Improved NP-inapproximability for 2-variable linear equations

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2Lin

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$$x_{10} = -x_3$$

$$x_{61} = -x_{24}$$

...

$$x_{48} = -x_5$$

(x_i = -1,1)

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2Lin(**2**) ∈ 2Lin(**q**) ≈ **UniqueGames**

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Folklore wisdom: get 2Lin(**2**) right and 2Lin(**q**) will follow.

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seems we're close, right?

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Suppose val($\mathcal{I}$) = (1 - **ε**).

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Usually called "Min-2Lin(2)-Deletion".

Let me just call this **2Lin**.

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[Rao]: If (**ε**, O(f(**q**)***ε**$^{1/2}$))-approx is **NP**-hard for 2Lin(**q**), for f(**q**) = Ω(1), then **UG** is true.

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[Us]: (ε, $^{11}/_{8}*\varepsilon$)-approx **NP**-hard (**and more!**)

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Step 2: gadget reduce 3Lin to 2Lin **[TSSW]**

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(see also **[OW12]**)

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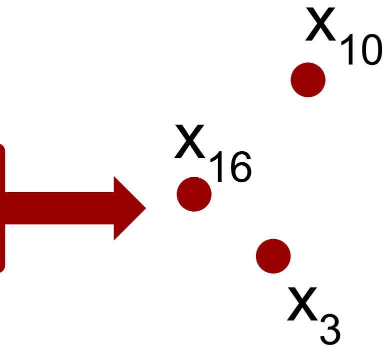

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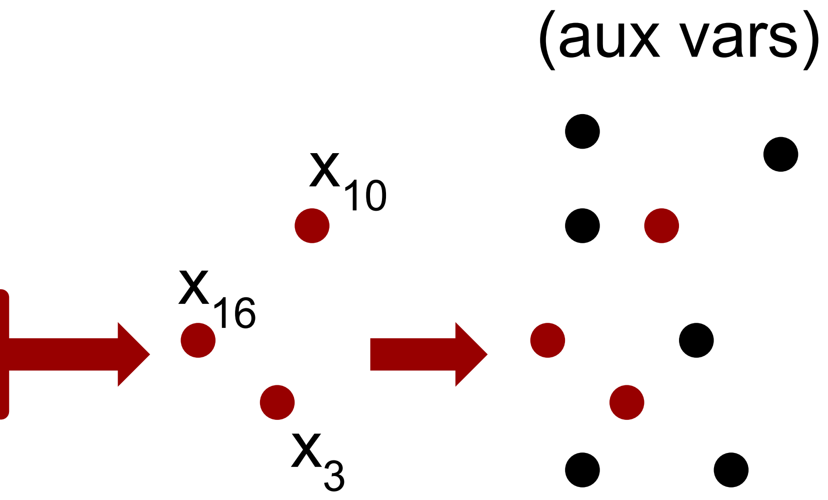

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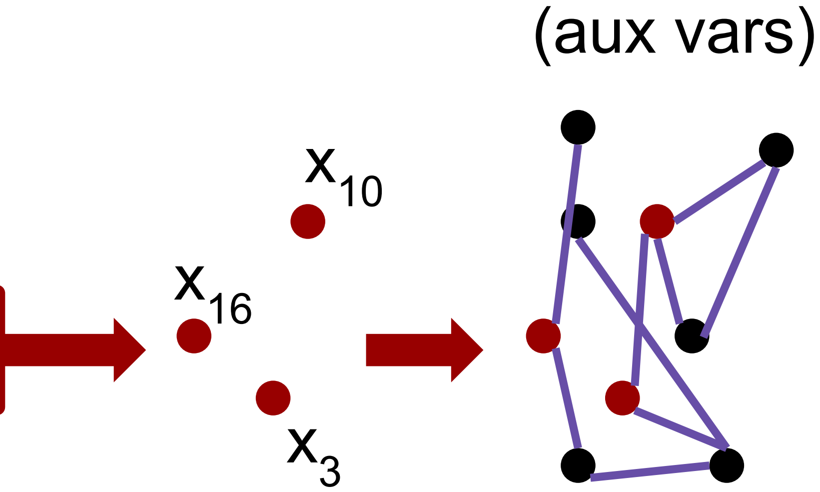

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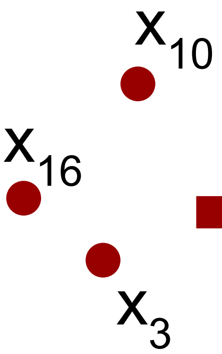

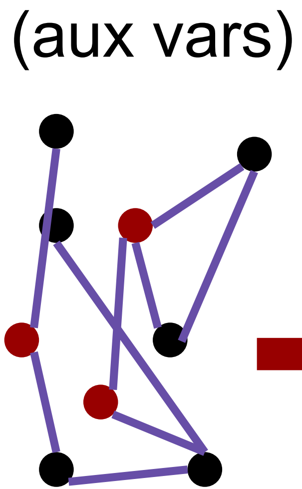

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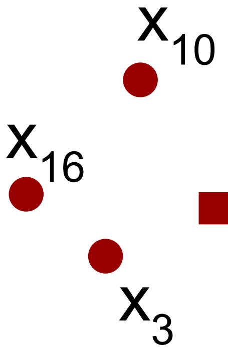
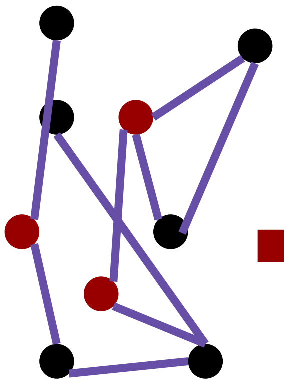


(aux vars)



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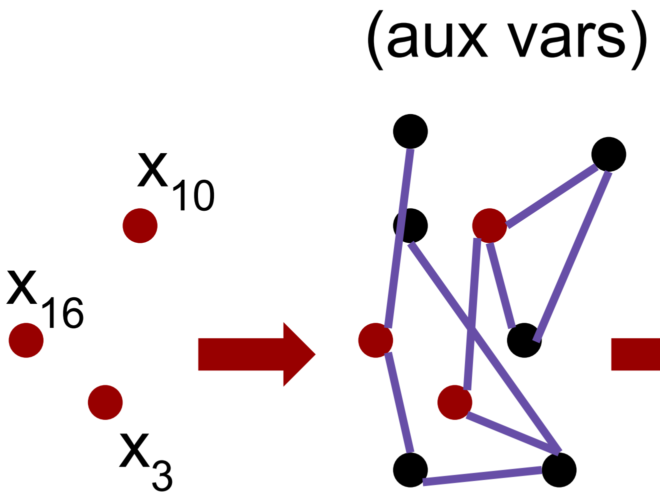

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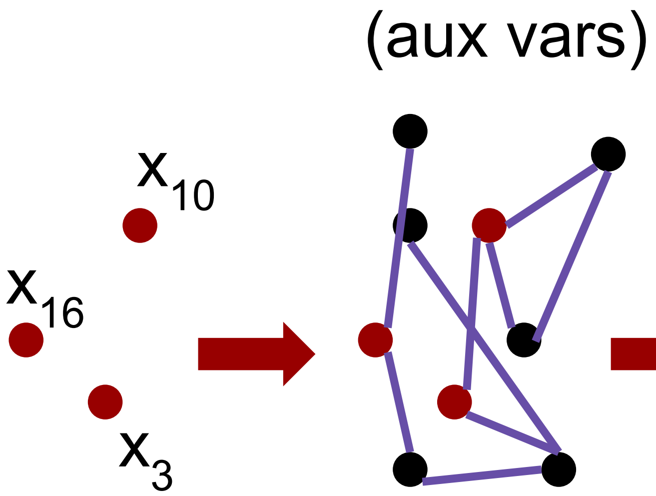

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- can certify optimality via **dual LP**.

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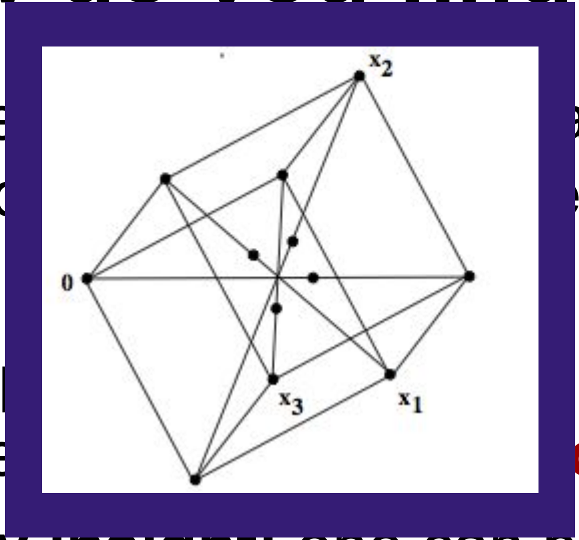

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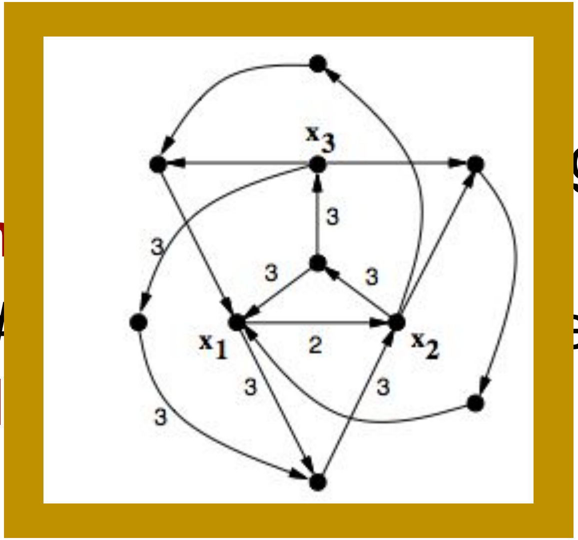

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Stronger **No** condition. Might help out the gadget.

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Downside: best 3Lin-to-2Lin gadget no better than in '97!

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We instantiate with **BPISP** = $\text{Had}_{\mathbf{k}}$, specifically **k** = 3.

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Contains many simultaneous 3Lin tests. Difficult to satisfy!

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$\mathbf{Had_3}(x_1, x_2, x_3, x_{1,2}, x_{1,3}, x_{2,3}, x_{1,2,3}) = 1$ iff

- x_i's allowed to be arbitrary
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- $x_a \cdot x_{\{1,2,3\}\setminus a} \cdot x_{1,2,3} = 1$ for all $a \in \{1,2,3\}$

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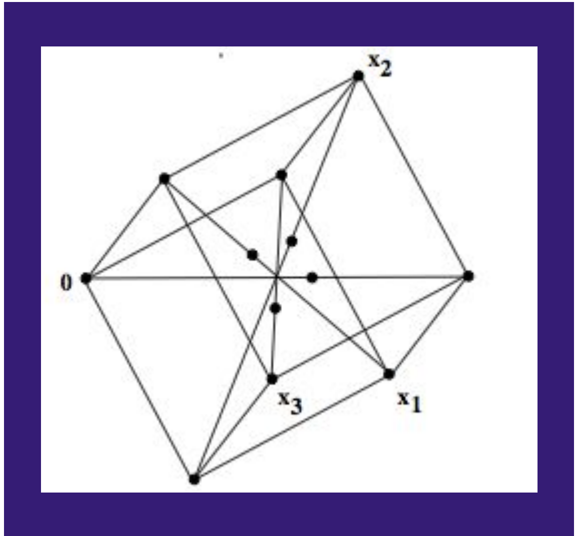
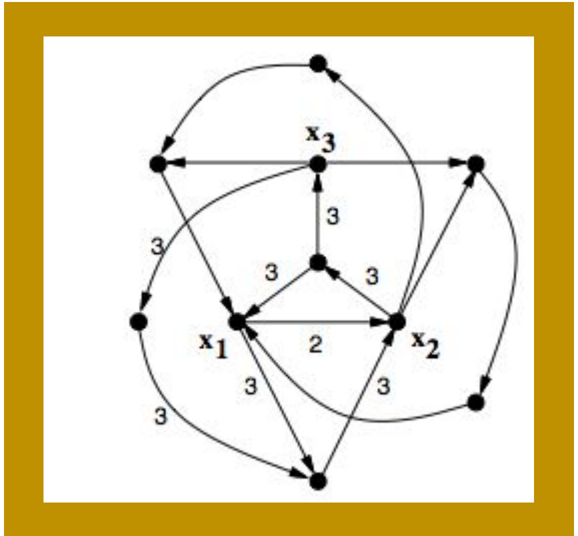
(so **no** pictures like these)





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but (spoiler alert) it all works out in the end.

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- can't beat (ε, $2.54 \cdot \varepsilon$)-hardness via a Chan gadget starting from ***any*** **BPISP** predicate

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We couldn't say anything about the normal hardness ratio.

Maybe you can?


Thanks!