Parametric Surfaces
Parametric Curves

Define curve as values at t along an interval \([u_0, u_n]\)
Parametric Surfaces

Extends idea of parametric curves:
Parameters \((u, v)\) define points along a surface \(S(u, v) = (x(u,v), y(u,v), z(u,v))\)
Example: Circle vs Sphere

Unit Circle:
\[ \gamma(t) = (\cos(t), \sin(t)) \]

Unit Sphere:
\[ \gamma(\phi, \Theta) = (\cos(\phi)\sin(\Theta), \sin(\phi)\sin(\Theta), \cos(\Theta)) \]
NURBS Revisited

- Basis splines form curves
- Curves form patches
- Complex shapes generated from little data

Subdivision surface  NURBS surfaces
Surfaces of Revolution

Idea: Rotate a 2D profile curve around an axis to create a surface

In-class activity: What shapes do the above curves (red) form around the axis (black)?
Parameterization

\( u = \text{axis of rotation} \)

\( v = \text{rotation} \)

Example: surface \( S(u, v) \) rotated around \( z \) axis

\[
\begin{align*}
x &= \text{radius}(u)\cos(v) \\
y &= \text{radius}(u)\sin(v) \\
z &= u
\end{align*}
\]
Properties

• Axial symmetry
• Easily computed surface area
• Simplified calculations
• Nice physical properties
Extruded Surfaces

Idea: Take a curve or patch in a plane and extend along an axis
Parameterization

\[ C(u) = \text{curve in plane} \]
\[ v = \text{axis of extrusion} \]

Example: surface \( S(u, v) \) from curve \( C(u) \) in xy-plane extruded along z axis

\[ x = C_x(u) \]
\[ y = C_y(u) \]
\[ z = v \]
Sweep Surfaces

Idea: Move profile curve along trajectory curve to create a surface
How to Orient?

Assume profile curve $C(u)$ lies in a coordinate system $(x_c, y_c)$ with origin $O_c$
For every point along trajectory curve $T(v)$, $O_c$ should coincide with $T(v)$

How to orient $C(u)$ at each point?
Fixed Frame

Translate $O_c$ along $T(v)$
Frenet Frame

- Smoothly varying orientation
- Must calculate TNB (tangent, normal, binormal) unit vectors
  - $C(u)$ in normal plane
  - $O_c$ at $T(v)$
  - $x_c$ aligned with $b$
  - $y_c$ aligned with $-n$
Frenet in Practice
Fixed Versus Frenet

(FreeCAD)
Sweeping with Rails

Common industry practice uses two guiding curves or “rails”
Other Variations

• Scale $C(u)$ as it moves along $T(v)$
  • Length of $T(v)$ can be scale factor

• Morph $C(u)$ into some other curve $C'(u)$ as it moves along $T(v)$
Constructive Solid Geometry

Create new objects from existing objects using boolean operations
Primitives

Simple shapes that form the basis of all constructed objects
  • Cube, prism, sphere, cylinder, cone, torus

Affine transformations can be applied
Boolean Operations

Set operations:
  • Union, intersect, difference (subtract)

Objects defined by boundary representations
  • Ray cast to determine overlap
  • BSP trees often used as acceleration structure
Spatial Partitioning with BSPs

Remember BSP trees?

- Binary Space Partitioning Trees
BSP Trees

- Natural fit for these sorts of operations
- Tree constructed based on geometry of object
  - Fast to create
  - Good depth and partitioning properties
  - Good traversal properties
CSG Uses

• Guarantees on water-tightness if primitives are water-tight
• Fast to calculate
• Arbitrary complexity from very simple shapes
• Common in:
  • CAD programs for engineering and manufacturing
    • Mathematical guarantees for physically-based systems
  • Game engines for level-building
    • BSP trees useful for world partitioning in games
Geometry Brushes in UE4
BSP primitives in UE4