A Machine-Checked Model of Safe Composition *

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Abstract

Programs of a software product line can be synthesized by composing *features* which implement some unit of program functionality. In most product lines, only some combination of features are meaningful; *feature models* express the high-level domain constraints that govern feature compatability. Product line developers also face the problem of *safe composition*– whether every product allowed by a feature model is type-safe when compiled and run. To study the problem of safe composition, we present Lightweight Feature Java (LFJ), an extension of Lightweight Java with support for features. We define a constraint-based type system for LFJ and prove its soundness using a full formalization of LFJ in Coq. In LFJ, soundness means that any composition of features that satisfies the typing constraints will generate a well-formed LJ program.

Categories and Subject Descriptors F.3.3 [*Studies of Program Constructs*]: Type structure

General Terms Design, Languages

Keywords Product lines, Type safety, Feature model

1. Introduction

Programs are typically developed over time by accumulation of new features. However, many programs break away from this linear view of feature development. In some cases a feature is removed when it is no longer useful. It is also common to create and maintain multiple versions of a product with different sets of features. The result is a *product line*, a family of related products.

The inclusion, exclusion, and composition of features in a product line is easier if each feature is defined as a modular unit. A given feature may involve configuration settings, user interface changes, persistent storage, and control logic. As such, features typically cut across the normal modularity boundaries of programs. Modularizing a program into features, or *feature modularity*, is quite difficult as a result.

AHEAD (3) is a simple yet powerful system for feature modularity based on Java. A feature in AHEAD is a collection of Java class definitions and *refinements*. A refinement is a modification to an existing class. When a feature is added to a product, the classes in the feature refine the corresponding classes in the base product. Figure 1 is a simple example of an AHEAD product line containing two features, Bank and Sync. The Bank feature in Figure 1a implements an elementary Account class with setBalance and update methods. Feature Sync in Figure 1b implements a synchronization feature so that accounts can be used in a multi-threaded environment. Sync includes a refinement of class Account that modifies update to use a lock, which is also introduced as a static variable.

```
feature Bank {
  class Account {
    int balance = 0;
    int getBalance() {
      return balance;
    }
    void update(int x) {
      int newBal = balance + x;
      balance = newBal;
    }
}}
```

(a) Bank Feature

```
feature Sync {
  refines class Account {
    static Lock lock
        = new Lock();
    refines void update(int x) {
      lock.lock();
      Super.update(x);
      lock.unlock();
    }
}}
```

(b) Synchronized Feature

```
class Account {
  int balance = 0;
  static Lock lock = new Lock();
  int getBalance() {
    return balance;
  }
  void setBalace(int x) {
    lock.lock();
    int newBal = balance + x;
    balance = newBal;
    lock.unlock();
  }
}
```

(c) A composed program: Sync•Bank

Figure 1: Buffer with synchronization feature

Method refinement in AHEAD is accomplished by inheritance; Super.update(x) indicates a call to (or substitution of) the prior definition of method update(x). By composing the refinement of Figure 1b with the class of Figure 1a, a class that is equivalent to that in Figure 1c is produced. The Bank feature can also be used on it own. While this example is simple, it exemplifies the feature-oriented approach to program synthesis: adding a feature means adding new members to existing classes and modifying existing methods. The following section presents a more complex example, and more details on the composition of features.

Not all features are compatible, and there may be complex dependencies among features. A *feature model* defines the legal combinations of features in a product line. A feature model can also

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```
feature A {
  class Base extends Class1 {
    int f1 = 0;
    int f1add (int x, int y) {
      f1 += y; return f1;
    }
  }
}

feature B {
  refines class Base extends Class2 {
    int f2 = 2;
    refines int f1add (int x, int y) {
      Super(); return x + f1;
    }
  }
}

feature C {
  refines class Base extends Class3 {
    int f2 = 0;
    int f1add (int x, int y) {
      f1 += x; return f1;
    }
  }
}

feature D {
  class T extends Class1 {
    Base b = new Base();
    int bmult (int x, int y, int z) {
      b.f2 = z;
      return b.f1add(x, y, z);
    }
  }
}
```

Figure 2: Definitions of four features A, B, C, and D

```
class Base extends Class2 {
  int f1 = 0;
  int f2 = 2;
  int f1add (int x, int y) {
    f1 += y; return x+ f1;
  }
}
```

Figure 3: Product B•A

```
class Base extends Class3 {
  int f1 = 0;
  int f2 = 0;
  int f1add (int x, int y) {
    f1 += x; return f1;
  }
}
```

Figure 4: Product C•A

```
class Base extends Class3 {
  int f1 = 0;
  int f2 = 0;
  int f1add (int x, int y) {
    f1 += x; return f1;
  }
}
class T extends Class1 {
  Base b = new Base();
  int bmult (int x, int y, int z) {
    b.f2 = z;
    return b.f1add(x, y, z);
  }
}
```

Figure 5: Product C•D•A

represent user-level domain constraints that define which combinations of features are useful.

In addition to domain constraints, there are low-level implementation constraints that must also be satisfied. For example, a feature can reference a class, variable, or method that is defined in another feature. *Safe composition* guarantees that a program synthesized from a composition of features does not refer to undefined classes, methods, and variables. While it is possible to check individual programs by building them, it is more desirable to ensure that all legal programs that can be synthesized are type safe. This requires a novel approach to type checking.

We formalize AHEAD using an object-oriented kernel language extended with features, called *Lightweight Feature Java* (LFJ). LFJ is based on Lightweight Java (10), a subset of Java that includes a formalization in Coq, using the Ott tool (9). A program in LFJ is a set of features containing classes and class refinements. Multiple products can be constructed by selecting and composing appropriate features according to a *product specification*– a composition of features.

We define a constraint-based type system for LFJ and prove its soundness. The type system and its safety are formalized in Coq. We then show how to relate the constraints produced by the type system to the constraints imposed by a feature model, using a reduction to propositional logic. This reduction mechanically verifies that a feature model will only allow safe compositions of features, guaranteeing that the resulting programs will be type safe.

Type checking feature modules would be simplified if the features modules were separated by well-defined interfaces. However, AHEAD does not specify explicit interfaces. Our constraint-based type system generates the necessary constraints between features, allowing a full product line to be checked for safety without generating each product individually.

2. The Problem of Safe Composition

Feature refinements can make significant changes to classes. Features can introduce new methods and fields to a class. Features refine existing methods by adding new statements before and after a method's body or by overwriting it altogether. Features can also alter the class hiearachy by changing the declared parent of a class.

The features in Figure 1 illustrate these modifications. Feature A introduces the `Base` class which extends `Class1` and which has the field `f1` and the method `f1add`. Feature B refines the `Base` class by updating its parent to `Class2`, introducing a new field, `f2`, and refining the `f1add` method by altering its return value. Feature C also refines the `Base` class by modifying the parent class and introducing `f2`, but it introduces a new definition for the `f1add` method.

An AHEAD model of a product line is an *algebra* that consists of a set of operations, where each operation implements a feature. We write $M = \{A, B, C, D\}$ to mean model M has the features (operations) A, B, C, and D. One possible realization of this set of features is given in 1. One or more features of a model are *constants* that build base programs through a set of class introductions:

- A a program containing only the `Base` class
- D a program containing only the `T` class

The remaining operations are *functions*, which are program refinements or extensions:

$B \bullet A$ adds feature B to program A
$C \bullet A$ adds feature C to program A

where $\bullet$ denotes function application and $B \bullet A$ is read as "feature B refines program A" or equivalently "feature B is added to program A". A refinement can extend the program with new definitions or modify existing definitions. The *design* of a product is a *product specification* specifying the composition of its features:

$P_1 = B \bullet A$	Program P_1 has features B and A	Fig. 1
$P_2 = C \bullet A$	Program P_2 has features C and A	Fig. 1
$P_3 = C \bullet A \bullet A$	Program P_3 has features D, C, A	Fig. 1

AHEAD is based on step-wise development: one begins with a simple program (e.g., constant feature A) and builds a more complex program by progressively adding features (e.g., adding features C and D to A in P_3).

The goal of safe composition is to ensure that the each product specification allowed by a feature model produces a well-typed program. The combinatorial nature of this development presents a number of problems to determining type safety of a product line. The members and methods of a class referenced in a feature might be introduced in several different features, as with Feature D which references the `f2` member of the `Base` class. In order for a composition including feature D to build a well-typed Java program, it must be composed with a feature that introduces this field, in this case either B or C. This requirement could also be met by a feature which adds the `f2` field to `Class1`, or which sets the parent of `Base` to a different class which includes `f2`. Subtyping also presents a problem: since the parent of a class can change through refinement, the final class hiearchy of a product depends on the included features. Each feature has a set of type-safety constraints which can be met by the combination of a number of different features, each with their own set of constraints.

3. Lightweight Feature Java

Lightweight Feature Java (LFJ) is a kernel language that captures the key concepts of the Jak language used in AHEAD. LFJ is based on Lightweight Java (LJ), a minimal imperative subset of Java (10). LJ supports classes, mutable fields, constructors, single inheritance, methods, dynamic method dispatch and method overloading. LJ does not include local variables, field hiding, interfaces, inner classes, or generics. This imperative kernel provides a minimal foundation for studying a type system for feature-oriented programming. LJ is more appropriate for this work than Featherweight Java (7) because of its treatment of constructors. When composing features, it is important to be able to add new member variables to a class during refinement. Featherweight Java requires all member variables to be initialized in a single constructor call. As a result, adding a new member variable causes all previous constructor calls to be invalid. Lightweight Java allows such refinements through its support of multiple constructors and more flexible initialization of member variables. In addition, Lightweight Java comes with a full formalization in Coq, which we extended to prove the soundness of LFJ.

The syntax that LFJ adds to that of LJ in order to support feature-oriented programming is given in Figure 6. A feature definition `FD` maps a feature name `F` to a list of class declarations `cld` and a list of class refinements `rcld`. A class refinement `rcld` includes a class name `dcl`, a set of LJ field and method introductions, $\overline{\texttt{fd}}$ and $\overline{\texttt{md}}$, a set of method refinements $\overline{\texttt{rmd}}$, and the name of the updated superclass `cl`. This differs from the current implementation of class refinements in AHEAD, which does not allow superclass refinement. A method refinement advises a method with signature `ms` with two lists of LJ statements $\overline{\texttt{s}}$ and an updated return value `y`. The *feature table* `FT` contains all the features available for a specific product line. A product specification `PS` is a sequence of distinct feature names. Any LJ program can be represented in LFJ as a feature definition containing the class definition of that program and no refinements.

Product specification
`PS` ::= $\overline{\texttt{F}}$
Feature declarations
`FD` ::= **feature** `F` $\{\overline{\texttt{cld}};\ \overline{\texttt{rcld}}\}$
Class refinement
`rcld` ::= **refines class** `dcl` **extending** `cl`$\{\overline{\texttt{fd}}; \overline{\texttt{md}}; \overline{\texttt{rmd}}\}$
Method refinement
`rmd` ::= **refines method** `ms` $\{$`rmb`$\}$
Method refinement
`rmb` ::= $\overline{\texttt{s}}$; **Super()**; $\overline{\texttt{s}}$; **return** `y`

Figure 6: Modified Syntax of Lightweight Feature Java.

3.1 Feature Composition

A LJ program is modelled as a partial function from class names to their definitions: $CT : dcl \rightarrow cld_{opt}$. In the operational semantics of LJ, this function is concretely realized as the path function specialized on the current program: $CT = \texttt{path}_P$. Features are themselves functions from LJ programs to LJ programs. Composition of a feature **feature** `FD` $\{\overline{\texttt{cld}}; \overline{\texttt{rcld}}\}$ with an LJ program P produces a new mapping, CT':

$$CT'(dcl) = \begin{cases} \texttt{path}_{\overline{cld}}(dcl) & dcl \in \overline{cld} \\ \overline{rcld} \bullet CT(dcl) & dcl \notin \overline{cld} \end{cases} \quad (1)$$

In the case that `FD` introduces a class named dcl, CT' returns this class, ignoring any previous declarations and refinements of that class. Otherwise, CT' finds the definition of dcl in the previous program using the original CT function and returns the resulting class definition, cld, refined by the $\overline{rcld}$. If a class refinement $rcld$ in $\overline{rcld}$ is named dcl, the $\bullet$ operator builds a refined class by first advising the methods of cld with the method refinements in $rcld$. The fields and methods introduced by $rcld$ are then added to this class and the parent of the resulting class is set to the class named in $rcld$. Refinement fails if cld lacks a method with a signature refined by $rcld$.

A product specification builds a LJ program by recursively composing the features it specifies in this manner, starting with the empty LJ program. Each LFJ feature table can construct a family of programs from its features in this manner; the classes included in each and their definitions is determined by the set of features which produced the program. The class hierarchy is also potentially different in each product: refinements can alter the parent of a class, and two mutually exclusive features can define the same class with a different parent.

3.2 Safe Composition

A product line is safe if every composition of product specifications in the line produces a well-formed LJ program. For any particular specification, this can be checked by composing the specification and then checking the safety of the resulting program using the standard LJ type system. This approach considers a potentially exponential number of programs and is burdened with the overhead of explicitly constructing each, making it a computationally expensive process. Instead, we propose a type system that can statically verify that all programs in a product line are type-safe without having to synthesize the entire family of programs.

The key difficulty with this approach is that features are typically program fragments which make use of class definitions made in other features; typing judgements which rely on these external objects can only be resolved during composition with other features. Every LJ construct has two categories of requirements which

must be met in order for it to be well-formed in the LJ type system. The first category consists of premises which only depend on the structure of the construct, e.g. the requirement that the parameters of a well-formed method be distinct. The remaining premises rely on accessing information from the surrounding program through the CT function, as with determining that the type of a variable y is a subtype of the type of variable x when assigning y to x in a method body. Intuitively, these premises define the structure of the programs in which LJ constructs are well-formed. In the standard LJ type system, the structure of the surrounding program is known. In feature-oriented programming, however, each feature can produce a number of programs, and the final makeup of the surrounding program is not static. Converting these kinds of premises into constraints provides an explicit interface for an LJ construct with any surrounding program. For a feature in a given feature table, this interface determines which features must be included with it in a product specification in order for its constructs to be well-formed in the final LJ program.

4. LFJ Type System

We present a constraint-based type system for LFJ based on a constraint-based type system we have developed for LJ. The constraint-based systems retain the premises that depend on the structure of the construct being typed and convert those that rely on external information into constraints. By using constraints, the external typing requirements for each feature are made explicit, separating derivation of these requirements from their satisfaction. This separation allows us to first generate an set of constraints for a feature and to then consider which product specifications have a combination of features which satisfy these constraints. The constraints used by our type system are given in Figure 7 and are divided into three categories. The two composition constraints guarantee successful composition of a feature F by requiring that refined classes and methods be introduced by a feature in a product line before F. The two uniqueness constraints ensure that member names are not overloaded within a class named dcl, a restriction in the LJ formalization. The structural constraints come from the standard LJ type system and constrain the set of members of a class and the class hierarchy in the final program. The subtype constraint is particularly important because the class hierarchy is malleable until composition time; if it were static, constraints that depend on subtyping class could be reduced to other constraints or eliminated entirely.

dcl **introduces** ms **before** F	**Composition Constraints**
dcl **introduced before** F	
$cl\ f$ **unique in** dcl	**Uniqueness Constraints**
$cl\ m\ (\overline{vd_k}^k)$ **unique in** dcl	
$cl_1 \prec cl_2$	
$cl_2 \prec \mathbf{ftype}(cl_1, f)$	
$\mathbf{ftype}(cl_1, f) \prec cl_2$	
$\overline{cl_k}^k \rightarrow cl \prec \mathbf{mtype}(cl, m)$	**Structural Constraints**
$\mathbf{defined}(cl)$	
$f \notin \mathbf{fields}(\mathbf{parent}(dcl))$	
$m \in \mathbf{mds}(\mathbf{parent}(dcl)) \rightarrow$ $\mathbf{mtype}(\mathbf{parent}(\mathbf{dcl}), m) =$ $\overline{cl_k}^k \rightarrow cl$	

Figure 7: Syntax of Lightweight Feature Java typing constraints.

The typing rules for LFJ are found in Figures 8-10 and rely on judgements of the form $\vdash J \mid \xi$, where J is a typing judgement from LFJ and ξ is a set of constraints, called a *signature*. The signature ξ provides an explicit interface which guarantees that J holds in any product specification that satisfies ξ. Typing rules for statements, methods, and classes are those from LJ augmented with signatures. Typing rules for class and method refinements in a feature F are similar to those for the objects they refine, but require that the refined class or method be introduced in a feature that comes before the F in a product specification. Method refinements do not have to check that the the names of their parameters are distinct and that their parameter types and return type are well-formed, since a method introduction which already performs these checks must preceed the refinement in order for it to be well-formed. The signature of a product specification PS is the union of the constraints on each of the features in PS.

$\boxed{\Gamma \vdash s \mid \mathcal{C}}$ Statement well-formed in context with constraints

$$\frac{\overline{\Gamma \vdash s_k \mid \mathcal{C}}^k}{\Gamma \vdash s_k \mid \bigcup_k \mathcal{C}_k} \quad \text{(WF-BLOCK)}$$

$$\frac{\Gamma(x) = \tau_1 \qquad \Gamma(var) = \tau_2}{\Gamma \vdash var = x; \mid \{\tau_1 \prec \tau_2\}} \quad \text{(WF-VAR-ASSIGN)}$$

$$\frac{\Gamma(x) = \tau_1 \qquad \Gamma(var) = \tau_2}{\Gamma \vdash var = x.f; \mid \{\mathbf{ftype}(\tau_1, f) \prec \tau_2\}} \quad \text{(WF-FIELD-READ)}$$

$$\frac{\Gamma(x) = \tau_1 \qquad \Gamma(y) = \tau_2}{\Gamma \vdash var = x.f; \mid \{\tau_2 \prec \mathbf{ftype}(\tau_1, f)\}} \quad \text{(WF-FIELD-WRITE)}$$

$$\frac{\begin{array}{c}\Gamma(x) = \tau_1 \qquad \Gamma(y) = \tau_2 \\ \Gamma \vdash s_1 \mid \mathcal{C}_1 \qquad \Gamma \vdash s_2 \mid \mathcal{C}_2 \\ \mathcal{C}_3 = \{\tau_2 \prec \tau_1 \vee \tau_1 \prec \tau_2\}\end{array}}{\Gamma \vdash \mathbf{if}\ x == y\ \mathbf{then}\ s_1\ \mathbf{else}\ s_2 \mid \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3} \quad \text{(WF-IF)}$$

$$\frac{\Gamma(var) = \tau_1 \qquad \mathbf{type}\ (cl) = \tau_2}{\Gamma \vdash var = \mathbf{new}\ cl() \mid \{\tau_2 \prec \tau_1\}} \quad \text{(WF-NEW)}$$

$$\frac{\begin{array}{c}\Gamma(x) = \tau \qquad \Gamma(var) = \pi \qquad \overline{\Gamma(y_k) = \pi_k}^k \\ \mathcal{C} = \{\overline{\pi_k}^k \rightarrow \pi \prec \mathbf{mtype}(\tau, meth)\}\end{array}}{\Gamma \vdash var = x.meth(\overline{y_k}^k)\ cl() \mid \mathcal{C}} \quad \text{(WF-MCALL)}$$

Figure 8: Typing Rules for LJ and LFJ statements.

Once the signature of a product specification PS is generated according to the rules in Figure 10, we evaluate whether it is satisfied by PS using the rules in Figure 11. Compositional constraints on a feature F are satisfied when a feature with the appropriate introductions preceeds F in PS. Uniqueness constraints are satisfied when no two distinct features in PS introduce the same member to a class dcl. In LFJ, satisfaction of structural constraints is evaluated as in LJ, replacing uses of the **path** with the CT function built by composition of the features in PS.

4.1 Soundness of the LFJ Type System

The soundness proof is based on successive refinements of the type systems of LJ and LFJ, reducing them to the proofs of progress and preservation of the original LJ type system given in (10). We first use our constraint-based type system for LJ, utilizing the structural constraints listed in Figure 7 and the corresponding judgements in Figure 11 to check constraint satisfaction. This type system is shown to be equivalent to the original LJ type system, in that a

$\boxed{\vdash_{\tau,\mathtt{F}} md \mid \mathcal{C}}$ Method well-formed in class with constraints

$$\frac{\begin{array}{c}\mathbf{distinct}(\overline{var_k}^{\,k}) \qquad \overline{\mathbf{type}(cl_k) = \tau_k}^{\,k} \qquad \mathbf{type}(cl) = \tau' \\ \Gamma = [\overline{var_k \mapsto \tau_k}^{\,k}][\mathbf{this} \mapsto \tau] \qquad \overline{\Gamma \vdash s_\ell \mid \mathcal{C}_\ell}^{\,\ell} \qquad \Gamma(y) = \tau''\end{array}}{\vdash_\tau cl\ meth\ (\overline{cl_k\ var_k}^{\,k})\ \{\overline{s_\ell}^{\,\ell}\ \mathbf{return}\ y;\} \mid \{\tau'' \prec \tau', \overline{\mathbf{defined}\ cl_k}^{\,k}\} \cup \bigcup_\ell \mathcal{C}_\ell} \quad \text{(WF-METHOD)}$$

$\boxed{\vdash cld \mid \mathcal{C}}$ Class well-formed with constraints

$$\frac{\begin{array}{c}\mathbf{distinct}(\overline{f_j}) \qquad \mathbf{distinct}(\overline{m_k}) \qquad dcl \neq cl \qquad \mathbf{type}(dcl) = \tau \qquad \overline{\vdash_\tau cl_k\ meth_k\ (\overline{cl_{\ell,k}\ var_{\ell,k}}^{\,\ell,k})\ mb_k \mid \mathcal{C}_k}^{\,k} \\ \xi = \bigcup_j \{f_j \notin \mathbf{fields}(\mathbf{parent}(dcl))\} \qquad \upsilon = \bigcup_j \{cl_j\ f_j\ \mathbf{unique\ in}\ dcl\} \qquad \upsilon' = \bigcup_k \{cl_k\ meth_k\ (\overline{cl_k\ var_k}^{\,k})\ \mathbf{unique\ in}\ dcl\} \\ \xi' = \bigcup_k \{meth_k \in \mathbf{mds}(\mathbf{parent}(dcl)) \rightarrow \mathbf{mtype}(\mathbf{type}(\mathbf{parent}(dcl)), meth_k) = \overline{cl_{\ell,k}}^{\,\ell,k} \rightarrow cl_k\}\end{array}}{\vdash \mathbf{class}\ dcl\ \mathbf{extends}\ cl\ \{\overline{cl_j\ f_j}^{\,j};\ \overline{cl_k\ meth_k\ (\overline{cl_{\ell,k}\ var_{\ell,k}}^{\,\ell,k})\ mb_k}^{\,k}\} \mid \bigcup_k \mathcal{C}_k \cup \{\mathbf{defined}\ cl, \overline{\mathbf{defined}\ cl_j}^{\,j}\} \cup \xi \cup \xi' \cup \upsilon \cup \upsilon'} \quad \text{(WF-CLASS)}$$

$\boxed{\vdash_{\tau,\mathtt{F}} rmd \mid \mathcal{C}}$ Refined method well-formed in class of feature with constraints

$$\frac{\begin{array}{c}\mathbf{type}(cl) = \tau' \qquad \Gamma = [\overline{var_k \mapsto \tau_k}^{\,k}][\mathbf{this} \mapsto \tau] \\ \Gamma(y) = \tau'' \qquad \overline{\Gamma \vdash s_j \mid \mathcal{C}_j}^{\,j} \qquad \overline{\Gamma \vdash s_\ell \mid \mathcal{C}_\ell}^{\,\ell} \\ \mathcal{C} = \{\tau'' \prec \tau', \tau\ \mathbf{introduces}\ cl\ meth\ (\overline{cl_k\ var_k}^{\,k})\ \mathbf{before}\ \mathtt{F}\} \cup \bigcup_j \mathcal{C}_j \cup \bigcup_\ell \mathcal{C}_\ell\end{array}}{\vdash_{\tau,\mathtt{F}} \mathbf{refines\ method}\ cl\ meth\ (\overline{cl_k\ var_k}^{\,k})\ \{\overline{s_j}^{\,j};\ \mathbf{Super}();\ \overline{s_\ell}^{\,\ell};\ \mathbf{return}\ y;\} \mid \mathcal{C}} \quad \text{(WF-REFINES-METHOD)}$$

$\boxed{\vdash_{\mathtt{F}} rcld \mid \mathcal{C}}$ Class refinement well-formed in feature with constraints

$$\frac{\begin{array}{c}dcl \neq cl \qquad \mathbf{type}(dcl) = \tau \qquad \overline{\vdash_\tau cl_k\ meth_k\ (\overline{cl_{\ell,k}\ var_{\ell,k}}^{\,\ell,k})\ mb_k \mid \mathcal{C}_k}^{\,k} \qquad \overline{\vdash_{\tau,\mathtt{F}} rmd_m \mid \mathcal{C}'_m}^{\,m} \\ \xi = \bigcup_j \{f_j \notin \mathbf{fields}(\mathbf{parent}(dcl))\} \qquad \upsilon = \bigcup_j \{cl_j\ f_j\ \mathbf{unique\ in}\ dcl\} \qquad \upsilon' = \bigcup_k \{cl_k\ meth_k\ (\overline{cl_k\ var_k}^{\,k})\ \mathbf{unique\ in}\ dcl\} \\ \xi' = \bigcup_k \{meth_k \in \mathbf{mds}(\mathbf{parent}(dcl)) \rightarrow \mathbf{mtype}(\mathbf{type}(\mathbf{parent}(dcl)), meth_k) = \overline{cl_{\ell,k}}^{\,\ell,k} \rightarrow cl_k\}\end{array}}{\begin{array}{c}\vdash_{\mathtt{F}} \mathbf{refines\ class}\ dcl\ \mathbf{extending}\ \mathtt{cl}\ \{\overline{cl_j\ f_j}^{\,j}; \overline{\vdash_\tau cl_k\ meth_k\ (\overline{cl_{\ell,k}\ var_{\ell,k}}^{\,\ell,k})\ mb_k}^{\,k};\ \overline{rmd_{\ell,k}}^{\,\ell,k}\} \mid \bigcup_k \mathcal{C}_k \cup \bigcup_m \mathcal{C'}_m \cup \\ \{\mathbf{defined}\ cl, \overline{\mathbf{defined}\ cl_j}^{\,j}, dcl\ \mathbf{introduced\ before}\ \mathtt{F},\} \cup \xi \cup \xi' \cup \upsilon \cup \upsilon'\end{array}} \quad \text{(WF-REFINES-CLASS)}$$

Figure 9: Typing Rules for LFJ method and class refinements.

program with unique class names and an acyclic class hierarchy satisfies its signature if and only if it is well-formed according to the original typing rules. We then show that if a single LFJ product specification is well-formed according to the constraint-based LFJ type system, it produces a LJ program that is also well-formed. We have formalized in the Coq proof assistant the syntax and semantics of LJ and LFJ presented in the previous section, as well as all of the soundness proofs that follow. For this reason, the following sections elide many of the bookkeeping details, instead presenting sketches of the major pieces of the proofs of soundness.

Theorem 4.1. *Let P be a LJ program with distinct class names and an acyclic, well-founded class hierarchy. Let $\mathcal{C}$ be the set of constraints generated by a class cld in P. cld is well-formed if and only if P satisfies $\mathcal{C}$: $P \vdash cld \leftrightarrow P \models \mathcal{C}$ where $\vdash cld \mid \mathcal{C}$.*

Proof. The two key pieces of this proof are: showing that satisfaction of each of the constraints guarantees that the corresponding judgement holds, and that there is a one-to-one correspondence between the constraints generated by the typing rules in Fig. 9 and the premises used in the declarative LJ type system. The former is straightfoward except for the subtyping constraint, which relies on the **path** function to check for satisfaction. We can prove their equivalance by induction on the derivation of the subtyping judgement in one direction and induction on the length of the path in the other. We can then show that the two type systems are equivalent by examination of the structure of P. At each level of the typing rules, the structural premises are identical and each of the external premises of the rules appears as a constraint in the signature. As a result of the previous argument, satisfaction of the signature guarantees that premises of the typing rules hold for each structure in P. Having shown the two type systems are equivalent, the proofs of progress and preservation for the constraint-based type system follow immediately.

□

Theorem 4.2. *[Soundness of the LFJ Type System] Let PS be a LFJ product specification and $\mathcal{C}$ be a set of constraints such that $\vdash PS \mid \mathcal{C}$. If $PS \models \mathcal{C}$ and **Object** is in the path of every class*

$\boxed{\vdash P \mid \mathcal{C}}$ Program well-formed with constraints

$$\frac{\overline{\vdash cld_k \mid \mathcal{C}_k}^{k} \qquad P = \overline{cld_k}^{k} \quad \textbf{distinct names}\ (P)}{\vdash P \mid \bigcup_k \mathcal{C}_k} \qquad \text{(WF-PROGRAM)}$$

$\boxed{\vdash \mathtt{F} \mid \mathcal{C}}$ Feature well-formed with constraints

$$\frac{\overline{\vdash cld_k \mid \mathcal{C}_k}^{k} \qquad \overline{\vdash_F rcld_\ell \mid \mathcal{C}_\ell}^{\ell}}{\vdash \textbf{feature}\ \mathtt{F}\ \{\overline{cld_k}^{k}\,\overline{rcld_\ell}^{\ell}\} \mid \bigcup_k \mathcal{C}_k \cup \bigcup_\ell \mathcal{C}_\ell} \qquad \text{(WF-FEATURE)}$$

$\boxed{\vdash \mathtt{PS} \mid \mathcal{C}}$ Product specification well-formed with constraints

$$\vdash \emptyset \mid \emptyset \qquad \text{(WF-SPECIFICATION-NIL)}$$

$$\frac{\vdash F \mid \mathcal{C} \qquad \vdash \overline{F_k}^{k} \mid \mathcal{C}'}{\vdash F, \overline{F_k}^{k} \mid \mathcal{C} \cup \mathcal{C}'} \qquad \text{(WF-SPECIFICATION)}$$

Figure 10: Typing Rules for LFJ Programs and Features.

introduced by a feature in PS, then the composition of the features in PS produces a valid, well-formed LJ program.

Proof. This proof can be decomposed into three key lemmas, corresponding to the three types of typing constraints:

(i) Composition of the features in PS produces a valid LJ program, P.

For each class or method refinement of a feature F in PS, a composition constraint is generated by the LFJ typing rules. Each of these are satisfied according to the definition in Fig. 11, allowing us to conclude that a feature with appropriate declarations appears before F in PS. Each of these declarations will appear in the program generated by the features preceeding F, allowing us to conclude that the composition of PS will succeed.

(ii) P is typable in the constraint-based LJ type system with constraints $\mathcal{C}'$.

In essence, we must show that the premises of the constraint-based LJ typing judgements hold. Our assumption that each class in PS is a descendent of **Object** ensures that P has an acyclic, well-founded class hierarchy. The premises for the LJ methods and statements are identical, leaving class typing rules for us to consider. The LJ typing rules require that the method and field names for a class be distinct, but these premises are removed by the LFJ typing rules, as the members of a class are not finalized until after composition. This requirement is instead enforced by the uniqueness constraints in Fig. 11, which are satisfied only when a method or field name is introduced by a single feature. Since $PS \models \mathcal{C}$, it follows that the premises of the LJ typing rules hold for P and that there exists some set of constraints $\mathcal{C}'$ such that $\vdash P \mid \mathcal{C}'$.

(iii) P satisfies the constraints in $\mathcal{C}'$ and is thus a well-formed LJ program.

We break this proof into two sublemmas:

(a) $\mathcal{C}' \subseteq \mathcal{C}$.

$$\frac{\mathbf{ftype}(P, \tau_1, f) = \tau_3 \qquad \tau_2 \in \mathbf{path}(P, \tau_3)}{P \models \tau_2 \prec \mathbf{ftype}(\tau_1, f)}$$

$$\frac{\mathbf{ftype}(P, \tau_1, f) = \tau_3 \qquad \tau_3 \in \mathbf{path}(P, \tau_2)}{P \models \mathbf{ftype}(\tau_1, f) \prec \tau_2}$$

$$\frac{\mathbf{mtype}(P, \tau, m) = \overline{\pi'_k}^{k} \to \pi' \qquad \pi \in \mathbf{path}(P, \pi') \quad \overline{\pi'_k \in \mathbf{path}(P, \pi_k)}^{k}}{P \models \overline{\pi_k}^{k} \to \pi \prec \mathbf{mtype}(\tau, m)}$$

$$\frac{\mathbf{type}(cl) \in \mathbf{path}(P, \mathbf{type}(cl))}{P \models \mathbf{defined}(cl)}$$

$$\frac{\tau_2 \in \mathbf{path}(P, \tau_1)}{P \models \tau_1 \prec \tau_2}$$

$$\frac{\mathbf{ftype}(P, \mathbf{parent}(dcl), f) = \bot}{P \models f \notin \mathbf{fields}(\mathbf{parent}(dcl))}$$

$$\frac{\mathbf{mtype}(P, \mathbf{parent}(dcl), m) = \bot \ \vee\ \mathbf{mtype}(P, \mathbf{parent}(dcl), m) = \overline{\pi_k}^{k} \to \pi}{P \models m \in \mathbf{mds}(\mathbf{parent}(dcl)) \to \mathbf{mtype}(\mathbf{parent}(dcl), m) = \overline{\pi_k}^{k} \to \pi}$$

$$\frac{FP = \overline{A_k}^{k} F \overline{B_\ell}^{\ell} H \overline{C_j}^{j} \qquad \tau.ms \in H \qquad \tau \notin \mathbf{introductions}(\overline{B_\ell}^{\ell})}{FP \models \tau\ \mathbf{introduces}\ ms\ \mathbf{before}\ F}$$

$$\frac{FP = \overline{A_k}^{k} F \overline{B_\ell}^{\ell} H \overline{C_j}^{j} \qquad \mathbf{dcl} \in H}{FP \models dcl\ \mathbf{introduced\ before}\ F}$$

$$\frac{\mathbf{type}(dcl) = \tau \qquad cl_1\ f \neq cl_2\ f \qquad \neg\exists A \in FP, \exists B \in FP, A \neq B \wedge \tau.cl_1\ f \in A \wedge \tau.cl_2\ f \in B}{FP \models cl\ f\ \mathbf{unique\ in}\ dcl}$$

$$\frac{\mathbf{type}(dcl) = \tau \qquad ms_1 = cl\ m\ (\overline{vd_k}^{k}) \qquad ms_2 = cl'\ m\ (\overline{vd'_k}^{k}) \qquad cl \neq cl' \vee \overline{vd_k}^{k} \neq \overline{vd'_k}^{k} \qquad \neg\exists A \in FP, \exists B \in FP, A \neq B \wedge \tau.ms_1 \in A \tau.ms_2 \in B}{FP \models cl\ m\ (\overline{vd_k}^{k})\ \mathbf{unique\ in}\ dcl}$$

Figure 11: Satisfaction of typing constraints.

The key observation for this proof is that every class, method, and statement in P originated from some feature in PS. The most interesting case is for the constraints generated by method bodies: a statement contained in a method body can come from either the initial introduction of that method or advice added by a method refinement. In either case, the statement was included in some feature in PS and thus generated some set of constraints in $\mathcal{C}$. Because method signatures are fixed across refinement, the context used in typing both kinds of statements is the same as that used for the method in the final composition. This does not entail that $\mathcal{C} = \mathcal{C}'$, however, as there could be some construct introduced in PS that is overwritten by an introduction in a subsequent feature.

(b) For any structural constraint $\mathcal{K}$, if $PS \models \mathcal{K}$, then $P \models \mathcal{K}$.

This reduces to showing that class declaration fetched by $CT(dcl)$ is the class with that identifier in P. This follows from tracing the definition of the CT function down to the final introduction of dcl in the product line. From here, we know that this class appears in the program synthesized from the product specification starting with this feature. Further refinements of this class are reflected in the $\bullet$ operator used recursively to build $CT(dcl)$; each refinement

succeeds by (i) above. Since the two functions are the same, the helper functions which call **path** in P (i.e. **ftype**, **mtype**) and those that use CT in PS return the same values. We can thus conclude that the satisfaction judgements for PS and P are equivalent.

All constraints in $\mathcal{C}'$ appear in $\mathcal{C}$, so $PS \models \mathcal{C}'$. By (b) above, it follows that $P \models \mathcal{C}'$. P must therefore be a well-formed LJ program by Theorem 4.1.

□

5. Feature Models

A *feature model* represents the dependencies and constraints between features that make up a product line. One common representation for feature models is *feature diagrams*. A feature diagram is a hierarchy of features where each node in the tree corresponds to a feature. Annotations on the tree represent constraints.

5.1 Feature Diagrams

Consider an elementary automotive product line that differentiates cars by transmission type (automatic or manual), engine type (electric or gasoline), and the option of cruise control. Figure 12 shows the feature diagram of this product line. A car has a body, engine, transmission, and optionally a cruise control. A transmission is either automatic or manual (choose one), and an engine is electric-powered, gasoline-powered, or both.

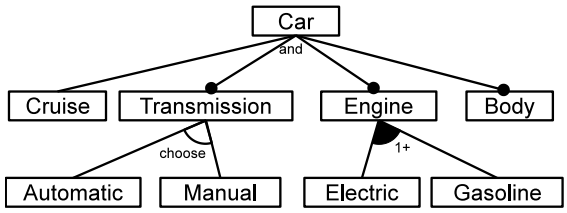

Figure 12: Feature diagram

Besides hierarchical relationships, feature models also allow cross- tree constraints, although these are more difficult to represent in a feature diagram. Such constraints are often inclusion or exclusion statements of the form: if feature F is included in a product, then features A and B must also be included (or excluded). A cross-tree constraint is that cruise control requires an automatic transmission.

Feature models are compact representations of propositional formulas (4). We exploit this representation in relating feature models to the constraint-based type system for LFJ.

5.2 Feature Models and AHEAD

A feature model determines the set of legal combinations of features in the AHEAD algebra. A given program specification can be tested for validity by checking if it satisfies the constraints expressed in a feature model. For example, the feature model *Auto* of the automotive product line is:

$$(\text{Cruise} \vee (\text{Body} \wedge (\text{Automatic} \vee \text{Manual}) \wedge (\text{Electric} \vee \text{Gasoline}))) \wedge (\text{Automatic} \rightarrow \neg\, \text{Manual}) \wedge (\text{Manual} \rightarrow \neg\, \text{Automatic})$$

where Body is the lone constant. Some products (i.e. legal expressions or sentences) of this product line are:

c1 = Automatic•Electric•Body

c2 = Cruise•Automatic•Electric•Gasoline•Body

$c1$ is a car with an electric engine and automatic transmission. $c2$ is a car with both electric and gasoline engines, automatic transmission, and cruise control.

5.3 Checking Feature Models

By the soundness of the LFJ type system, satisfaction of the signature of every feature in a product specification is sufficient to guarantee that its composition is a well-formed program. Given a feature table FT, the signature of a feature F provides an interface with that table. This interface can be translated into a propositional formula describing the minimal structural requirements that any product specification which includes F and is built from FT must satisfy in order for the constructs in F to be well-formed. The conjunction of these formulas builds a formula ϕ_{safe} which any product specification must satisfy in order to be well-formed. The safety of a feature model can then be statically verified by using a SAT solver to check that its propositional representation implies this minimal formula.

The propositional variables of ϕ_{safe} have three basic forms, described in Figure 13. Note that any satisfying assignment to the **In** and **Prec** variables which obeys the properties of the precedence and subtyping relations describes a unique product specification. These properties are enforced by the first seven constraints in Figure 15. WF_{Spec} is the conjunction of these constraints. The rules in Figure 14 generate a formula ϕ_F from the signature of a feature F. ϕ_{safe} is constructed by first building a clause for each feature F stating its inclusion implies ϕ_F: $\mathbf{In}_F \rightarrow \phi_F$. The constraints generated by STY_WF in Figure 15 are then added to this formula to ensure that the class hierarchy of a product specification is acyclic. These constraints require that each class included in a product specification be a subtype of `Object`. A satisfying assignment to ϕ_{safe} and WF_{Spec} corresponds to a product specification which satisfies the constraints on each of its features. The synthesis of those features will therefore produce a well-formed program by Theorem **??**. Since the representation of the feature model in propositional logic and the minimal well-formedness formula share the same variables, it is possible to check whether $FM \wedge WF_{Spec} \rightarrow \phi_{safe}$ is valid. If so, the guarantees of the feature model are strictly stronger than those of ϕ_{safe}. Thus, the composition of any product specification which satisfies the syntactic constraints of the feature model is well-formed.

$\mathbf{In}_A$: Feature A is included.
$\mathbf{Prec}_{A,B}$: Feature A precedes Feature B.
$\mathbf{Sty}_{\tau_1,\tau_2}$: τ_1 is a subtype of τ_2.

Figure 13: Description of propositional formulas.

A satisfying assignment to a formula in Figure 14 which obeys WF_{Spec} represents a product specification which satisfies the associated constraint. The **Final** and **FinalIn** abbreviations ensure that introductions and refinements in features appearing before the feature with the final introduction are ignored. The composition and uniqueness constraints have straightforward propositional representations that govern the valid orderings and makeup of product lines. The translations of the structural constraints rely on the mutability of the class hierarchy: since the class hierarchy of the product line is flexible, any class cl_1 that has a required field or method could ultimately satisfy a constraint on the members of another class, cl_2, if $cl_2 \prec cl_1$ in the final product specification.

6. Soundness of ϕ_{safe}

As stated previously, each assignment to the variables of $WF_{Spec} \rightarrow \phi_{safe}$ describes a unique product specification. We argue that a satisfying assignment to the propositional formula built from the signature of a feature F corresponds to a product specification which satisfies the constraints on F. It suffices to show that the formula generated by a constraint ensures its satisfaction by a product line.

$$
\begin{array}{ll}
\tau_1 \prec \tau_2 & \Rightarrow \mathbf{Sty}_{\tau_1,\tau_2} \\
\tau_2 \prec \mathbf{ftype}(\tau_1, f) & \Rightarrow \bigvee\{\mathbf{Sty}_{\tau_2,cl} \wedge \mathbf{Sty}_{\tau_1,\mathbf{type}(cld)} \wedge \mathbf{FinalIn}_{\mathbf{name}(cld),F} \mid \exists cld \in \mathbf{clds}(F), \exists cl, cl\ f \in \mathbf{fds}(cld)\}\vee \\
 & \quad \bigvee\{\mathbf{Sty}_{\tau_2,cl} \wedge \mathbf{Sty}_{\tau_1,\mathbf{type}(rcld)} \wedge \mathbf{Final}_{\mathbf{name}(rcld),F} \mid \exists rcld \in \mathbf{rclds}(F), \exists cl, cl\ f \in \mathbf{fds}(rcld)\} \\
\mathbf{ftype}(\tau_1, f) \prec \tau_2 & \Rightarrow \bigvee\{\mathbf{Sty}_{cl,\tau_2} \wedge \mathbf{Sty}_{\tau_1,\mathbf{type}(rcld)} \wedge \mathbf{FinalIn}_{\mathbf{name}(cld),F} \mid \exists cld \in \mathbf{clds}(F) \exists cl, cl\ f \in \mathbf{fds}(cld)\}\vee \\
 & \quad \bigvee\{\mathbf{Sty}_{cl,\tau_2} \wedge \mathbf{Sty}_{\tau_1,\mathbf{type}(rcld)} \wedge \mathbf{Final}_{\mathbf{name}(rcld),F} \mid \exists rcld \in \mathbf{rclds}(F), \exists cl, cl\ f \in \mathbf{fds}(rcld)\} \\
\overline{\pi_k}^k \to \pi \prec \mathbf{mtype}(\tau, m) & \Rightarrow \bigvee\{\mathbf{Sty}_{cl,\pi} \wedge \bigwedge_k \mathbf{Sty}_{\pi_k,cl_k} \wedge \mathbf{FinalIn}_{\mathbf{name}(cld),F} \mid \exists cld \in \mathbf{clds}(F), \\
 & \qquad \exists cl, \overline{cl_k}^k, \overline{v_k}^k cl\ m(\overline{cl_k v_k}^k) \in \mathbf{mds}(cld)\}\vee \\
 & \quad \bigvee\{\mathbf{Sty}_{cl,\pi} \wedge \bigwedge_k \mathbf{Sty}_{\pi_k,cl_k} \wedge \mathbf{Final}_{\mathbf{name}(rcld),F} \mid \exists rcld \in \mathbf{rclds}(F), \\
 & \qquad \exists cl, \overline{cl_k}^k, \overline{v_k}^k cl\ m(\overline{cl_k v_k}^k) \in \mathbf{mds}(rcld)\} \\
\mathbf{defined}(cl) & \Rightarrow \bigvee\{\mathbf{In}_F \mid \exists cld \in \mathbf{clds}(F), \mathbf{name}(cld) = cl\} \\
\tau\ \mathbf{introduces}\ ms\ \mathbf{before}\ F & \Rightarrow \bigvee\{\mathbf{In}_G \wedge \mathbf{Prec}_{G,F} \wedge \bigwedge\{\mathbf{In}_H \to \mathbf{Prec}_{F,H} \vee \mathbf{Prec}_{H,G} \mid \exists cld' \in \mathbf{clds}(H)), \mathbf{type}(\mathbf{name}(cld')) = \tau\} \\
 & \quad \mid \exists cld \in \mathbf{clds}(G), \mathbf{type}(\mathbf{name}(cld)) = \tau \wedge ms \in \mathbf{methods}(\mathbf{mds}(cld))\}\vee \\
 & \quad \bigvee\{\mathbf{In}_G \wedge \mathbf{Prec}_{G,F} \wedge \bigwedge\{\mathbf{In}_H \to \mathbf{Prec}_{F,H} \vee \mathbf{Prec}_{H,G} \mid \exists cld' \in \mathbf{clds}(H)), \mathbf{type}(\mathbf{name}(cld')) = \tau\} \\
 & \quad \mid \exists rcld \in \mathbf{rclds}(G), \mathbf{type}(\mathbf{name}(rcld)) = \tau \wedge ms \in \mathbf{methods}(\mathbf{mds}(rcld))\} \\
dcl\ \mathbf{introduced\ before}\ F & \Rightarrow \bigvee\{\mathbf{In}_G \wedge \mathbf{Prec}_{G,F} \mid \exists cld \in \mathbf{clds}(F), \mathbf{name}(cld) = dcl\} \\
cl\ f\ \mathbf{unique\ in}\ dcl & \Rightarrow \bigwedge\{\neg\mathbf{In}_F \mid \exists cld \in \mathbf{clds}(F), \mathbf{name}(cld) = dcl \wedge \exists cl', cl' f \in \mathbf{fds}(cld) \wedge cl \neq cl'\}\wedge \\
 & \quad \bigwedge\{\neg\mathbf{In}_F \mid \exists rcld \in \mathbf{rclds}(F), \mathbf{name}(rcld) = dcl \wedge \exists cl', cl' f \in \mathbf{fds}(rcld) \wedge cl \neq cl'\} \\
cl\ m\ (\overline{vd_k}^k)\ \mathbf{unique\ in}\ dcl & \Rightarrow \bigwedge\{\neg\mathbf{In}_F \mid \exists cld \in \mathbf{clds}(F), \mathbf{name}(cld) = dcl \wedge \exists cl', \overline{vd'_k}^k, cl' m\ (\overline{vd'_k}^k) \in \mathbf{mds}(cld) \wedge cl \neq cl' \vee \\
 & \qquad (\bigvee_k vd_k \neq vd'_k)\}\wedge \\
 & \quad \bigwedge\{\neg\mathbf{In}_F \mid \exists rcld \in \mathbf{rclds}(F), \mathbf{name}(rcld) = dcl \wedge \exists cl', \overline{vd'_k}^k, cl' m\ (\overline{vd'_k}^k) \in \mathbf{mds}(rcld) \wedge cl \neq cl' \vee \\
 & \qquad (\bigvee_k vd_k \neq vd'_k)\} \\
f \notin \mathbf{fields}(\mathbf{parent}(dcl)) & \Rightarrow \bigwedge\{\mathbf{In}_F \wedge \mathbf{FinalIn}_{\mathbf{name}(cld),F} \to \neg\mathbf{Sty}_{\mathbf{type}(dcl),cl} \mid \\
 & \quad \exists cld \in \mathbf{clds}(F), \mathbf{name}(cld) = cl \wedge dcl \neq cl \wedge \exists cl', cl' f \in \mathbf{fds}(cld)\}\wedge \\
 & \quad \bigwedge\{\mathbf{In}_F \wedge \mathbf{Final}_{\mathbf{name}(rcld),F} \to \neg\mathbf{Sty}_{\mathbf{type}(dcl),cl} \mid \\
 & \quad \exists rcld \in \mathbf{rclds}(F), \mathbf{name}(rcld) = cl \wedge dcl \neq cl \wedge \exists cl', cl' f \in \mathbf{fds}(rcld)\} \\
m \in \mathbf{mds}(\mathbf{parent}(dcl)) \to \mathbf{mtype}(\mathbf{parent}(dcl), m) = \overline{\pi_k}^k \to \pi & \Rightarrow \bigwedge\{\mathbf{In}_F \wedge \mathbf{FinalIn}_{\mathbf{name}(cld),F} \to \neg\mathbf{Sty}_{\mathbf{type}(dcl),cl} \mid \exists cld \in \mathbf{clds}(F), \mathbf{name}(cld) = cl \\
 & \quad \wedge dcl \neq cl \wedge m \in \mathbf{methods}(cld) \wedge \exists \tau', \overline{\tau_k}^k, \mathbf{mty}(cld, m) = \overline{\tau_k}^k \to \tau' \wedge \pi \neq \tau' \vee (\bigvee_k \pi_k \neq \tau_k)\} \\
 & \quad \bigwedge\{\mathbf{In}_F \wedge \mathbf{Final}_{\mathbf{name}(cld),F} \to \neg\mathbf{Sty}_{\mathbf{type}(dcl),cl} \mid \exists rcld \in \mathbf{rclds}(F), \mathbf{name}(rcld) = cl \\
 & \quad \wedge dcl \neq cl \wedge m \in \mathbf{methods}(cld) \wedge \exists \tau', \overline{\tau_k}^k, \mathbf{mty}(cld, m) = \overline{\tau_k}^k \to \tau' \wedge \pi \neq \tau' \vee (\bigvee_k \pi_k \neq \tau_k)\}
\end{array}
$$

Figure 14: Translation of constraints to propositional formulas, where $\mathbf{FinalIn}_{cl,F}$ stands for $\mathbf{In}_F \wedge \bigwedge\{\mathbf{In}_G \to \mathbf{Prec}_{G,F} \mid cl \in \mathbf{names}(\mathbf{clds}(G)) \wedge G \neq F\}$ and $\mathbf{Final}_{cl,F}$ stands for $\mathbf{In}_F \wedge \bigwedge\{\mathbf{In}_G \to \mathbf{Prec}_{G,F} \mid cl \in \mathbf{names}(\mathbf{clds}(G))\}$.

$$
\begin{array}{rl}
\textsc{Prec\_Total}: & \forall A, B, A \neq B, \mathbf{In}_A \wedge \mathbf{In}_B \leftrightarrow (\mathbf{Prec}_{A,B} \vee \mathbf{Prec}_{B,A}) \\
\textsc{Prec\_ASym}: & \forall A, B, \mathbf{Prec}_{A,B} \to \neg\mathbf{Prec}_{B,A} \\
\textsc{Prec\_Irrefl}: & \forall A, \neg\mathbf{Prec}_{A,A} \\
\textsc{Sty\_Refl}: & \forall \tau, \mathbf{Sty}_{\tau,\tau} \leftrightarrow \bigvee\{\mathbf{In}_F \mid cld \in \mathbf{clds}(F) \wedge \mathbf{type}(\mathbf{name}(cld)) = \tau\} \\
\textsc{Sty\_Obj}: & \mathbf{Sty}_{\texttt{Object},\texttt{Object}} \\
\textsc{Sty\_ASym}: & \forall \tau_1, \tau_2, \mathbf{Sty}_{\tau_1,\tau_2} \to \neg\mathbf{Sty}_{\tau_2,\tau_1} \\
\textsc{Sty\_Total}: & \forall \tau_1, \tau_2, \tau_3, \mathbf{Sty}_{\tau_1,\tau_2} \leftrightarrow ((\mathbf{Sty}_{\tau_1,\tau_3} \wedge \mathbf{Sty}_{\tau_3,\tau_2})\vee \\
 & \bigvee\{\mathbf{In}_F \mid \exists cld \in \mathbf{clds}(F), \mathbf{type}(\mathbf{name}(cld)) = \tau_1 \wedge \mathbf{type}(\mathbf{parent}(cld)) = \tau_2\}\wedge \\
 & \bigwedge\{\mathbf{In}_G \to \mathbf{Prec}_{G,F} \mid G \neq F \wedge \exists cld \in \mathbf{clds}(G), \mathbf{type}(\mathbf{name}(cld)) = \tau_1\}\wedge \\
 & \bigwedge\{\mathbf{In}_G \to \mathbf{Prec}_{G,F} \mid G \neq F \wedge \exists rcld \in \mathbf{rclds}(G), \mathbf{type}(\mathbf{name}(rcld)) = \tau_1\}\vee \\
 & \bigvee\{\mathbf{In}_F \mid \exists rcld \in \mathbf{rclds}(F), \mathbf{type}(\mathbf{name}(rcld)) = \tau_1 \wedge \mathbf{type}(\mathbf{parent}(cld)) = \tau_2 \wedge \\
 & \mathbf{name}(rcld) \notin \mathbf{names}(\mathbf{clds}(F))\}\wedge \\
 & \bigwedge\{\mathbf{In}_G \to \mathbf{Prec}_{G,F} \mid G \neq F \wedge \exists cld \in \mathbf{clds}(G), \mathbf{type}(\mathbf{name}(cld)) = \tau_1\}\wedge \\
 & \bigwedge\{\mathbf{In}_G \to \mathbf{Prec}_{G,F} \mid G \neq F \wedge \exists rcld \in \mathbf{rclds}(G), \mathbf{type}(\mathbf{name}(rcld)) = \tau_1\}\ ) \\
\textsc{Sty\_WF}: & \forall A, \forall c \in \mathbf{clds}(A), \mathbf{In}_A \to \mathbf{Sty}_{\mathbf{ty}(\mathbf{name}(c)),\texttt{Object}}
\end{array}
$$

Figure 15: Additional constraints.

Again, the interesting case is the **Sty** constraint used for the subtyping relation. In effect, the STY_TOTAL rule builds the transitive closure of the subtyping relation, starting with the parent/child relationships established in the program by the last definition of a class; this mirrors the construction of the subtyping relation used in the original LJ type system. By construction, if a satisfying assignment to ϕ_{safe} includes a feature F, ϕ_F must also be satisfied. It follows that any satisfying assignment to $WF_{Spec} \rightarrow \phi_{safe}$ represents a product specification which satisfies the signatures of each of the features in it, and thus produces a well-formed LJ program.

In effect, the feature model describes the assignments that represent legitimate specifications of a product line, guiding exploration of the family of programs it represents. In order for $FM \wedge WF_{Spec} \rightarrow \phi_{safe}$ to be valid, it must be the case that every member of the product family satisfies ϕ_{safe}, allowing us to conclude that FM only produces well-typed LJ programs.

7. Related Work

Our strategy of representing feature models as propositional formulas in order to verify their consistency was first proposed in (5). The authors checked the feature models against a set of user-provided feature dependences of the form $F \rightarrow A \vee B$ for features F, A, and B. This approach was adopted by Czarnecki and Pietroszek (6) to verify software product lines modelled as feature-based model templates. The product line is represented as an UML specification whose elements are tagged with boolean expressions representing their presence in an instantiation. These boolean expressions correspond to the inclusion of a feature in a product specification. These templates typically have a set of well-formedness constraints which each instantiation should satisfy. In the spirit of (5), these constraints are converted to a propositional formula; feature models are then checked against this formula to make sure that they do not allow ill-formed template instantiations.

The previous two approaches relied on user-provided constraints when validating feature models. The genesis for this work was a system developed by Batory et. al. (11) which generated the implementation constraints of a product line of Java programs by examining field, method, and class references in feature definitions. Analysis of existing product lines using this system detected previously unknown errors in the feature models of these product lines. This system relied on a set of rules for generating these constraints with no formal proof showing they were necessary and sufficient for well-formedness, which we have addressed here.

If features are thought of as modules, the feature model used to describe a product line is a *module interconnection language* (8). Normally, the typing requirements for a module would be explicitly listed by a "requires-and-provides interface" for each module. We infer this interface automatically by considering the minimum structural rules required of a feature 'module' by the type system. We verify that these interface constraints are satisfied by the implicit interface given to each module by the feature module. If composition is a linking process, we are guaranteeing that there will be no linking errors. The difference with normal linking is that we check all combinations of linkings allowed by the feature model.

The existing work on type-checking feature-oriented languages has focused on checking a single product specification, as opposed to checking an entire product line. Apel et al. (2) propose a type system for a model of feature-oriented programming based on Featherweight Java (7) and prove soundness for it and some further extensions of the model. *g*DEEP (1) is a language-independent calculus designed to capture the core ideas of feature refinement. The type system for *g*DEEP transfers information across feature boundaries and is combined with the type system for an underlying language to type feature compositions.

8. Conclusion

In feature-oriented programming, a feature model is a set of constraints describing how a set of features may be composed. This feature model is safe if it only allows the construction of well-formed programs. Simply iterating all the programs described by the feature model is computationally expensive. In order to statically verify that a feature model is safe, we have developed a calculus for studying feature composition in Java and a constraint-based type system for this language. The constraints generated by the typing rules provide an interface for each feature. We have shown that this set of constraints is sound with respect to LJ's type system. We construct SAT-instances for the interfaces of each feature which, when satisfied, ensure the product specification corresponding to the satisfying assignment will generate a well-typed LJ program. Using the feature model to guide the SAT solver, we are able to type check all the members of a product line, guaranteeing safe composition for all programs described by that feature model.

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