CS344M
Autonomous Multiagent Systems

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Good Afternoon, Colleagues

Are there any questions?

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Logistics

- Next week's readings up

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- Progress reports due in 1 week

T-tests

- Student's T-test history

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- Who's better at video games in general?

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- Can you run multiple times until you get a significant result?

Other testing concepts

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- Is the number of wins/ties/losses different vs. UvA than in self play?

Examples from your projects

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- Chi-squared test?

Mixed strategy equilibrium

		Player 2	
		Action 1	Action 2
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  p = 2/7

Correlated Equilibria

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Sometimes mixing isn't enough: Bach/Stravinsky

		Wife	
		S	B
Me	S	2,1	0,0
	B	0,0	1,2

Want only S,S or B,B - 50% each

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- Where would you meet in Austin?

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- Where would you meet in Austin? New York City?

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Incomplete Information Games

- We each get one of 3 cards: 1,2,3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum

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	F	-1,1	0,0

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  > With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium

Discussion

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- What can't game theory simulate?