CS313K: Logic, Sets and Functions
Fall 2010

Problem Set 8: Logical Notation

To express an assertion in logical notation means to write it using the propositional connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ and the quantifiers $\forall$ ("for all"), $\exists$ ("exists"). Here are examples of assertions about sets expressed both in set-theoretic notation and in logical notation:

English	Set-theoretic notation	Logical notation
A is a subset of B	$A \subseteq B$	$\forall x(x \in A \rightarrow x \in B)$
A is empty	$A = \emptyset$	$\neg\exists x(x \in A)$
A is non-empty	$A \neq \emptyset$	$\exists x(x \in A)$
A is the union of B and C	$A = B \cup C$	$\forall x(x \in A \leftrightarrow x \in B \vee x \in C)$
a is the only element of A	$A = \{a\}$	$\forall x(x \in A \leftrightarrow x = a)$

8.1. Write in logical notation:

(a) $A \subseteq B \cup C$,

(b) $A \subseteq \{a, b\}$,

(c) $A = \{a, b\}$.

8.2. Write in logical notation:

(a) A is disjoint from B,

(b) A is a proper subset of B,

(c) A is a singleton.

Here are some assertions about relations and functions expressed in logical notation:

English	Logical notation
R is reflexive	$\forall x(xRx)$
R is symmetric	$\forall xy(xRy \rightarrow yRx)$
a belongs to the range of f	$\exists x(f(x) = a)$
f is onto	$\forall y\exists x(f(x) = y)$

8.3. Express in logical notation:

(a) R is irreflexive,

(b) R is transitive,

(c) f is one-to-one,

(d) $f \circ g = g \circ f$.

8.4. If a binary relation R satisfies $\forall x \exists y(xRy)$ then it satisfies $\exists y \forall x(xRy)$. True or false?

8.5. Using variables for nonnegative integers, write each of the following conditions in logical notation:

(a) number n is a multiple of 3,

(b) number n can be represented as the sum of three complete squares;

(c) number n is composite.