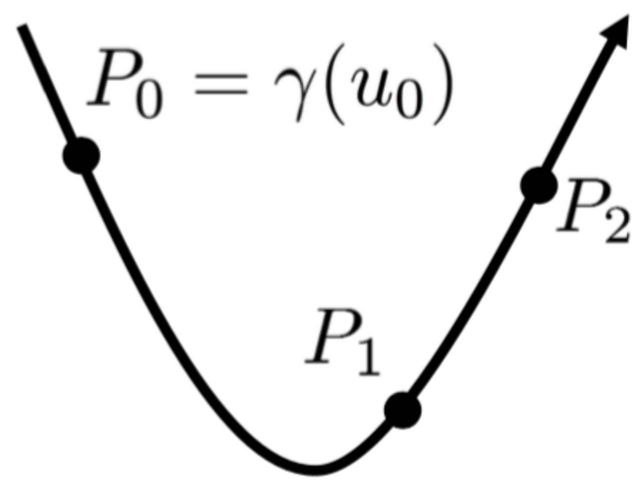
Parametric Surfaces

Parametric Curves

Define curve as values at t along an interval [u_0, u_n]



Parametric Surfaces

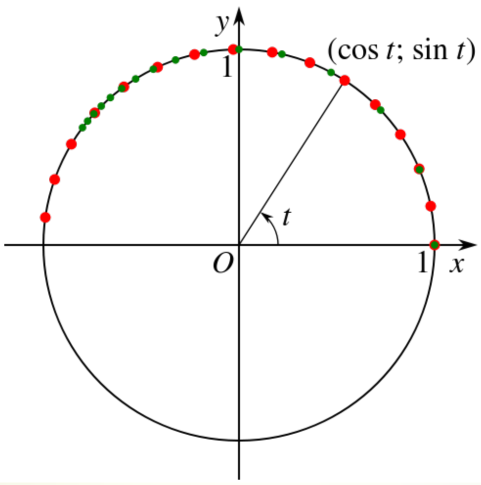
Extends idea of parametric curves:

Parameters (u, v) define points along a surface $S(u, v) = (x(u,v), y(u,v), z(u,v))$

Example: Circle vs Sphere

Unit Circle:

$\gamma(t) = (cos(t), sin(t))$

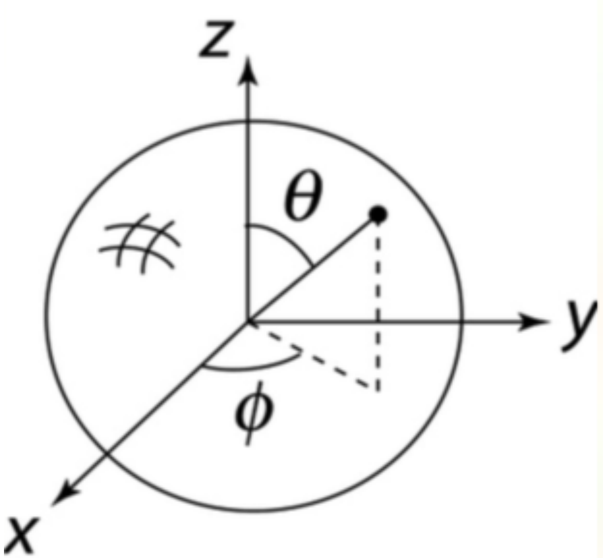


Unit Sphere:

$\gamma(\phi, \Theta) = (cos(\phi)sin(\Theta),$

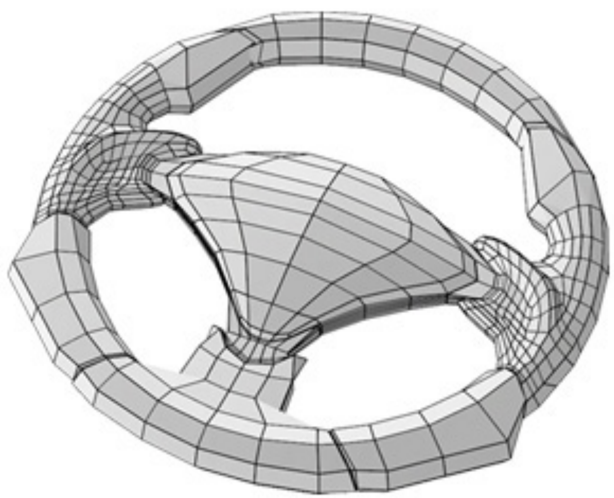
$sin(\phi)sin(\Theta),$

$cos(\Theta))$

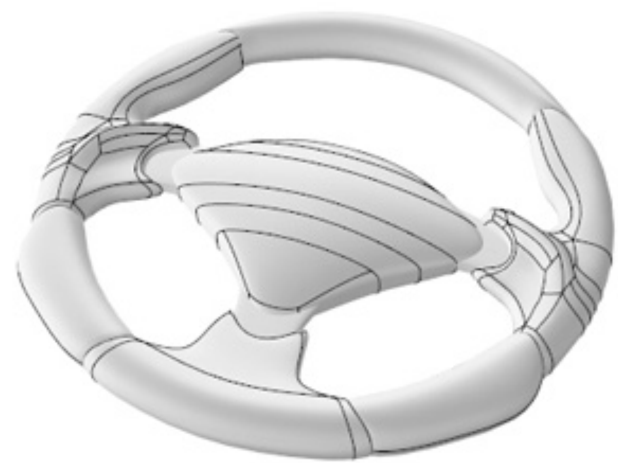


NURBS Revisited

- Basis splines form curves
- Curves form patches
- Complex shapes generated from little data



Subdivision surface

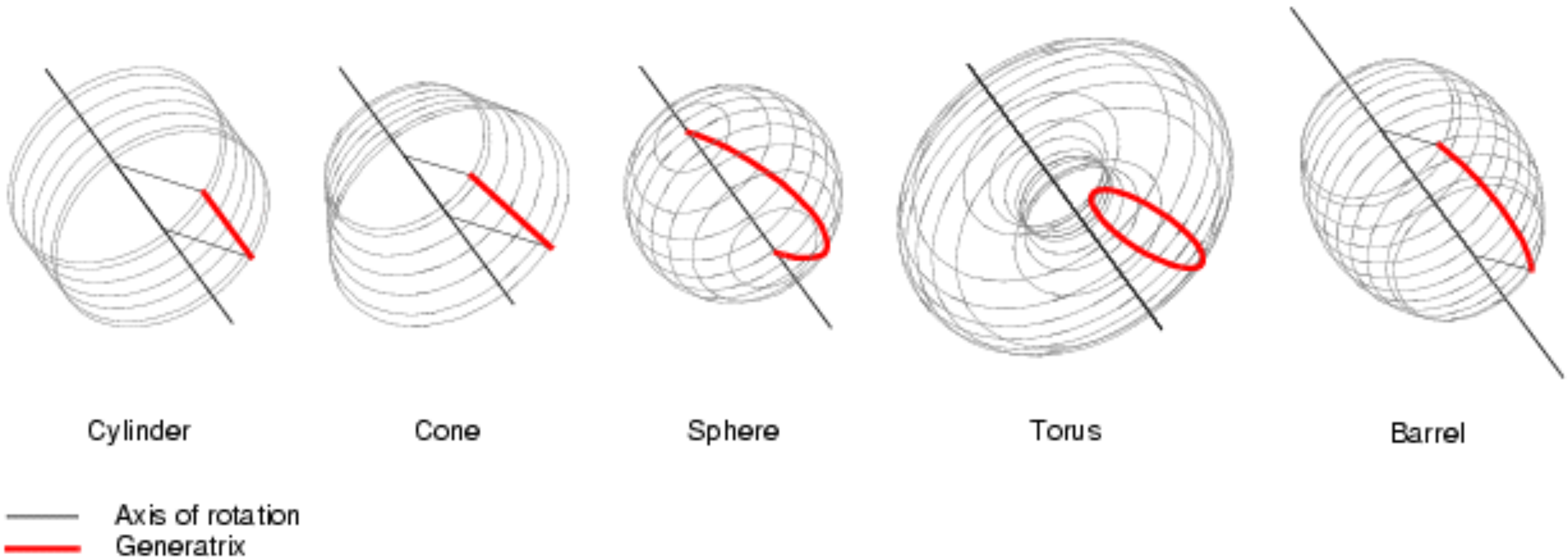


NURBS surfaces

Surfaces of Revolution

Idea: Rotate a 2D profile curve around an axis to create a surface

In-class activity: What shapes do the above curves (red) form around the axis (black)?


Cylinder
Cone
Sphere
Torus
Barrel
Axis of rotation
Generatrix

Parameterization

u = axis of rotation

v = rotation

Example: surface S(u, v) rotated around z axis

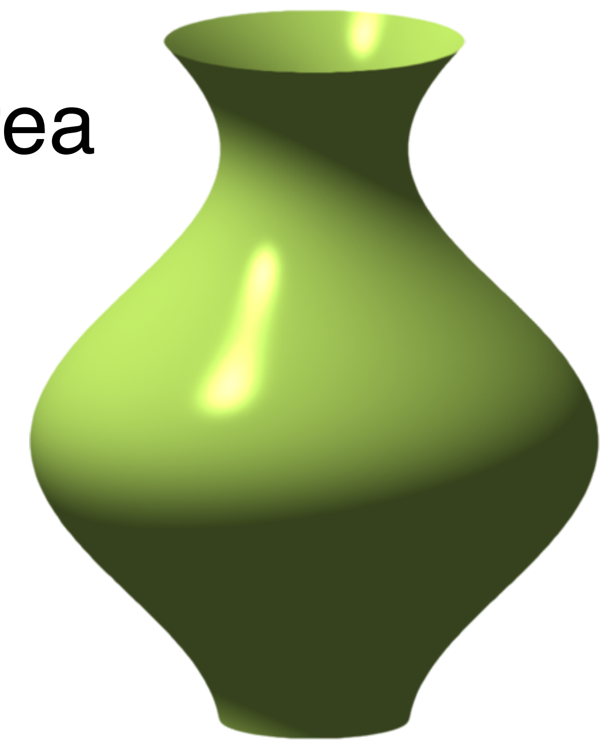
$x = \text{radius}(u)\cos(v)$

$y = \text{radius}(u)\sin(v)$

$z = u$

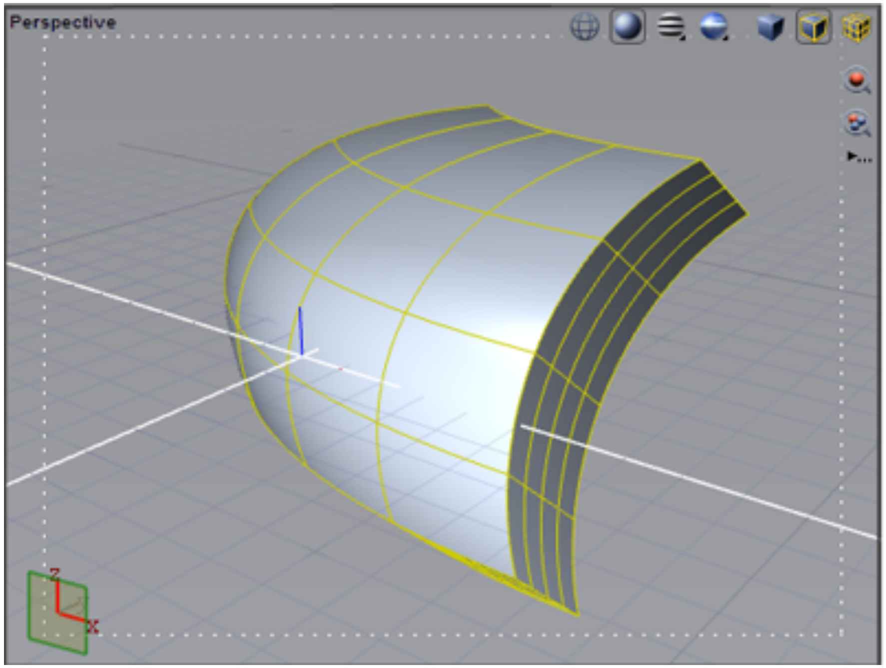
Properties

- Axial symmetry
- Easily computed surface area
- Simplified calculations
- Nice physical properties



Extruded Surfaces

Idea: Take a curve or patch in a plane and extend along an axis



Parameterization

C(u) = curve in plane

v = axis of extrusion

Example: surface S(u, v) from curve C(u) in xy-plane extruded along z axis

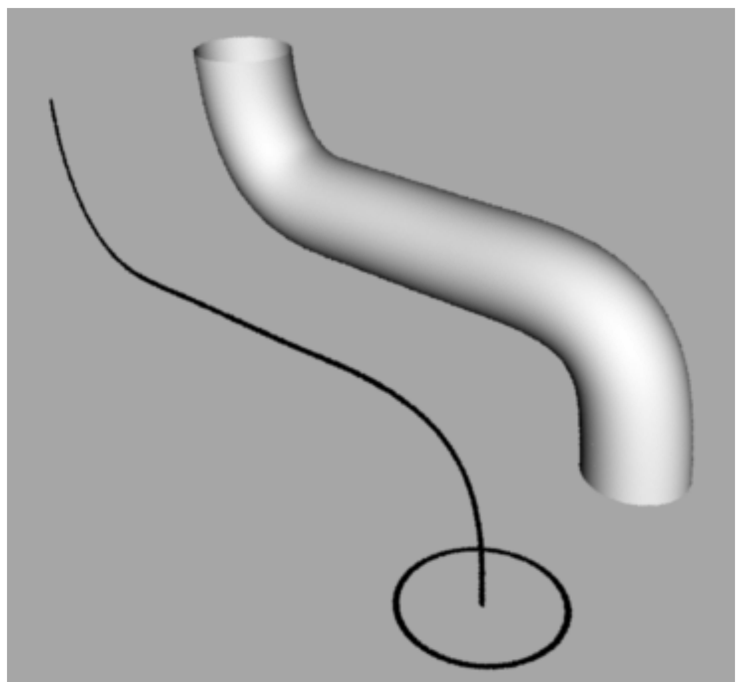
$x = C_x(u)$

$y = C_y(u)$

$z = v$

Sweep Surfaces

Idea: Move profile curve along trajectory curve to create a surface



How to Orient?

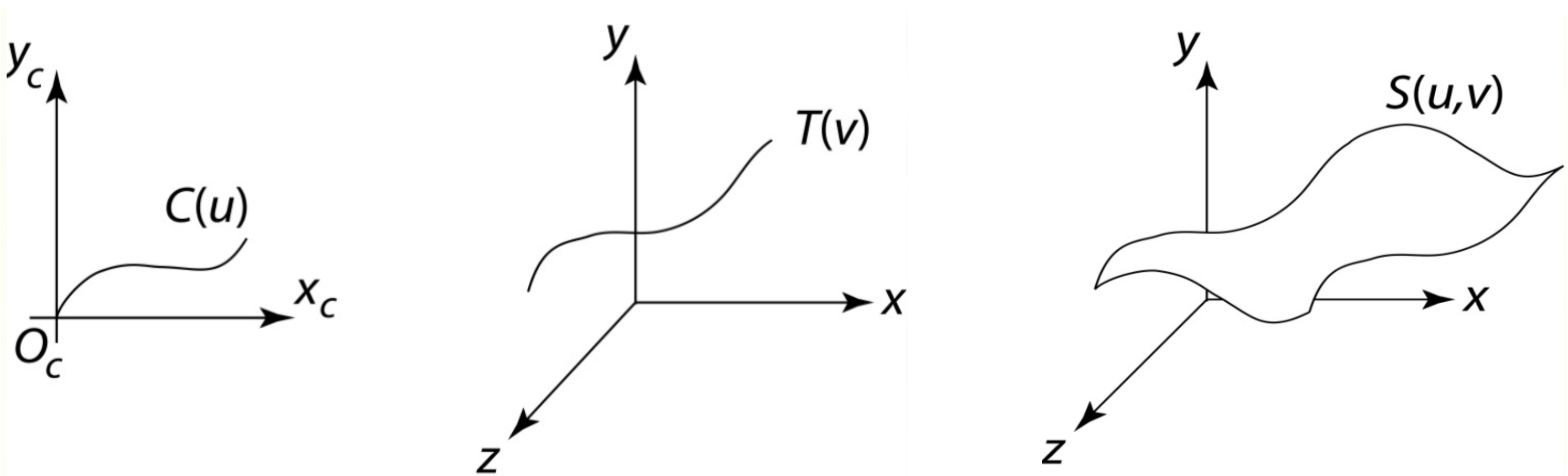
Assume profile curve $C(u)$ lies in a coordinate system (x_c, y_c) with origin O_c

For every point along trajectory curve $T(v)$, O_c should coincide with $T(v)$

How to orient $C(u)$ at each point?

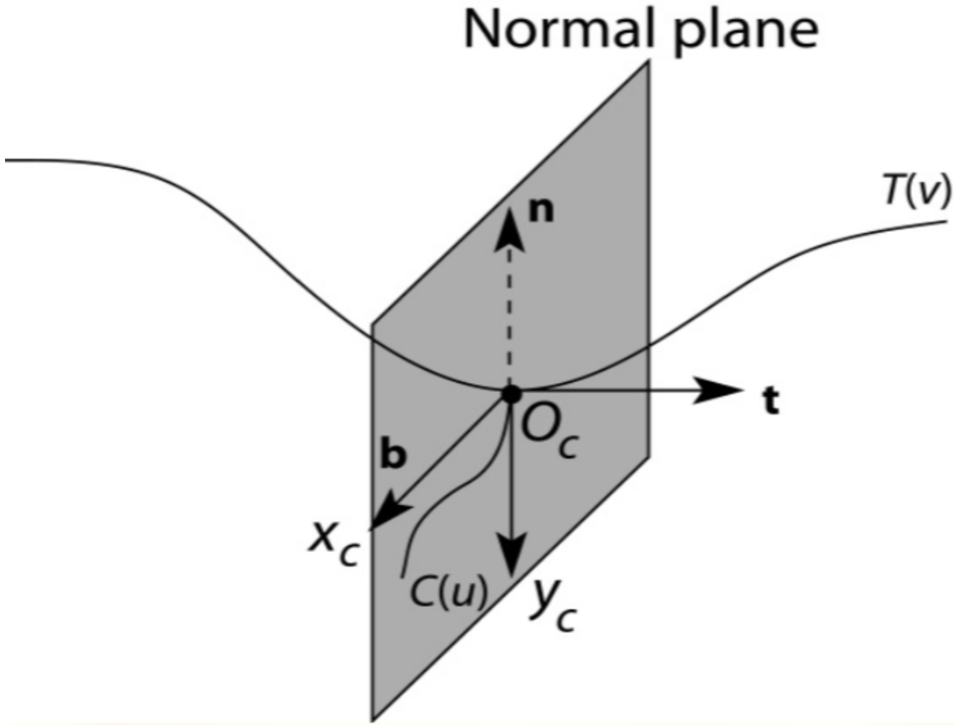
Fixed Frame

Translate O_c along T(v)

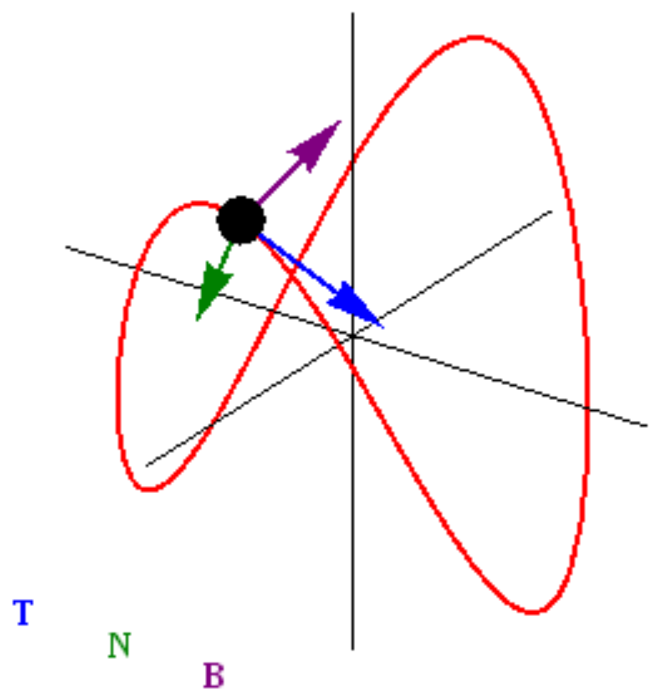


Frenet Frame

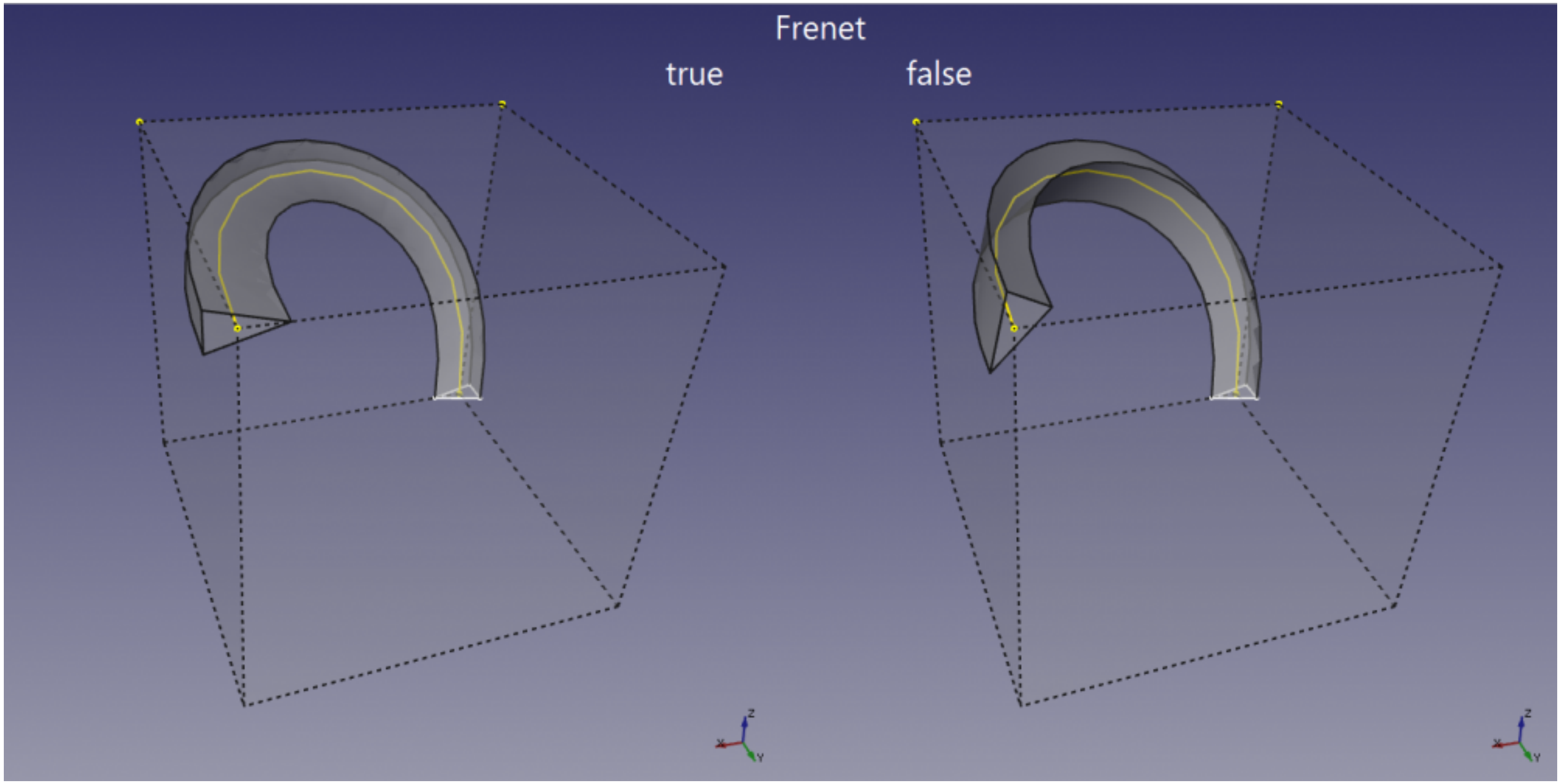
- Smoothly varying orientation
- Must calculate TNB (tangent, normal, binormal) unit vectors
  - C(u) in normal plane
  - O_c at T(v)
  - x_c aligned with b
  - y_c aligned with -n



Frenet in Practice



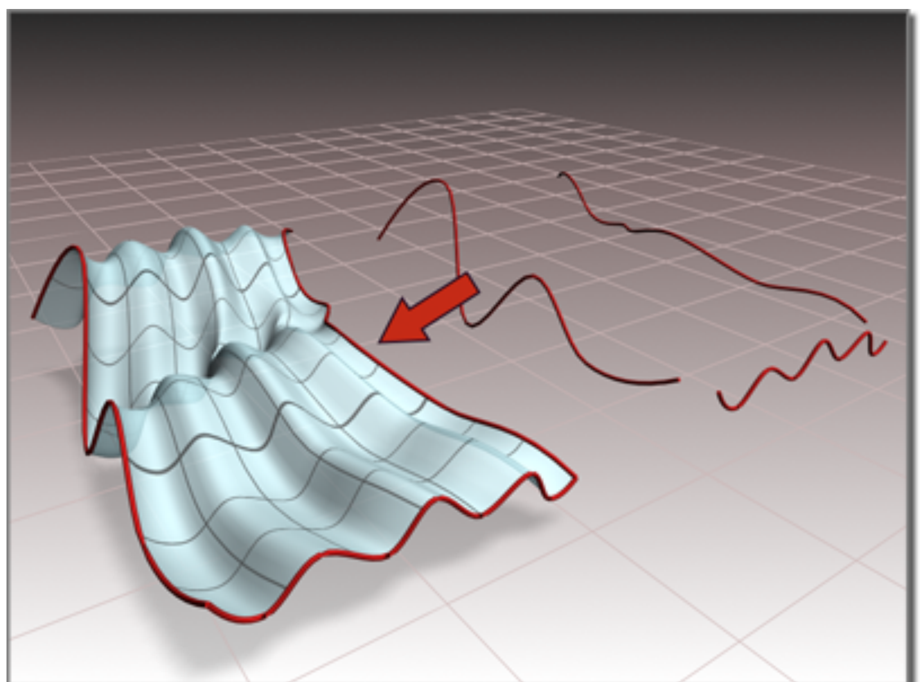
Fixed Versus Frenet



(FreeCAD)

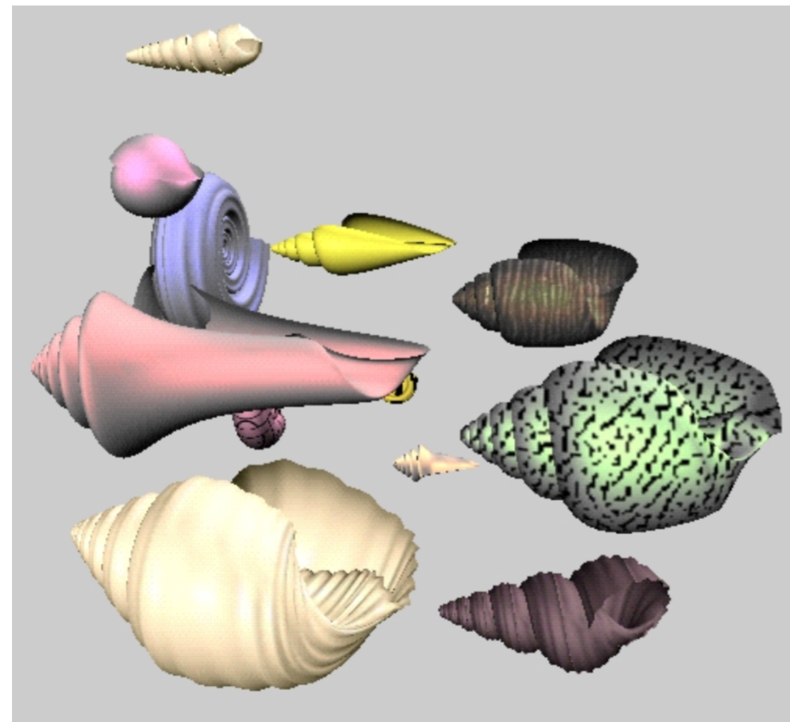
Sweeping with Rails

Common industry practice uses two guiding curves or "rails"



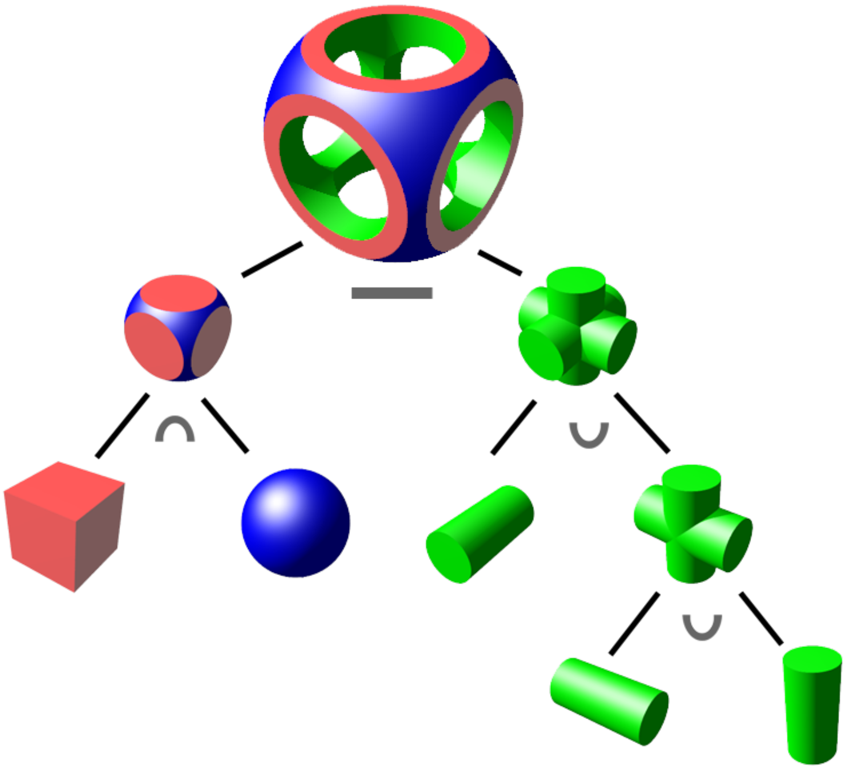
Other Variations

- Scale C(u) as it moves along T(v)
  - Length of T(v) can be scale factor
- Morph C(u) into some other curve C'(u) as it moves along T(v)



Constructive Solid Geometry

Create new objects from existing objects using boolean operations



Primitives

Simple shapes that form the basis of all constructed objects

- Cube, prism, sphere, cylinder, cone, torus

Affine transformations can be applied

Boolean Operations

Set operations:

- Union, intersect, difference (subtract)

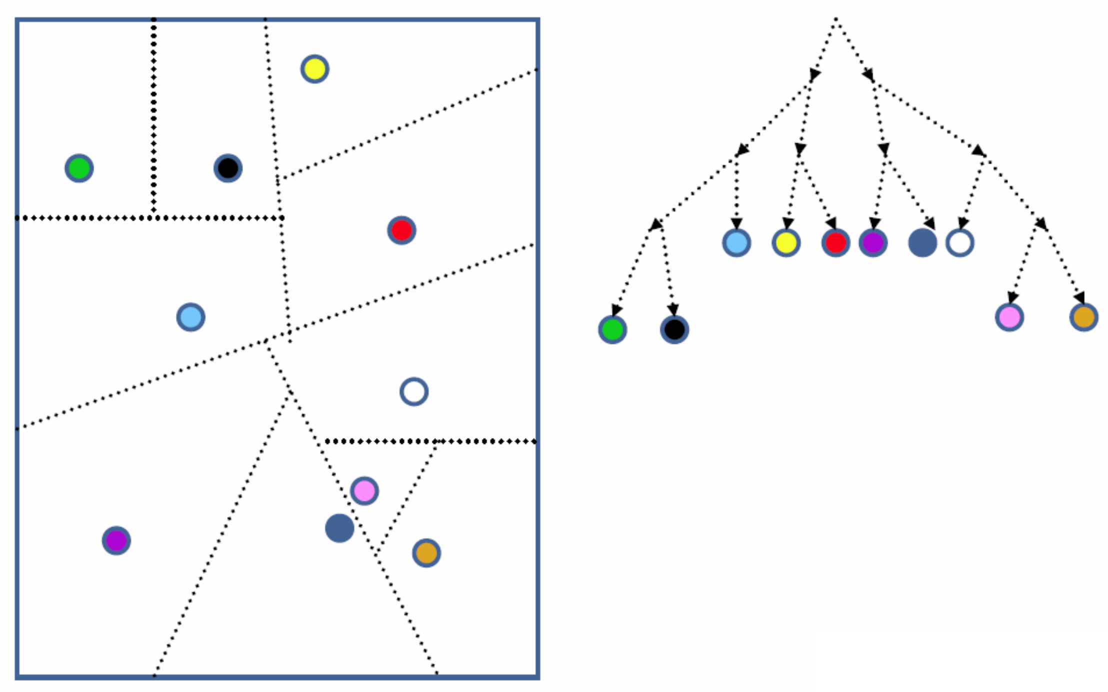
Objects defined by **boundary representations**

- Ray cast to determine overlap
- BSP trees often used as acceleration structure

Spatial Partitioning with BSPs

Remember BSP trees?

- Binary Space Partitioning Trees



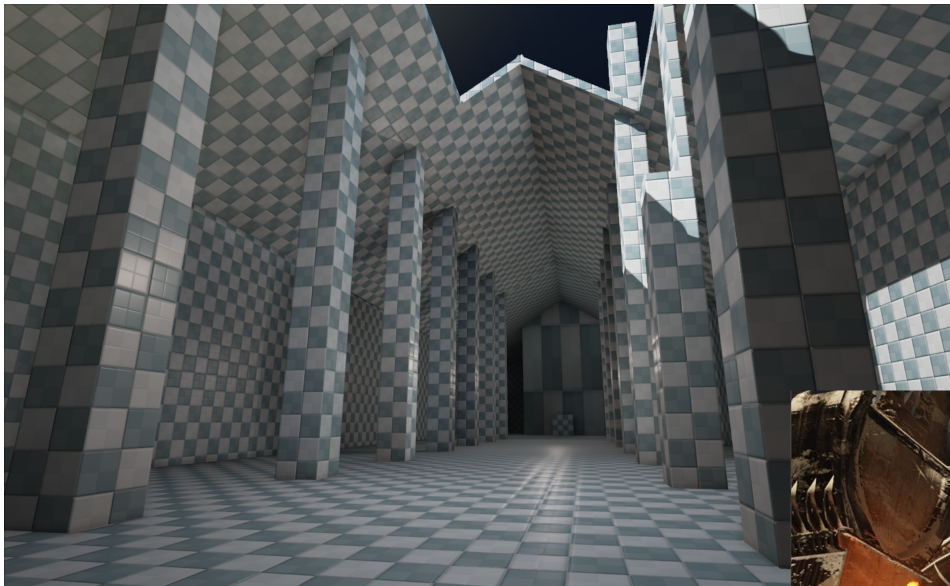
BSP Trees

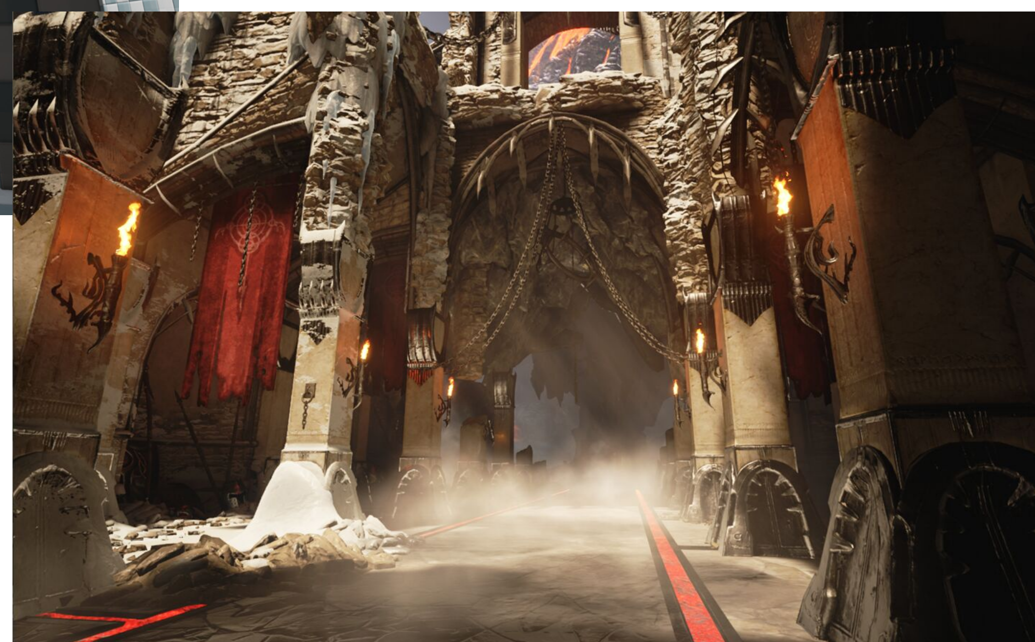
- Natural fit for these sorts of operations
- Tree constructed based on geometry of object
  - Fast to create
  - Good depth and partitioning properties
  - Good traversal properties

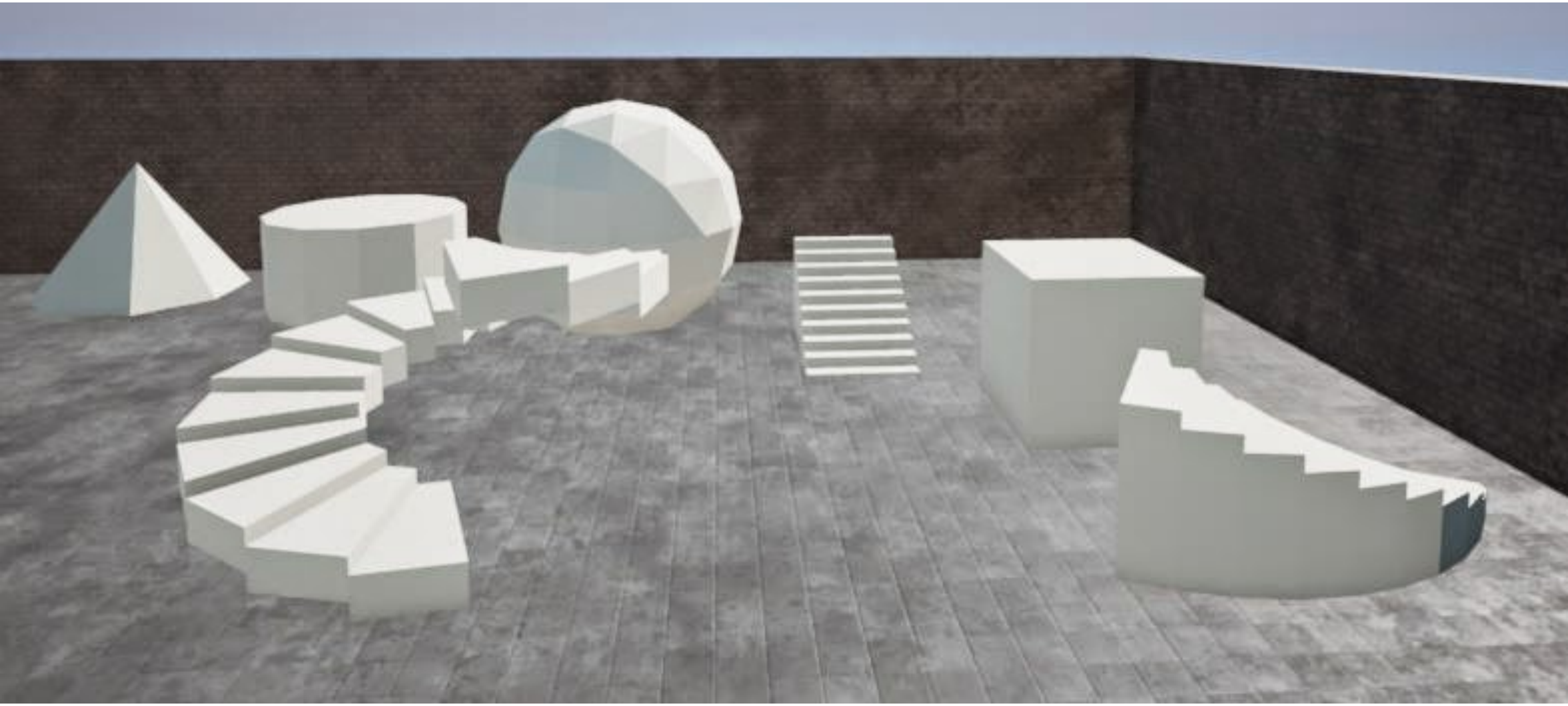
CSG Uses

- Guarantees on water-tightness if primitives are water-tight
- Fast to calculate
- Arbitrary complexity from very simple shapes
- Common in:
  - CAD programs for engineering and manufacturing
    - Mathematical guarantees for physically-based systems
  - Game engines for level-building
    - BSP trees useful for world partitioning in games

Geometry Brushes in UE4







BSP primitives in UE4