Introduction to Mathematical Logic, Handout 9
Logical Validity and Entailment

In predicate logic, we say about a sentence that it is *logically valid* if it is satisfied by all interpretations.

Problem 9.1 Determine whether the sentences

$$\exists x(P(x) \land Q(x)) \to (\exists x P(x) \land \exists x Q(x)),$$
$$(\exists x P(x) \land \exists x Q(x)) \to \exists x(P(x) \land Q(x))$$

are logically valid.

The *universal closure* of a formula F is the sentence $\forall v_1 \cdots v_n F$, where $v_1, \dots, v_n$ are all free variables of F. About a formula with free variables we say that it is *logically valid* if its universal closure is logically valid.

Problem 9.2 For each of the formulas

$$P(x) \to \exists x P(x),$$
$$P(x) \to \forall x P(x)$$

determine whether it is logically valid.

A formula F is *equivalent* to a formula G if the formula $F \leftrightarrow G$ is logically valid.

Problem 9.3 Determine whether the formula

$$\forall x \exists y P(x, y)$$

is equivalent to

$$\exists y \forall x P(x, y).$$

We say that a set Γ of sentences *entails* a sentence F, or that F is a *logical consequence* of Γ, if every interpretation that satisfies all sentences in Γ satisfies F.

Problem 9.4 Determine whether

$$\exists x P(x, x)$$

is a logical consequence of the sentences

$$\forall x \exists y P(x, y),\ \forall xyz((P(x, y) \land P(y, z)) \to P(x, z)).$$