Review of Propositional Logic

A *propositional signature* is a nonempty set of symbols called *atoms*. *Formulas* of a propositional signature σ are formed from the atoms of σ using the propositional connectives $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction) and $\supset$ (implication). For any formulas F and G, $F \equiv G$ stands for $(F \supset G) \wedge (G \supset F)$.

An *interpretation* of a propositional signature σ is a function from σ into the set of truth values $\{\mathsf{f}, \mathsf{t}\}$. The familiar recursive definition characterizes, for any formula F and any interpretation I, the truth value F^I that is *assigned* to F by I. For instance, if $\sigma = \{p, q, r\}$, the interpretation I is defined by the table

p	q	r
f	t	f

and F is

$$(p \vee q) \supset r \tag{1}$$

then $F^I = \mathsf{f}$.

If $F^I = \mathsf{t}$, we say that the interpretation I *satisfies* F, or is a *model* of F, and write $I \models F$. If a formula has a model, we say that it is *satisfiable*. This terminology applies also to sets of formulas: a *model* of a set Γ of formulas is an interpretation that satisfies all formulas in Γ, and a set of formulas is *satisfiable* if it has a model. We say that a set Γ of formulas *entails* a formula F, and write $\Gamma \models F$, if every model of Γ satisfies F.

The *satisfaction problem* for a set Γ of formulas is the problem of finding a model of Γ or establishing that Γ is unsatisfiable. The problem of checking whether F is entailed by Γ can be reduced to a satisfaction problem, because $\Gamma \models F$ iff the set $\Gamma \cup \{\neg F\}$ is unsatisfiable.

Two formulas, or sets of formulas, are *equivalent* to each other if they have the same models.

A *literal* is an atom or the negation of an atom. For any atom A, the literals A, $\neg A$ are *complementary* to each other. For any literal L, the literal complementary to L will be denoted by $\overline{L}$.

A *clause* is a finite set of literals. We will identify a clause C with the disjunction of its elements, and with a fixed unsatisfiable formula if $C = \emptyset$. A formula is in *conjunctive normal form (CNF)* if it has the form $C_1 \wedge \cdots \wedge C_n$

($n \geq 1$), where $C_1, \ldots, C_n$ are clauses. Using familiar transformations, we can convert any formula into an equivalent CNF. This process consists of three steps:

(i) eliminating $\supset$ in favor of $\vee$,

(ii) distributing $\neg$ over other connectives until it applies to atoms only,

(iii) distributing $\vee$ over $\wedge$ until the formula turns into a CNF.

Problem 1. (a) Convert formula (1) into its CNF. (b) The *size* of a formula is the number of occurrences of atoms in it. In the process of converting a formula into its CNF, can the size of the formula go up after step (i)? After step (ii)? After step (iii)?