1. In this problem, \( A = \{1, 2, \ldots, 10\} \), \( B = \{10, 11, \ldots, 20\} \), \( C = \{2, 4, 6, \ldots 20\} \). Find the cardinalities of the sets
   (a) \( A \cup C \),
   (b) \( A \cap C \),
   (c) \( (A \cup B) \setminus C \),
   (d) \( (A \cap B) \setminus C \),
   (e) \( (A \cap B) \times C \).

   **Solution:**
   (a) \( |A \cup C| = |\{1, 2, \ldots, 9, 10, 12, 14, 16, 18, 20\}| = 15 \).
   (b) \( |A \cap C| = |\{2, 4, 6, 8, 10\}| = 5 \).
   (c) \( |(A \cup B) \setminus C| = |\{1, 3, 5, \ldots, 19\}| = 10 \).
   (d) \( |(A \cap B) \setminus C| = |\emptyset| = 0 \).
   (e) \( |(A \cap B) \times C| = |A \cap B| \cdot |C| = 1 \cdot 10 = 10 \).

2. Find the cardinality of the set
   \( (\{1, 2, \ldots, 100\} \times \{1, 2, \ldots, 101\}) \setminus (\{1, 2, \ldots, 101\} \times \{1, 2, \ldots, 100\}) \).

   **Solution:** Denote
   the set \( \{1, 2, \ldots, 100\} \times \{1, 2, \ldots, 101\} \) by \( X \),
   the set \( \{1, 2, \ldots, 101\} \times \{1, 2, \ldots, 100\} \) by \( Y \).

   Set \( X \) consists of the pairs \( \langle m, n \rangle \) such that \( m \) is between 1 and 100, and \( n \) is between 1 and 101; there are 10,100 such pairs. Such a pair \( \langle m, n \rangle \) belongs to \( Y \) if \( n \) is between 1 and 100; there are 10,000 such pairs. Consequently the cardinality of \( X \setminus Y \) is 10, 100—10,000, which equals 100.

3. Find sets \( A \) and \( B \) such that
   \[
   A \setminus B = \{1, 5, 7, 8\}, \\
   B \setminus A = \{2, 10\}, \\
   A \cap B = \{3, 6, 9\}.
   \]

   **Answer:** \( A = \{1, 3, 5, 6, 7, 8, 9\} \), \( B = \{2, 3, 6, 9, 10\} \).
4. Can you conclude that $A = B$ if $A, B, C$ are sets such that

(a) $A \cup C = B \cup C$?

Answer: No. Counterexample: $A = \{1, 2, 3\}, B = \{1, 2\}, C = \{3\}$.

(b) $A \cap C = B \cap C$?

Answer: No. Counterexample: $A = \{1, 2, 3\}, B = \{3, 4, 5\}, C = \{3\}$.

5. For any sets $A$ and $B$, if $|A \times B| = 91$ then at least one of the sets $A, B$ is a singleton. True or false?

Answer: False. Example: $A = \{1, 2, \ldots, 7\}$; $B = \{1, 2, \ldots, 13\}$.

6. Consider the relation $x = 2y + 1$ between real numbers $x, y$. Is it reflexive? Is it symmetric? Is it transitive?

Solution: Denote the given relation by $R$, so that

$$xRy \leftrightarrow x = 2y + 1.$$ 

This relation is not reflexive, because the condition $1R1$ does not hold.

This relation is not symmetric, because the condition $1R3$ holds, but the condition $3R1$ doesn’t.

This relation is not transitive, because the conditions $1R3$ and $3R7$ hold, but the condition $1R7$ doesn’t.

7. What is the total number of binary relations on the set $\{1, \ldots, 10\}$? How many of them are reflexive?

Solution: A binary relation is an arbitrary subset of the set $\{1, \ldots, 10\} \times \{1, \ldots, 10\}$. So the total number of binary relations is $2^{100}$. Such a subset is a reflexive relation if it contains 10 pairs of the form $(n, n)$. So the number of reflexive relations is $2^{90}$. 