Literal Completion

The concept of literal completion, proposed by Norm McCain and Hudson Turner in the paper *Causal Theories of Action and Change*, is the foundation of the computational mechanism of CCALC.

Consider a finite program Π, possibly with constraints. For any literal H, by $Bodies(H)$ we denote the set of the bodies of all rules in Π whose head is H. By $Bodies(False)$ we denote the set of the bodies of all constraints in Π. For any finite set X of rule elements, the propositional formula $pf(X)$ is defined by the equation

$$pf(L_1, \dots, L_m, not\ L_{m+1}, \dots, not\ L_n) = L_1 \wedge \dots \wedge L_m \wedge \overline{L_{m+1}} \wedge \dots \wedge \overline{L_n}.$$

The *literal completion* of Π consists of the formulas

$$H \equiv \bigvee_{X \in Bodies(H)} pf(X)$$

for all H. The range of values of H includes all literals of the underlying language, even those that do not occur in Π, and the formula $False$.

Theorem on Literal Completion. *For any finite tight program Π and any consistent complete set X of literals, X is an answer set for Π iff X satisfies the literal completion of Π.*

As an example, consider the program from Problem 24:

$$\begin{aligned}
&p \leftarrow not\ \neg p,\\
&\neg p \leftarrow not\ p,\\
&q \leftarrow not\ \neg q,\\
&\neg q \leftarrow not\ q,\\
&r \leftarrow q.
\end{aligned}$$

The literal completion of this program is

$$\begin{aligned}
&p \equiv p,\\
&\neg p \equiv \neg p,\\
&q \equiv q,\\
&\neg q \equiv \neg q,\\
&r \equiv q,\\
&\neg r \equiv False.
\end{aligned}$$

This set of formulas is equivalent to $q \wedge r$. Consequently, it is satisfied by two interpretations: $\{p, q, r\}$ and $\{\neg p, q, r\}$. These are the complete consistent answer sets for the given program.

Problem 27. Use literal completion to find the causally explained transitions for the domain description from Problem 18′:

$$\begin{array}{l} \textbf{default } P, \\ A \textbf{ causes } \neg P. \end{array}$$

Problem 28. Use literal completion to find the causally explained transitions for the example from Problems 16–18:

$$\begin{array}{l} A \textbf{ causes } P \textbf{ if } Q, \\ \textbf{inertial } P, \neg P, Q, \neg Q. \end{array}$$