Using logic programming to talk about a directed graph:

$e(x, y)$: there is an edge from x to y.

$$e(1,2),\ e(2,3),\ e(3,4),$$
$$e(1,5),\ e(2,6),\ e(5,6).$$

$p(x, y)$: there is a path from x to y.

$$p(x,x),$$
$$p(x,y) \leftarrow e(x,z), p(z,y).$$

$t(x)$: x is a terminal node.

$$nt(x) \leftarrow e(x,y),$$
$$t(x) \leftarrow \textit{not}\ nt(x).$$

$w(x)$: x is a winning position.

$$w(x) \leftarrow e(x,y), \textit{not}\ w(y).$$

$$e(1,2) \quad e(2,3) \quad e(3,4)$$
$$e(1,5) \quad e(2,6) \quad e(5,6)$$

$$p(x,x)$$
$$p(x,y) \leftarrow e(x,z), p(z,y)$$

$$nt(x) \leftarrow e(x,y)$$
$$t(x) \leftarrow \textit{not } nt(x)$$

$$w(x) \leftarrow e(x,y), \textit{not } w(y)$$

Written in the syntax of Prolog:

```
e(1,2). e(2,3). e(3,4).
e(1,5). e(2,6). e(5,6).

p(X,X).
p(X,Y) :- e(X,Z), p(Z,Y).

nt(X) :- e(X,_).
t(X) :- \+ nt(X).

w(X) :- e(X,Y), \+ w(Y).
```

Using Prolog to answer queries:

```
| ?- p(1,3).

yes
| ?- p(3,1).

no
| ?- t(5).

no
| ?- t(6).

yes
| ?- w(1).

no
| ?- w(2).

yes
```

Queries with variables:

```
| ?- p(2,X).

X = 2 ? ;

X = 3 ? ;

X = 4 ? ;

X = 6 ? ;

no
| ?- w(X).

X = 3 ? ;

X = 2 ? ;

X = 5 ? ;

no
```

A Dark Secret of Prolog:

```
| ?- t(X).

no
```

This is called "floundering."

Semantically, rules in logic programming are different from propositional formulas.

Given the rule

$$p \leftarrow \textit{not}\ q$$

Prolog answers *yes* to p and *no* to q. For the rule

$$q \leftarrow \textit{not}\ p$$

this is the other way around.

The symbol *not* represents "negation as failure." One of the theories of negation as failure is based on the notion of an "answer set" (or "stable model").

Program	Answer sets
$p \leftarrow \textit{not } q$	$\{p\}$
$q \leftarrow \textit{not } p$	$\{q\}$
$p \leftarrow \textit{not } q$ $q \leftarrow \textit{not } p$	$\{p\}$, $\{q\}$
$p \leftarrow \textit{not } q$ $q \leftarrow \textit{not } p$ $r \leftarrow p$ $r \leftarrow q$	$\{p, r\}$, $\{q, r\}$

Prolog answers

yes to the queries that belong to all answer sets,

no to the queries that do not belong to any of them

(when it terminates and doesn't flounder).