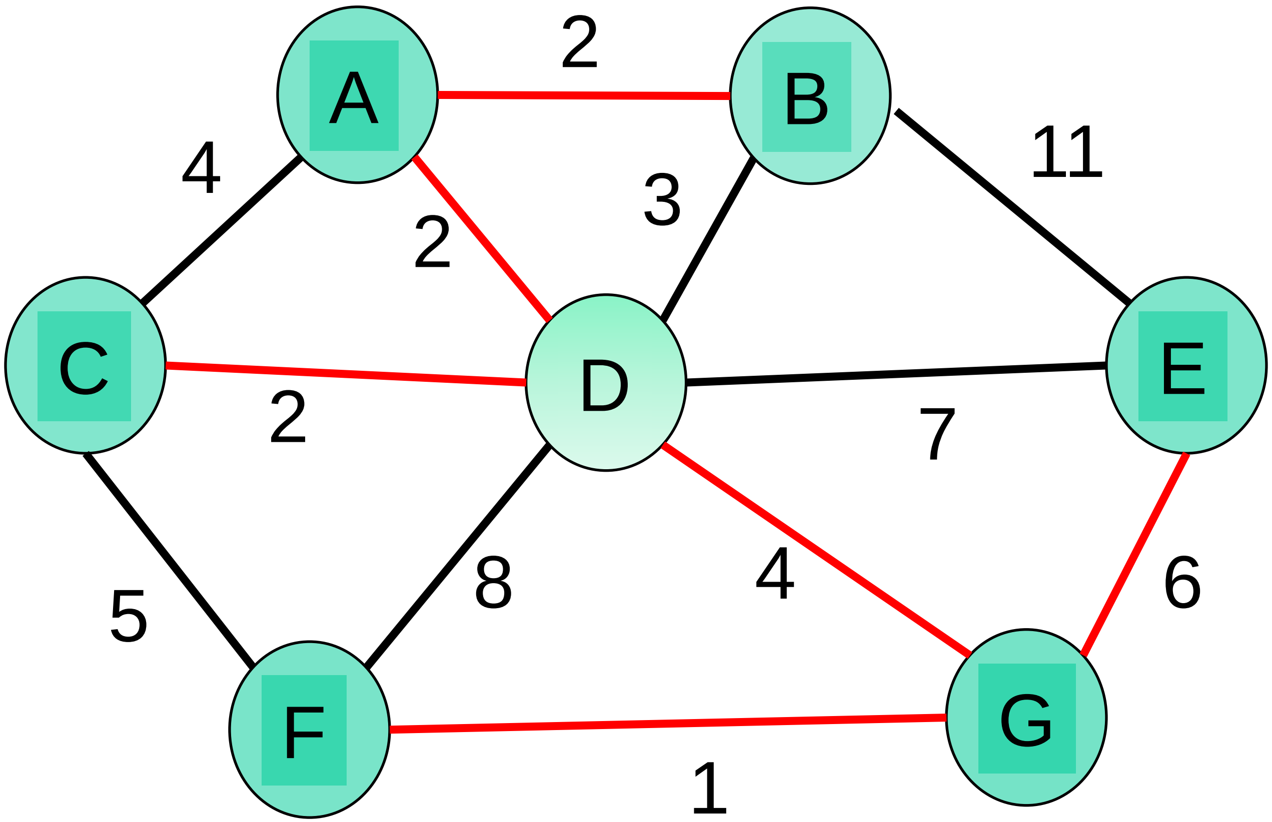
Lowest cost edge from tree to vertex not in Tree?
5 from D to G

Prim's Algorithm



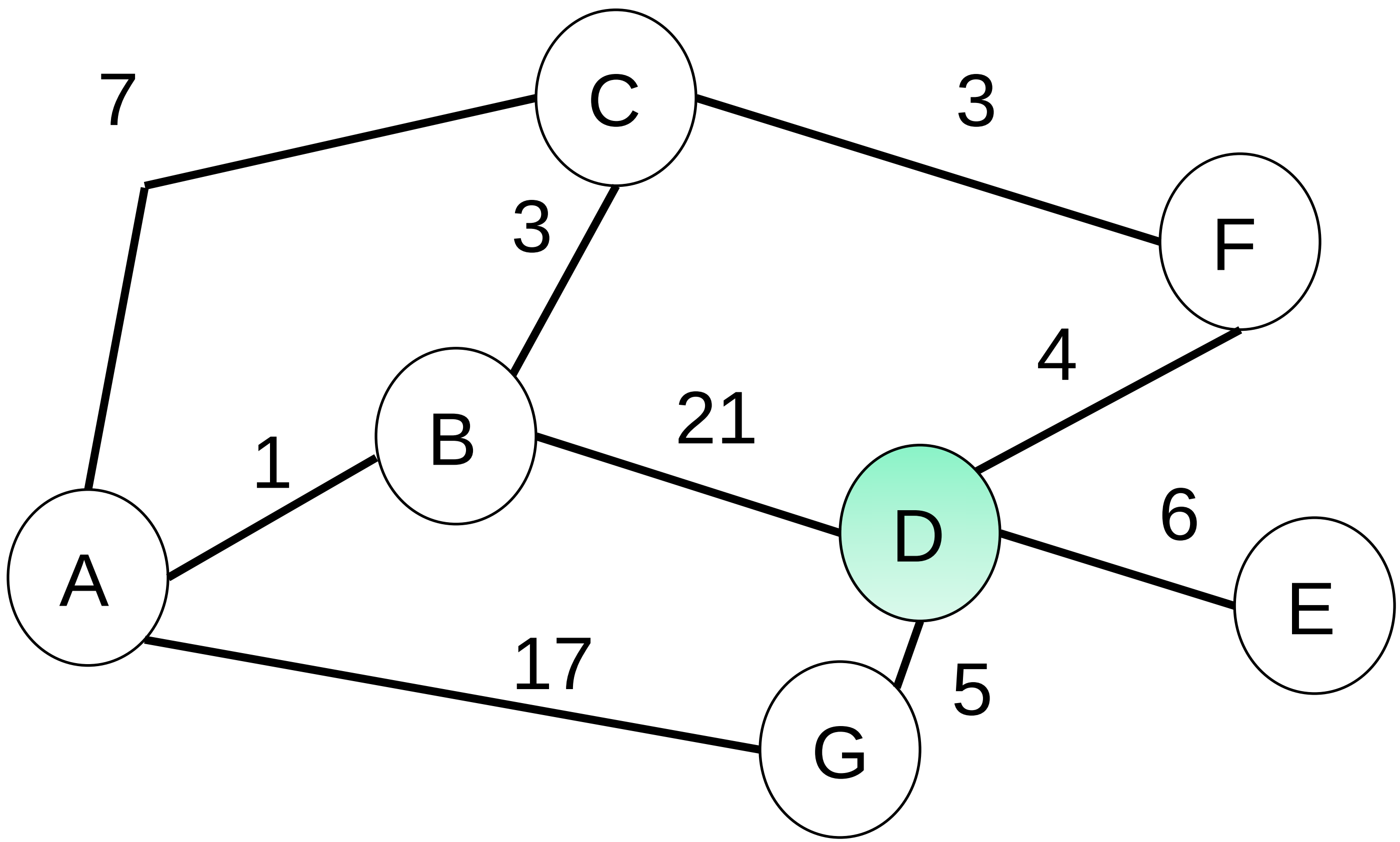
Lowest cost edge from tree to vertex not in Tree?
1 from G to F

Prim's Algorithm



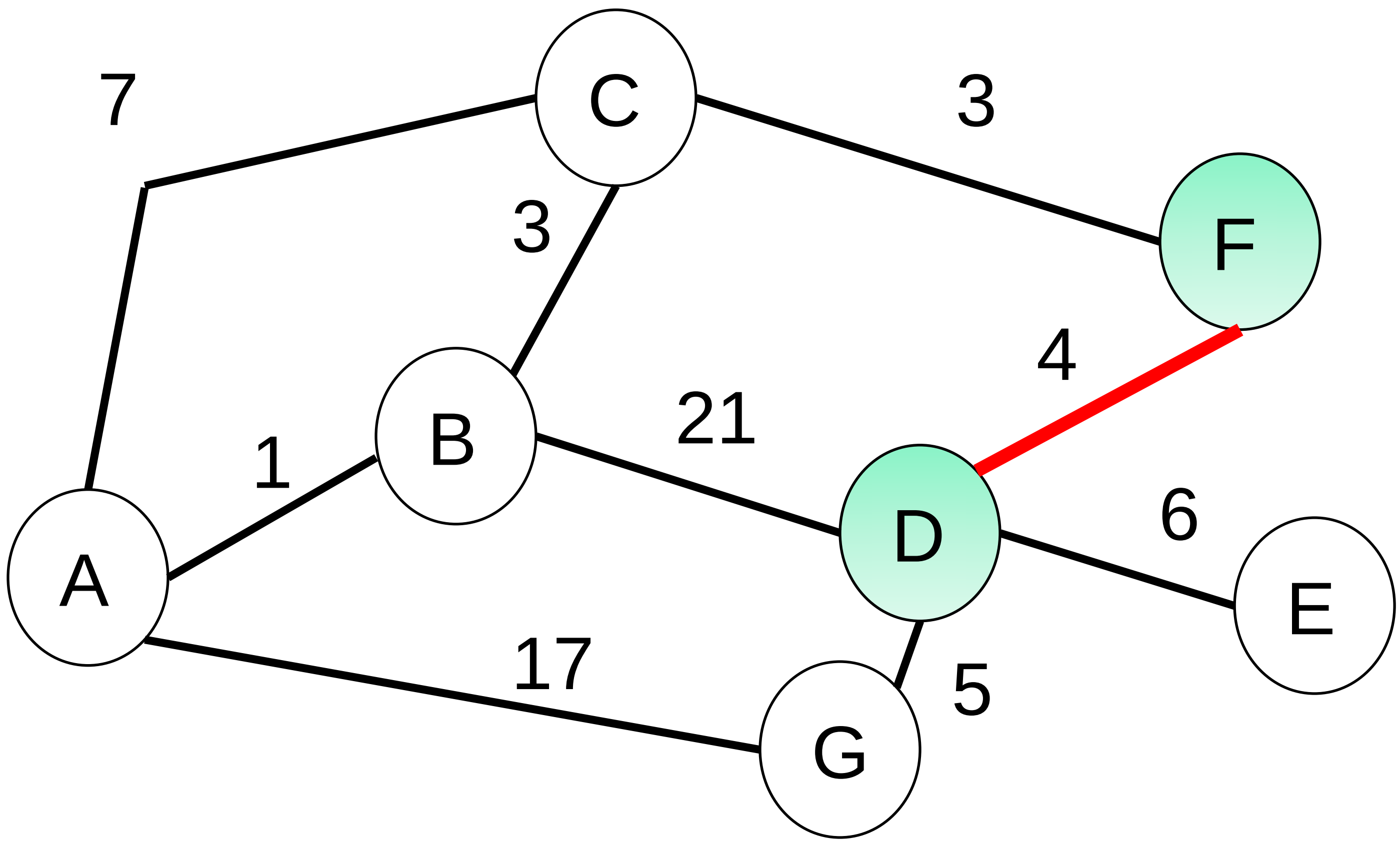
Lowest cost edge from tree to vertex not in Tree?
6 from G to E

Prim's Algorithm



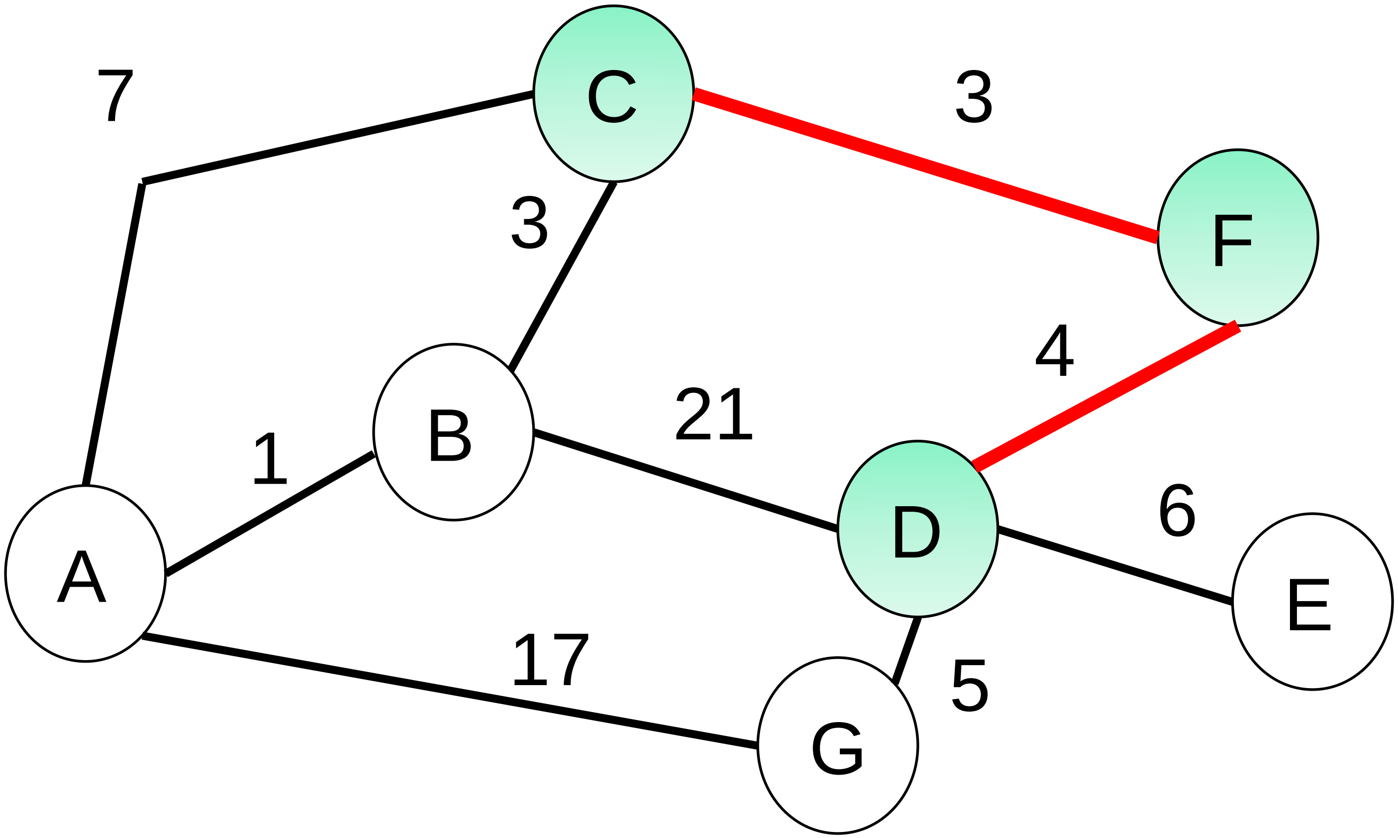
Pick D as root

Prim's Algorithm



Lowest cost edge from tree to vertex not in Tree?
4 from D to F

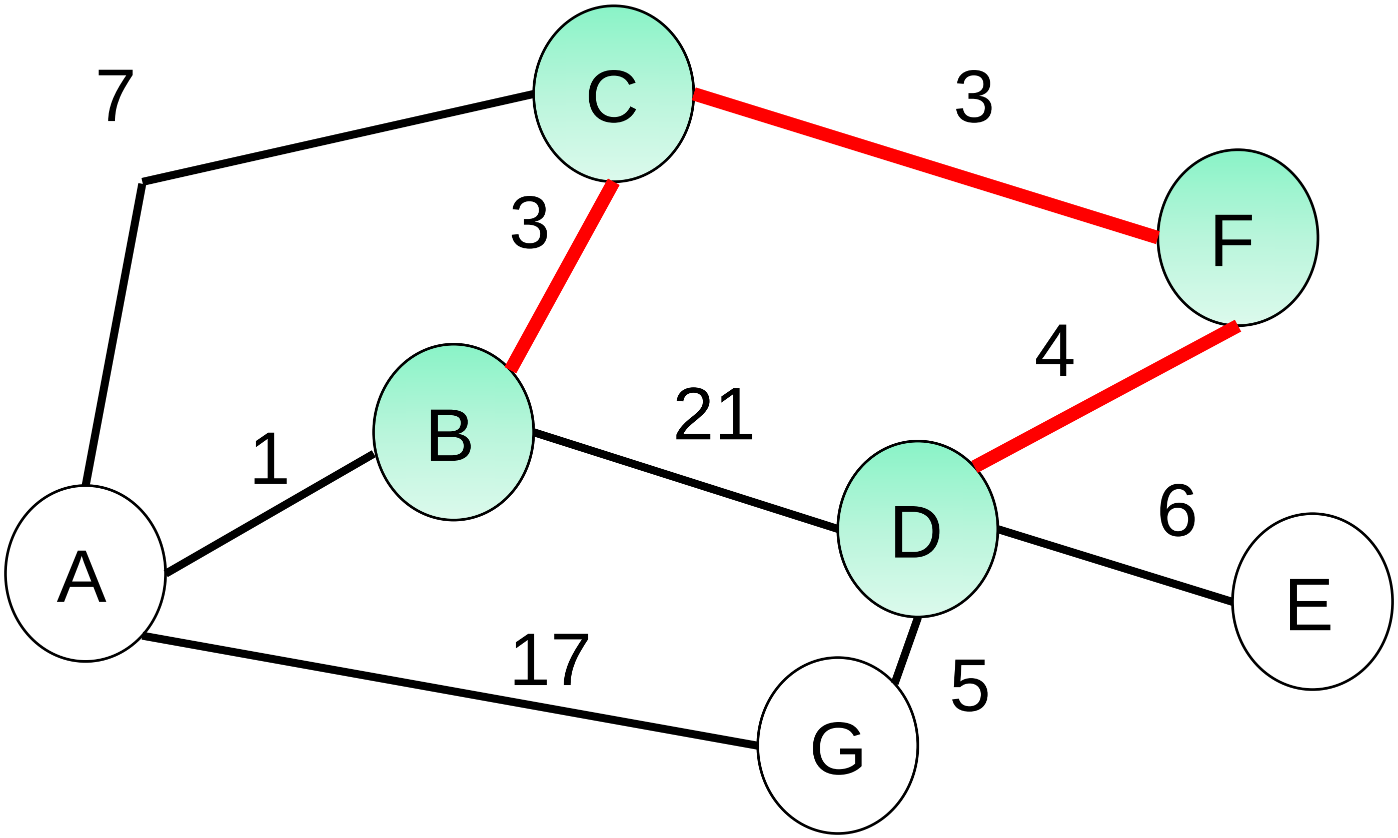
Prim's Algorithm



Lowest cost edge from tree to vertex not in Tree?

3 from F to C

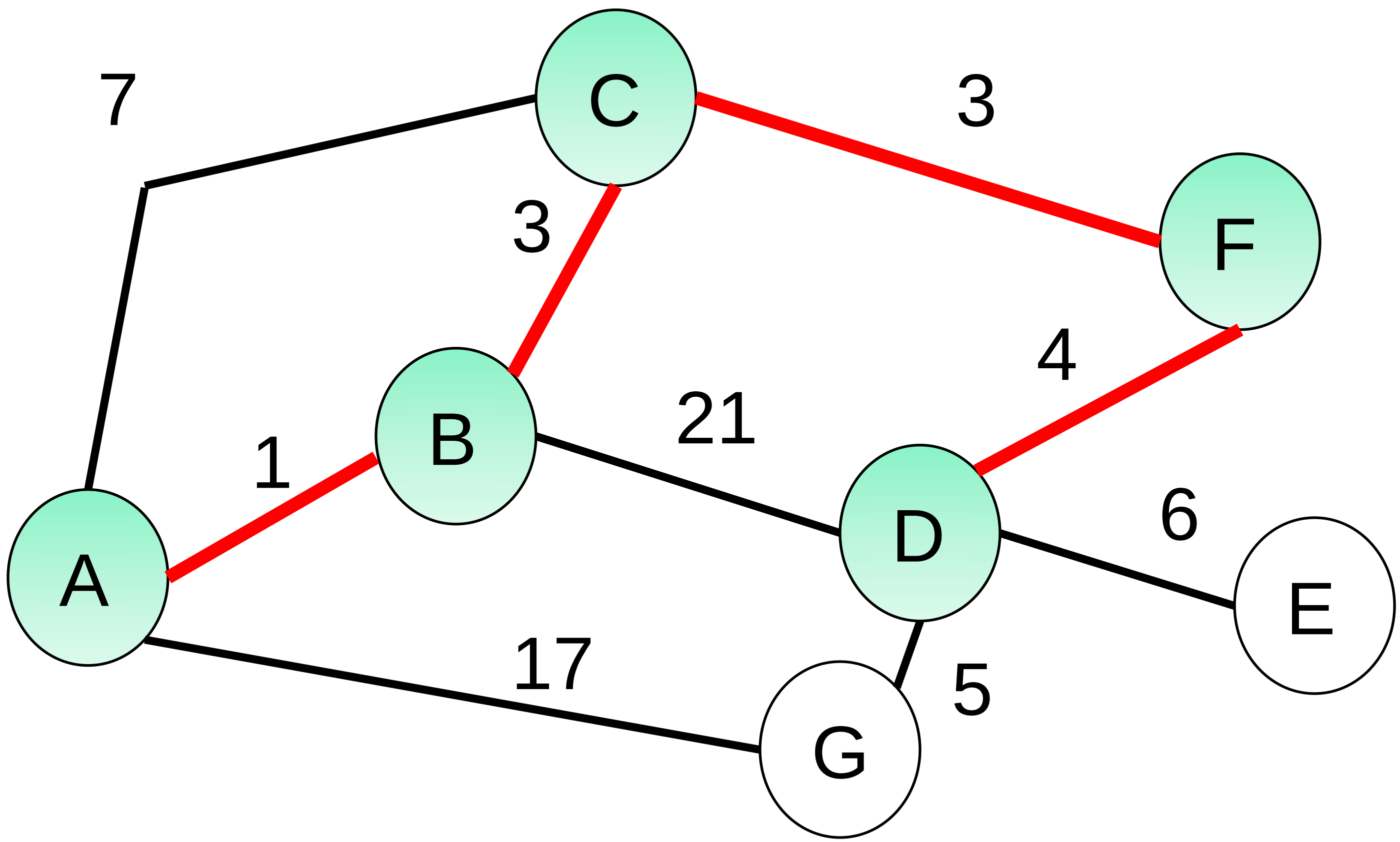
Prim's Algorithm



Lowest cost edge from tree to vertex not in Tree?

3 from C to B

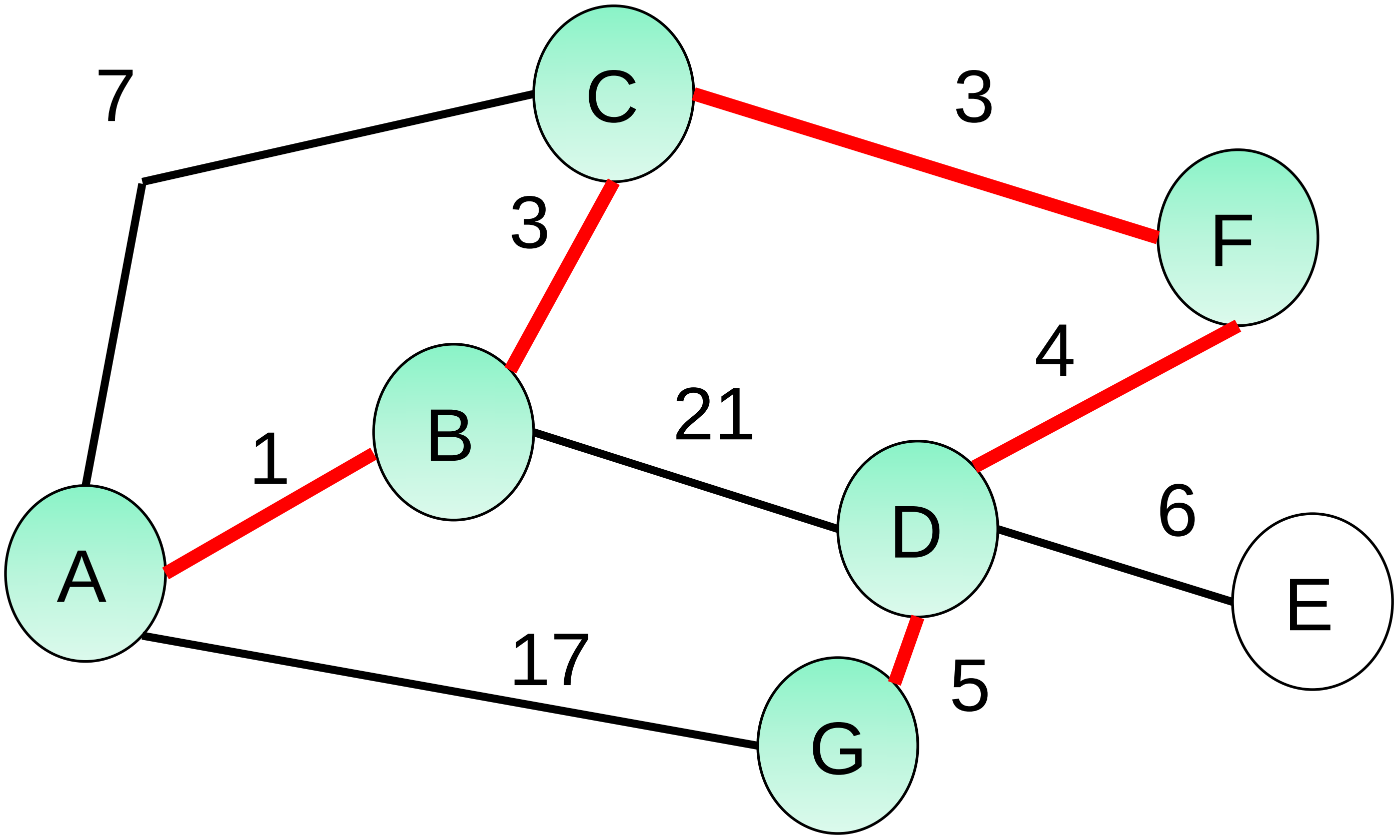
Prim's Algorithm



Lowest cost edge from tree to vertex not in Tree?

1 from B to A

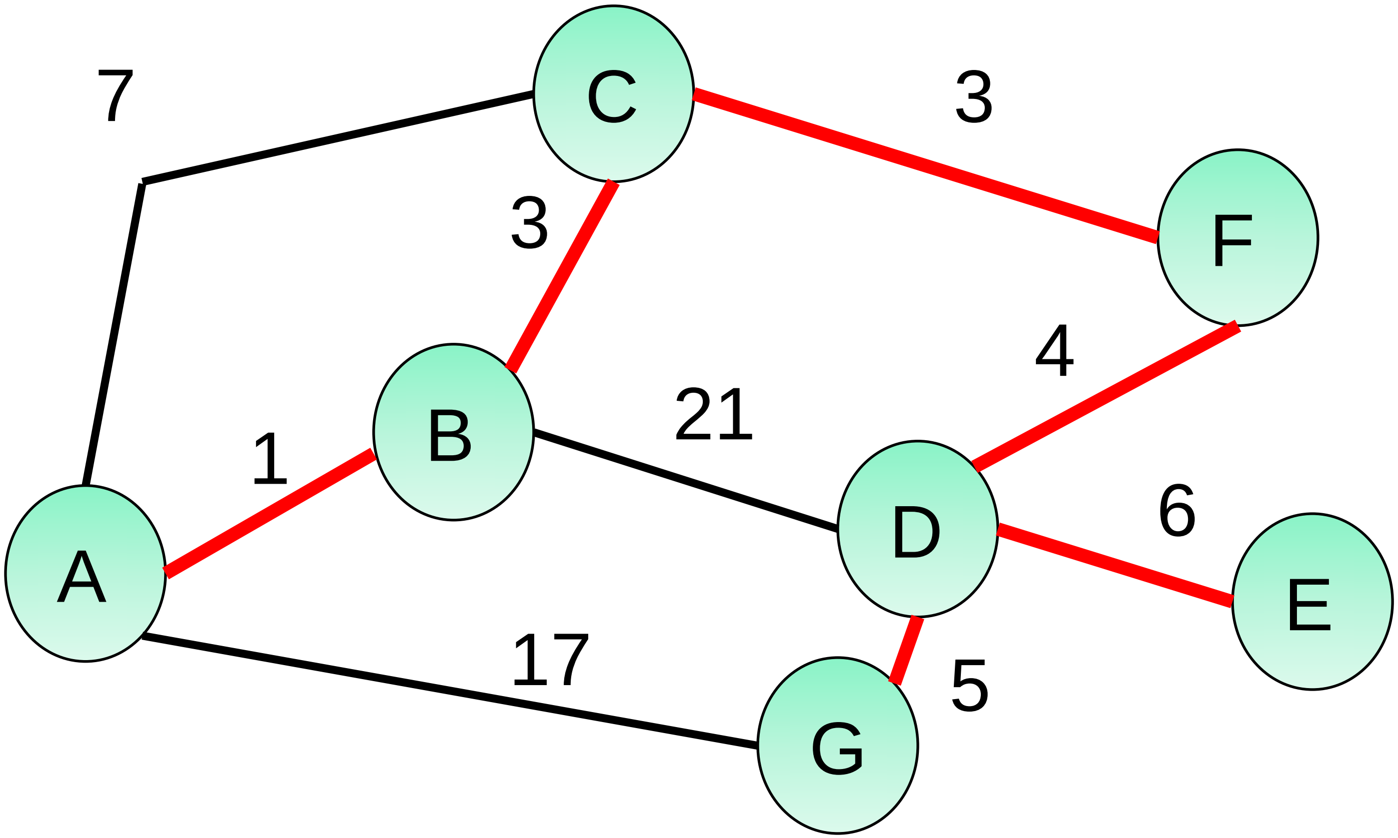
Prim's Algorithm



Lowest cost edge from tree to vertex not in Tree?

5 from D to G

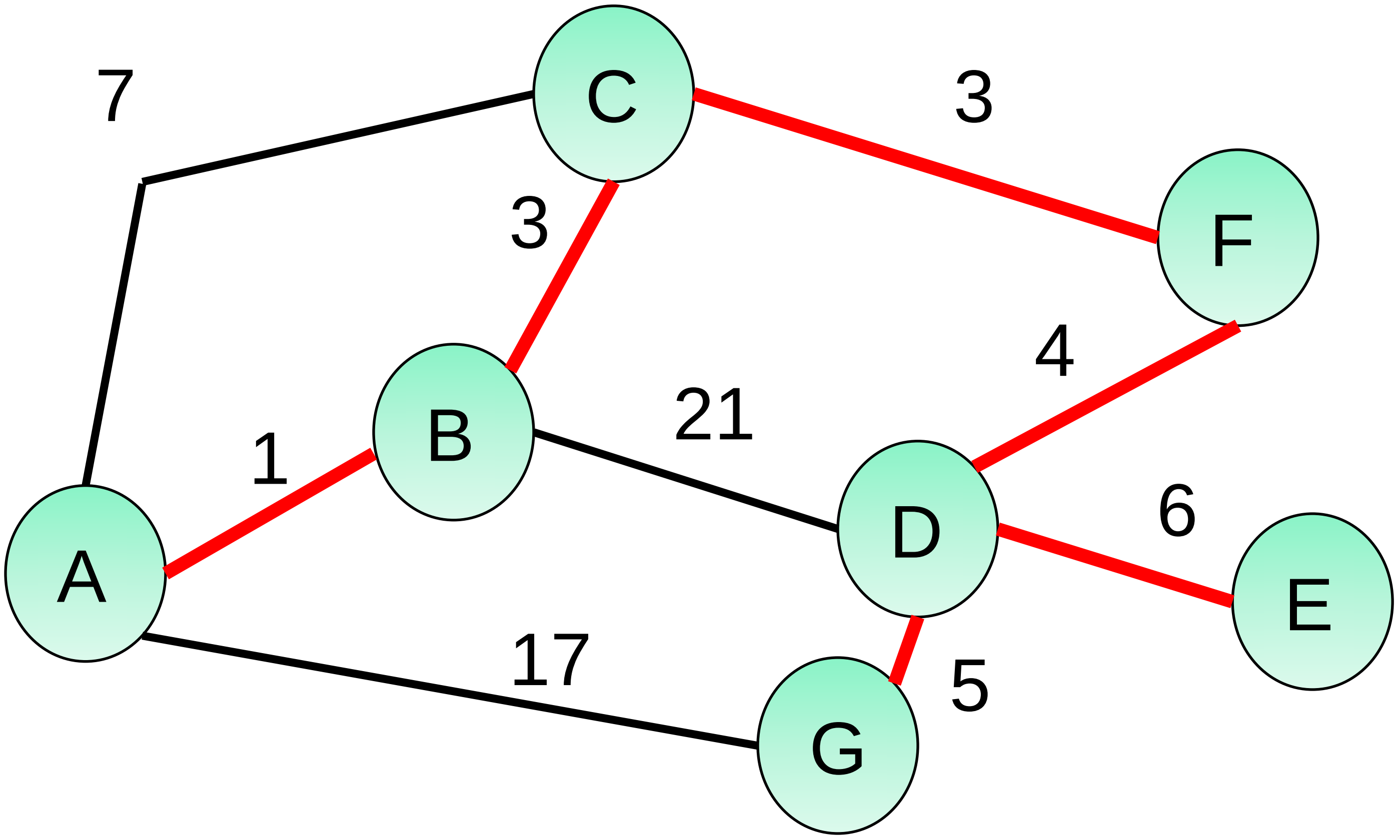
Prim's Algorithm



Lowest cost edge from tree to vertex not in Tree?

6 from D to E

Prim's Algorithm



Cost of Spanning Tree?

Other Graph Algorithms

- Lots!
- http://en.wikipedia.org/wiki/Category:Graph_algorithms