Inductive Invariant Generation via Abductive Inference

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Loop Invariants

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- Intuitively, a loop invariant summarizes the behavior of an unbounded number of computations in one formula.

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assert(Q);
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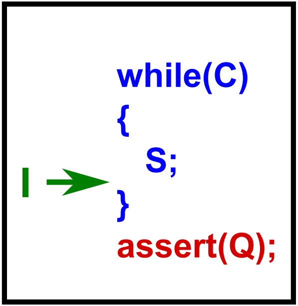

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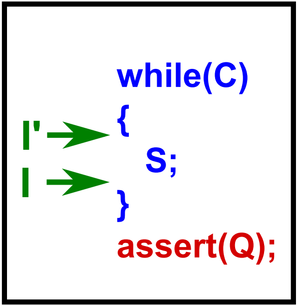

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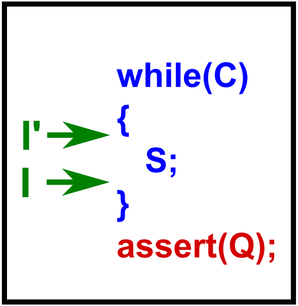

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Only way to prove a loop invariant is to show it is inductive.

Loop Invariant Example

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int x = 0;
int y = 0;

while(x < n)
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   x = x+1;
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assert( x + y >= 3*n);
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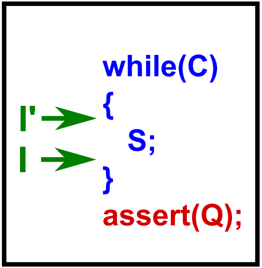
  - We need stronger invariant

This Talk



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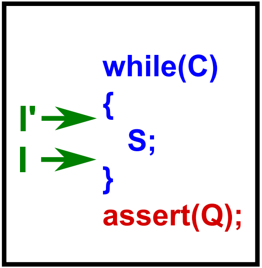

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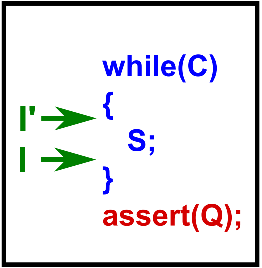

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Key Insight:
Use logical abduction to find inductive invariants

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- Abduction: Infers missing premise to explain a given conclusion
- Given known facts Γ and desired outcome ϕ, abductive inference finds "simple" explanatory hypothesis ψ such that

$$\Gamma \land \psi \models \phi \quad \text{and} \quad \mathrm{SAT}(\Gamma \land \psi)$$

Simple Example

- Facts: “If it rains, then it is wet and cloudy”, “If it is wet, then it is slippery”: $R \Rightarrow W \wedge C \ \wedge W \Rightarrow S$

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- Conclusion: "It is cloudy and slippery", i.e., $C \wedge S$
- Abductive explanation: R, i.e., "It is rainy"

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- Corresponds to missing inductive loop invariant

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- Trivial solution: ϕ, but generally not inductive
- So, what kind of solutions do we want to compute?

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- Generality: If explanation A is logically weaker than explanation B, always prefer A
- Simplicity: Prefer solutions with fewest number of variables
- Intuition: Most likely to generalize behavior of a loop

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- If all VCs are valid, found inductive invariants sufficient to verify program
- Otherwise, strengten LHS using abduction

Using Abduction for Loop Invariant Generation, cont.

- If $I \land \neg C \Rightarrow Q$ is invalid, abduction produces auxiliary invariant ψ such that $I \land \psi$ is strong enough to show Q

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- If $I \land \neg C \Rightarrow Q$ is invalid, abduction produces auxiliary invariant ψ such that $I \land \psi$ is strong enough to show Q
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- In either case, strengthen invariant to $I \land \psi$ and try to prove correctness

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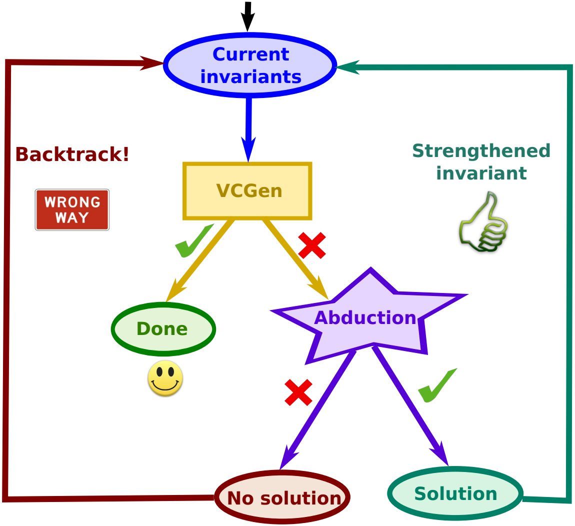
- Therefore, generate sequence of abductive solutions with increasing number of variables

$$I_0 \rightarrow I_1 \rightarrow I_2 \rightarrow I_3 \ldots$$

Full Algorithm



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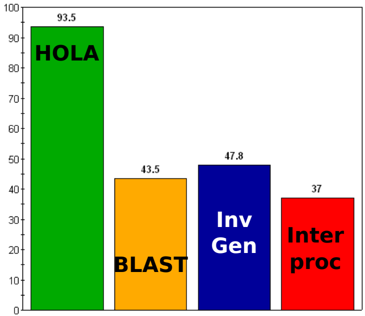

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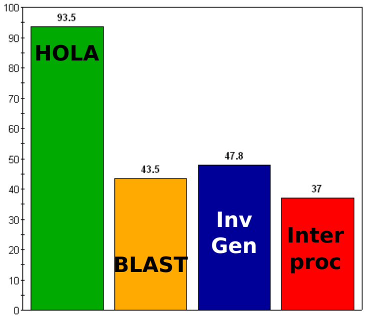

- But not strictly better: cannot prove two benchmarks at least one tool can show

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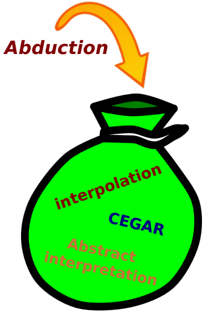

Abduction-based approach useful addition to known techniques for loop invariant generation

Questions?