1. Find the partition of the set \( \{1, 2, \ldots, 6\} \) corresponding to the equivalence relation \(|m - 3| = |n - 3|\).

Answer: \( \{\{1, 5\}, \{2, 4\}, \{3\}, \{6\}\} \).

2. Consider the equivalence relation between non-empty subsets \( A, B \) of \( \{1, 2, \ldots, 100\} \) defined by the condition: the greatest element of \( A \) is the same as the greatest element of \( B \). Let \( P \) be the partition corresponding to this equivalence relation. (a) Find the cardinality of \( P \). (b) Find an element of \( P \). (c) Find one more element of \( P \).

Solution. Each equivalence class of this relation consist of the non-empty subsets of \( \{1, 2, \ldots, 100\} \) that have the same greatest element. (a) There are 100 equivalence classes, because there are 100 choices for the greatest element. So the cardinality of \( P \) is 100. (b) \( \{\{1\}\} \) is an element of \( P \). (c) \( \{\{1, 2\}, \{2\}\} \) is another element of \( P \).

3. Find a partition of \( \mathbb{N} \) that consists of one infinite set and infinitely many finite sets.

Solution: One possible answer is \( \{\{0, 2, 4, 6, \ldots\}, \{1\}, \{3\}, \{5\}, \ldots\} \).

4. For each of the following relations between positive integers \( m, n \), determine whether it is a partial order, and whether it is a total order:

   (a) \( m|n \).

Solution: This relation is reflexive (every number evenly divides itself), anti-symmetric (if \( m \) divides \( n \) and \( n \) divides \( m \) then \( m = n \)), and transitive (if \( k \) divides \( m \) and \( m \) divides \( n \) then \( k \) divides \( n \)). Consequently this is a partial order. But it is not total: for example, 2 doesn’t divide 3 and 3 doesn’t divide 2.

   (b) \( m|n^2 \).

Solution: This relation is not anti-symmetric; for instance, 2 divides \( 4^2 \) and 4 divides \( 2^2 \). So it is not a partial order and hence not a total order.

   (c) \( m^2|n \).

Solution: This relation is not reflexive; for instance, \( 2^2 \) doesn’t divide 2. So it is not a partial order and hence not a total order.

   (d) the first digit of \( m \) in decimal notation is less than or equal to the first digit of \( n \).

Solution: This relation \( R \) is not anti-symmetric; for instance, \( 15R16 \) and \( 16R15 \). So it is not a partial order and hence not a total order.