CS 395T, Spring 2005
Answer Set Programming

Problems on Strong Equivalence

A formula F is *strongly equivalent* to a formula G if, for every formula H, $F \wedge H$ and $G \wedge H$ have the same answer sets. The theorem on intuitionistic equivalence and the replacement theorem for intuitionistic logic show that intuitionistically equivalent formulas are strongly equivalent.

Problem 1 The program

$$\begin{array}{l} \{p\}^c \\ \leftarrow \textit{not } p \end{array}$$

is strongly equivalent to

$$\begin{array}{l} \{p\}^c \\ p. \end{array}$$

Problem 2 Determine whether the program

$$\begin{array}{l} p \leftarrow q \\ p \leftarrow \textit{not } q \end{array}$$

is strongly equivalent to p.

The *logic of here-and-there* can be obtained from system **N** by replacing the law of the excluded middle

$$\Rightarrow F \vee \neg F$$

with the axiom schema

$$\Rightarrow F \vee (F \rightarrow G) \vee \neg G.$$

Problem 3 For any formula F, the weak law of the excluded middle $\neg F \vee \neg\neg F$ is provable in the logic of here-and-there.

Problem 4 For any formulas F, and G, formulas $\neg(F \wedge G)$ and $\neg F \vee \neg G$ are equivalent to each other in the logic of here-and-there.

Problem 5 Rule

$$p \leftarrow \mathit{not}\ \mathit{not}\ p$$

is strongly equivalent to $\{p\}^c$.

Problem 6 Program

$$\begin{array}{l} p; q \\ \leftarrow p, q \end{array}$$

is strongly equivalent to

$$\begin{array}{l} p \leftarrow \mathit{not}\ q \\ q \leftarrow \mathit{not}\ p \\ \leftarrow p, q. \end{array}$$

Problem 7 Prove the assertions of Problems 5 and 6 using reducts, instead of the logic of here-and-there.